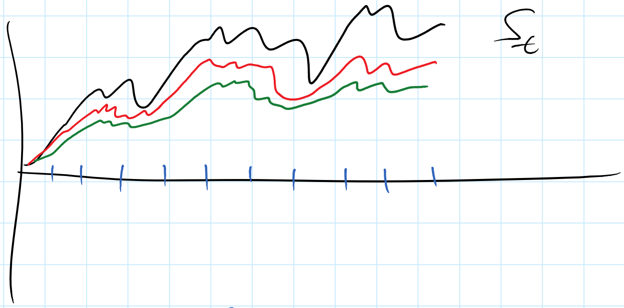
20 March 2013 14:35

$\phi \longrightarrow \phi\,\sigma^2$



$$dS_t = \sigma\, dW_t - b\, v_t\, dt$$

$$\hat{S}_t = S_t - a v_t$$

$$dX_t = \hat{S}_t\, v_t\, dt \quad ?$$

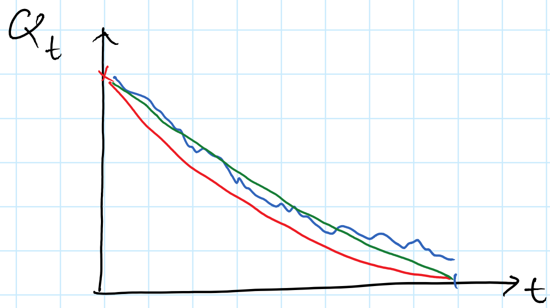
$$V_T = X_T + Q_T\,(S_T - \cancel{\alpha}\, Q_T)$$

$\alpha_0 = 0.05$

$S_0 = 100, \quad \sigma = 1, \quad T = 1$

several ϕ, + α

Optimal Liquidation with LOs



$\ell_t = \nu_t \Delta t$

M.O. arrive at rate λ

$N_t = \#$ of M.O. by t

$\mathbb{E}[\Delta N_t] = \lambda \Delta t$

$v =$ vol of M.O. $\exp(c)$

$\mathbb{E}[v] = \frac{1}{c}$

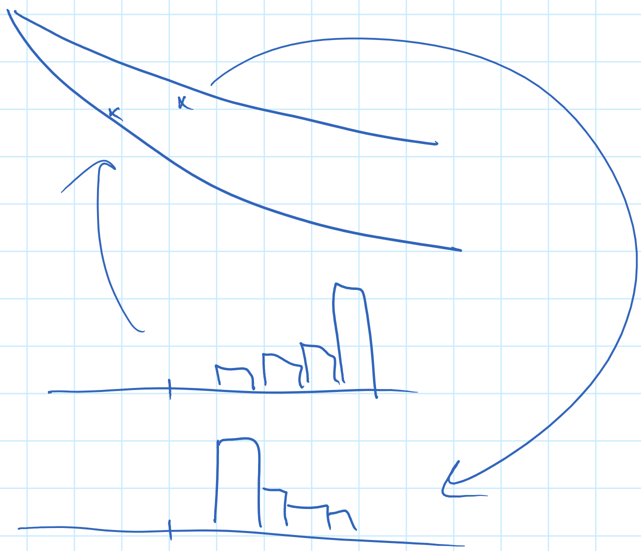
$V_t = \sum_{n=1}^{N_t} v_n \Rightarrow \mathbb{E}[\Delta V_t] = \frac{\lambda \Delta t}{c}$

if $\frac{\lambda \Delta t}{c} \gg \nu_t \Delta t$

then can just post ℓ_t

if $\nu_t \Delta t \sim \frac{\lambda \Delta t}{c}$

$\nu_t \Delta t \gg \frac{\lambda \Delta t}{c}$

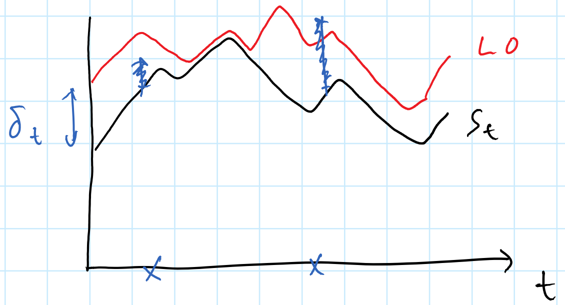


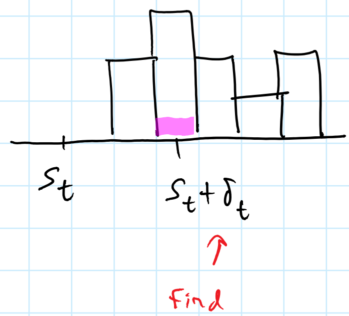
Apr 20
16 15
19

Apr 10, 12

Apr 28 →

Post optimally LOs to liquidate assets



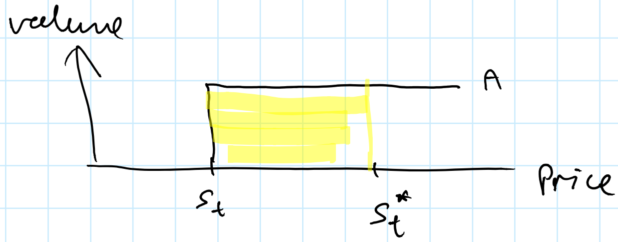


M.O. M_t counting process (Poisson process λ)

$$V_t = \sum_{n=1}^{M_t} v_n, \qquad v_1, \dots \text{ iid } \exp(C)$$

$$\mathbb{P}(\text{filled} \mid \text{M.O.}) = \mathbb{P}(S_t^* > S_t + \delta_t)$$

max price M.O. pays



LOB is flat

$$A(S_t^* - S_t) = v$$

$$\Rightarrow \quad S_t^* = S_t + \frac{v}{A}$$

$$\mathbb{P}(\text{filled} \mid \text{M.O.}) = \mathbb{P}(v > A\,\delta_t) = e^{-\left(\frac{A}{C}\right)\delta_t}$$

