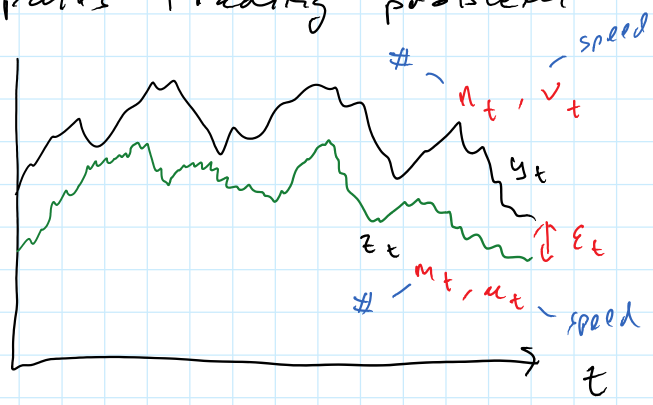
12 April 2013 14:13

1 pairs trading problem



$$\begin{cases} y_t - z_t = \varepsilon_t \\ d\varepsilon_t = \kappa(\theta - \varepsilon_t)\,dt + \eta\, dB_t \\ y_t = y_0 + \sigma W_t \end{cases}$$

mid-price

fundamental price

AM

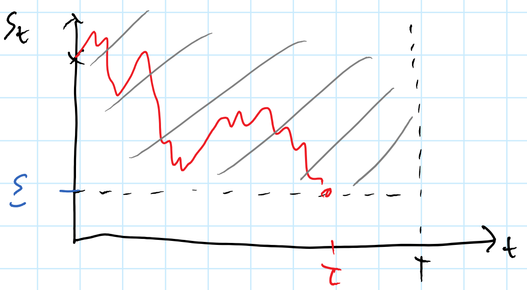
Ziwei

Bill

execution: $\hat{y}_t = y_t + a\,\nu_t$

$\hat{z}_t = z_t + b\,\mu_t$

2 Optimal liquidation with a price limiter



~~Adam~~

Zhen

Siying

$$S_t = S_0 + \sigma W_t$$

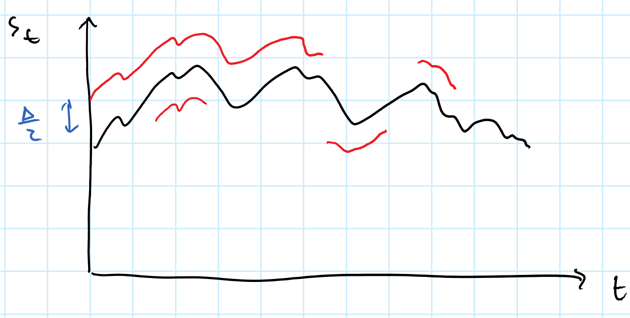
$$\hat{S}_t = S_t + a\,\nu_t$$

$$\sup_{\nu \in \mathcal{A}} \mathbb{E}\left[X_\tau + q_\tau (\overset{\underline{S}}{\hat{S}_\tau} - \alpha\, q_\tau) - \phi \int_0^\tau q_s^2\, ds \right]$$

a. $\tau = \inf\{u : S_u = \underline{S}\} \wedge \inf\{u : q_u = 0\} \wedge T$

b. $\tau = \inf\{u : \frac{x_u}{q_u} = \underline{S}\}$ . . . .

3 market making with LO & MO



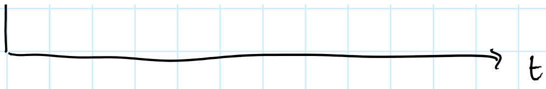
Erik, Bill, Robert, Siying, Adam

$$dX_t = \ell_t^+ (S_t + \tfrac{\Delta}{2})\, dN_t^+$$

$$- \ell_t^- (S_t - \tfrac{\Delta}{2})\, dN_t^-$$

$\tau_1^\pm, \tau_2^\pm, \ldots$ stopping times at which you place Buy/Sell MOs.

MO buy



you place Buy/Sell MOs.

MO buy

$$X_{\tau_k^+} = X_{\tau_k^+ -} - \left(S_{\tau_k^+} + \frac{\Delta}{2} \right)$$

$$X_{\tau_k^-} = X_{\tau_k^- -} + \left(S_{\tau_k^-} - \frac{\Delta}{2} \right)$$

4: MM with adverse selection using LOs.

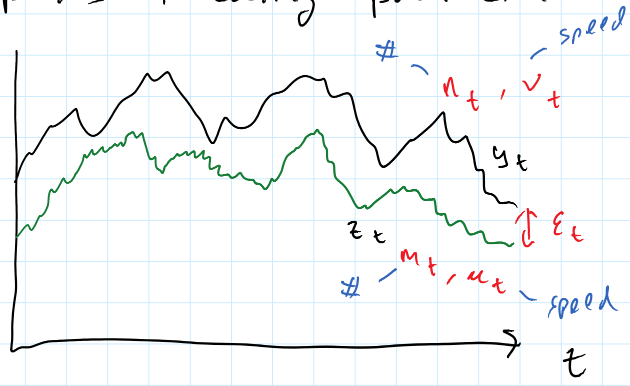
Adam, Zhen, Wei

$$dS_t = \varepsilon_t \, dt + \sigma \, dW_t$$

$$d\varepsilon_t = -\kappa \, \varepsilon_t \, dt \cancel{+ \eta \, dB_t} + \gamma \, dM_t^+ - \gamma \, dM_t^-$$

1

1 pairs trading problem



$$\begin{cases} y_t - z_t = \varepsilon_t \\ d\varepsilon_t = \kappa(\theta - \varepsilon_t)\,dt + \eta\,dB \\ y_t = y_0 + \sigma W_t \end{cases}$$

↪ mid-price

fundamental price

execution: $\hat{y}_t = y_t + a\nu_t$

$\hat{z}_t = z_t + b\mu_t$

$$H(t,x,y,\varepsilon,n,m) = \mathbb{E}\Big[X_T - \cancel{\phi \int_t^T (n_s^2 + m_s^2)\,ds}$$
$$+ n_T(y_T - \alpha n_T) + m_T(\underbrace{z_T}_{\hookrightarrow\, y_T + \varepsilon_T} - \beta m_T)\Big]$$

$$dX_t = -\nu_t \hat{y}_t\,dt - \mu_t \underbrace{\hat{z}_t}_{y_t + \varepsilon_t}\,dt$$

$$\partial_t H + \sup_{\nu,\mu} \mathcal{L}^{\nu,\mu} H = 0$$

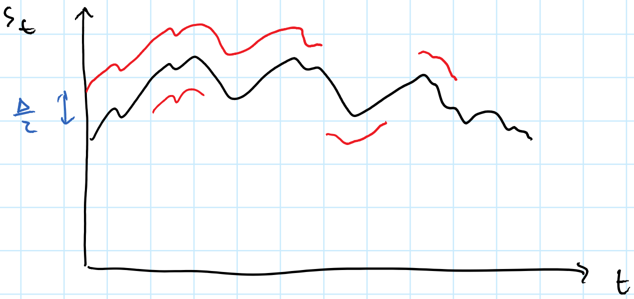
$$H(T,x,\dots) = x + n(y - \alpha n) + m(z - \beta m)$$

$$\mathcal{L}^{\nu,\mu} = -(\nu \underbrace{\hat{y}}_{y + a\nu} + \mu \underbrace{\hat{z}}_{z + b\mu})\partial_x + \tfrac{1}{2}\sigma^2 \partial_{yy}$$
$$+ \kappa(\theta - \varepsilon)\partial_\varepsilon + \tfrac{1}{2}\eta^2 \partial_{\varepsilon\varepsilon} + \nu \partial_n + \mu \partial_m$$

$$H = x + ny + mz + h(t,n,m,\varepsilon)$$

↪ (classical Almgren-Chriss: $q^2 h(t)$)

3 market making with LO & MO



$$dX_t = l_t^+ \left(S_t + \frac{\Delta}{2} \right) dN_t^+ \quad \lambda^+$$

$$- l_t^- \left(S_t - \frac{\Delta}{2} \right) dN_t^- \quad \lambda^-$$

$\tau_1^\pm, \tau_2^\pm, \ldots$ stopping times at which you place Buy/Sell MOs.

MO buy

$$X_{\tau_k^+} = X_{\tau_k^+ -} - \left(S_{\tau_k^+} + \frac{\Delta}{2} \right)$$

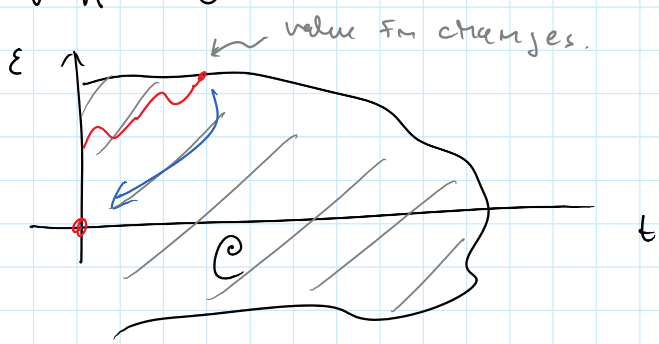
$$X_{\tau_k^-} = X_{\tau_k^- -} + \left(S_{\tau_k^-} - \frac{\Delta}{2} \right)$$

$$H(t, x, \varepsilon) = \sup_{\tau_1 \ldots} \mathbb{E}[X_\tau]$$

$$X_{\tau_k} = X_{\tau_k -} + \varepsilon_t \quad , \quad d\varepsilon_t = -\kappa \varepsilon_t dt + \sigma dW_t$$

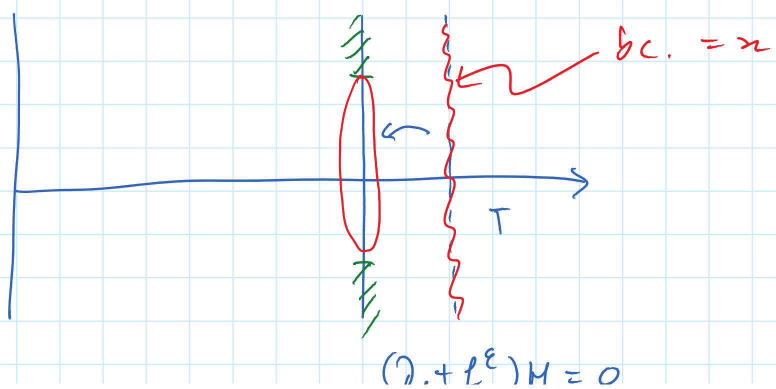
$$\varepsilon_\tau = 0 .$$

$$\cancel{\partial_t H + \sup \mathcal{L} H = 0}$$



$$(\partial_t + \mathcal{L}^\varepsilon) H = 0 \quad , \quad \varepsilon \in \mathcal{C}$$

$$H(t, x, \varepsilon) \to H(t, x + \varepsilon, 0)$$



$$(\partial_t + \mathcal{L}^{\varepsilon}) H = 0$$

$$\mathcal{C} = \{ (x, \varepsilon) : H(t, x, \varepsilon) > H(t, x+\varepsilon, 0) \}$$

$$H(t, x, S, q) = \sup_{\tau, \ell} \mathbb{E}\Big[X_T - \phi \int_t^T q_s^2 \, ds + q_T (S_T - \alpha q_T) \Big]$$

$\ell = (1,0)$ or $(0,0)$ if $q = \bar{q}$ $(+N)$

$(0,1)$ or $(0,0)$ if $q = \underline{q}$ $(-N)$

$$\max \Big\{ \sup_{\ell} (\partial_t + \mathcal{L}^{\ell}) H \, , \, \max\big(H(t, x - (S + \tfrac{\Delta}{2}), q+1, S) - H \, ; \, H(t, x + (S - \tfrac{\Delta}{2}), q-1, S) - H \big) \Big\}$$

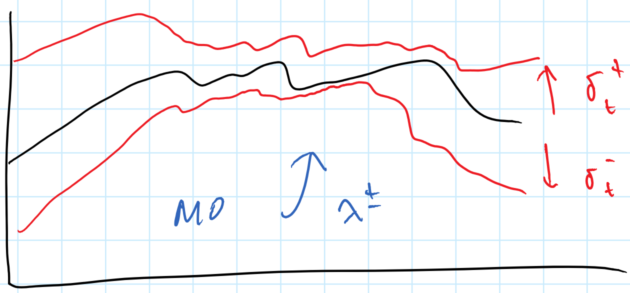
$$\mathcal{L}^{\ell} H = \tfrac{1}{2}\sigma^2 \partial_{SS} H + \lambda^+ [H(t, x + \ell^+ (S + \tfrac{\Delta}{2}), q - \ell^+, S) - H] + \lambda^- [H(t, x - \ell^- (S - \tfrac{\Delta}{2}), q + \ell^-, S) - H]$$

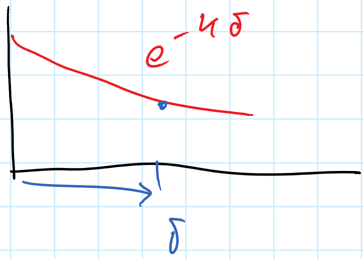
$$H = x + qS + h(t, q)$$

4: MM with adverse selection using LOs.

$$dS_t = \varepsilon_t \, dt + \sigma \, dW_t$$

$$d\varepsilon_t = -\kappa \varepsilon_t \, dt \cancel{+ \eta \, dB_t} + \gamma \, dM_t^+ - \gamma \, dM_t^-$$





$$H(t, x, S, \varepsilon, q) = \sup_{\delta^\pm} \mathbb{E}\left[X_T + q_T (S_T - \alpha q_T) - \phi \int_t^T q_s^2 \, ds \right]$$

$$dX_t = (S_t + \delta_t^+) \, dN_t^+ - (S_t - \delta_t^-) \, dN_t^-$$

$$q \in (\bar{q}, \underline{q})$$

$$\sup_\delta (\partial_t + \mathcal{L}^\delta) H - \phi q^2 = 0$$

$$H(t, x, S, \varepsilon, q) = x + qS + h(t, \varepsilon, q)$$

$$\mathcal{L}^\delta H = \left(\varepsilon \partial_S + \tfrac{1}{2}\sigma^2 \partial_{SS} - \kappa \varepsilon \partial_\varepsilon \right) H$$

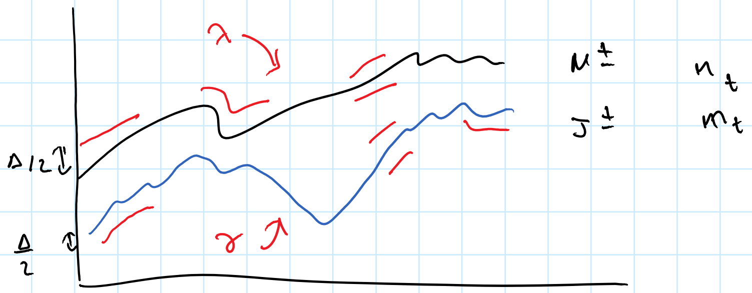
$$+ \lambda^\pm e^{-\kappa \delta^\pm} \left(H(t, x + (S + \delta^+), S, \varepsilon + \gamma, q - 1) - H \right)$$

$$+ \; (+ \leftrightarrow -)$$

$$+ \lambda^\pm (1 - e^{-\kappa \delta^\pm}) \left(H(t, x, S, \varepsilon + \gamma, q) - H \right)$$

$$y_t = y_0 + \sigma W_t$$

$$y_t - z_t = \varepsilon_t \quad , \qquad d\varepsilon_t = -\kappa\, \varepsilon_t\, dt + \eta\, dB_t$$



$$dX_t = \ell^+ (y_t + \tfrac{\Delta}{2})\, dN_t^+ - \ell^- (y_t - \tfrac{\Delta}{2})\, dN_t^-$$
$$+ g^+ (z_t + \tfrac{\Delta}{2})\, dJ_t^+ - g^- (z_t - \tfrac{\Delta}{2})\, dJ_t^-$$

$$H(t, x, y, \varepsilon, n, m) = \sup_{\ell, g} \mathbb{E}\Big[X_T + n_T (y_T - \alpha n_T) + m_T (z_T - \beta m_T) - \varphi \int_t^T (m_s^2 + n_s^2)\, ds \Big]$$

$n \in [\underline{n}, \bar{n}]$
$m \in [\underline{m}, \bar{m}]$

$$\partial_t H + \sup_{\ell, g} \mathcal{L}^{\ell, g} H = 0$$

$$\mathcal{L}^{\ell, g} H = \left(\tfrac{1}{2}\sigma^2 \partial_{yy} + -\kappa \varepsilon \partial_\varepsilon + \tfrac{1}{2}\eta^2 \partial_{\varepsilon\varepsilon} \right) H$$
$$+ \lambda^+ \left[H(t, x + \ell^+ (y + \tfrac{\Delta}{2}), y, \varepsilon, n - \ell^+, m) - H \right]$$
$$+ \lambda^- \left[H(t, x - \ell^- (y - \tfrac{\Delta}{2}), y, \varepsilon, n + \ell^-, m) - H \right]$$
$$+ \gamma^+ \left[H(t, x + g^+ (z + \tfrac{\Delta}{2}), y, \varepsilon, n, m - g^+) - H \right]$$
$$+ \gamma^- \left[H(t, x - g^- (z - \tfrac{\Delta}{2}), y, \varepsilon, n, m + g^-) - H \right]$$

$$H = x + ny + mz + h(t, \varepsilon, n, m)$$