κ

N_t^δ counting process for your filled L.O.s

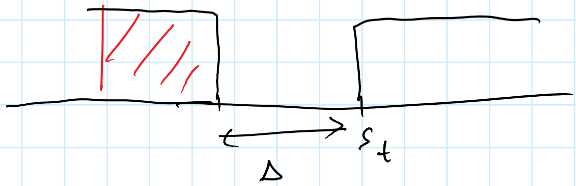
rate of arrival of M_t is $\lambda e^{-\kappa \delta_t}$

M_t is a doubly stochastic Poisson process.

$$dQ_t^\delta = -dN_t^\delta \quad \text{or} \quad Q_t^\delta = Q_0 - N_t^\delta \quad \text{inventory}$$

$$dS_t = \sigma\, dW_t \quad \text{best ask (best offer)}$$

$$dX_t^\delta = (S_t + \delta_t)\, dN_t^\delta$$

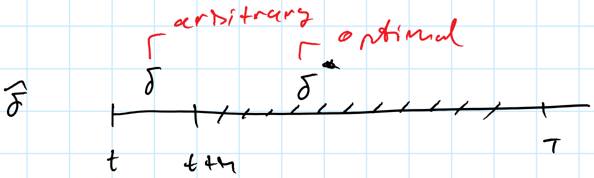


$$\mathcal{X}_T = X_T + Q_T(S_T - \Delta - \alpha Q_T)$$

$\tau = \inf\{t : q_t = 0\} \wedge T$

$$H(t,x,S,q) = \sup_{\delta \in \mathcal{A}} \mathbb{E}_{t,x,S,q}\Big[\mathcal{X}_{\underset{\hookrightarrow \tau}{T}} - \phi\sigma^2 \int_t^{T \to \tau} q_s^2 \, ds\Big]$$

$X_t = x, \quad S_t = S, \quad Q_t = q$



$$H^{\hat\delta} = \mathbb{E}_{t,x,s,q}\Big[\mathcal{X}_T^{\hat\delta} - \phi\sigma^2 \int_t^T (q_s^{\hat\delta})^2 \, ds\Big]$$

$$= \mathbb{E}_{t,x,s,q}\Big[\mathbb{E}\Big[\mathcal{X}_T^{\hat\delta} - \phi\sigma^2 \int_t^T (q_s^{\hat\delta})^2 \, ds \,\Big|\, \mathcal{F}_{t+h}\Big]\Big]$$

$$\hookrightarrow \underline{\int_t^{t+h} (q_s^{\hat\delta})^2 \, ds} + \int_{t+h}^T (q_s^{\hat\delta})^2 \, ds$$

$$= \mathbb{E}_{t,x,s,q}\Big[\underbrace{\mathbb{E}\Big[\mathcal{X}_T^{\hat\delta} - \phi\sigma^2 \int_{t+h}^T (q_s^{\hat\delta})^2 \, ds \,\Big|\, \mathcal{F}_{t+h}\Big]}_{\hookrightarrow H(t+h,\, X^\delta_{t+h},\, S_{t+h},\, q^\delta_{t+h})} - \phi\sigma^2 \int_t^{t+h} (q_s^{\hat\delta \to \delta})^2 \, ds\Big]$$

$$\int_t^{t+h}$$

$$dH = (\partial_t + \tfrac{1}{2}\sigma^2 \partial_{SS}) H \, dt + \sigma \partial_S H \, dW_t$$

$$+ \Big[H(t, X_{t^-} + (S_t + \delta_t), S_t, q_{t^-} - 1) - H(t, X_{t^-}, S_t, q_{t^-})\Big] \, dN_t$$

$\lambda_u = \lambda e^{-\kappa \delta_u}$

$$\Rightarrow H^{\hat\delta} = \mathbb{E}_{t,x,s,q}\Big[\int_t^{t+h} \Big[(\partial_t + \tfrac{1}{2}\sigma^2 \partial_{SS}) H_u + \lambda \big(H(u, X_{u^-} + (S_u + \delta_u), S_u, q_{u^-} - 1) - H_u\big)\Big] du - \phi\sigma^2 \int_t^{t+h} (q_u^\delta)^2 \, du + H_t\Big]$$

$\sup \delta \quad , \quad \frac{1}{\kappa} \quad , \quad \lim \kappa \downarrow 0$

$$\boxed{\begin{aligned} 0 = \sup_{\delta} \Big\{ & (\partial_t + \tfrac{1}{2}\sigma^2 \partial_{ss}) H(t,x,S,q) \\ & + \lambda e^{-\kappa\delta} \left[H(t, x+(S+\delta), S, q-1) - H(t,x,S,q) \right] \\ & - \phi \sigma^2 q^2 \Big\} \\ & (X_{t^-} = x, \; S_t = S, \; q_{t^-} = q) \\ & H(T,x,S,q) = x + q(S - \Delta - \alpha q) \end{aligned}}$$

if $T \longrightarrow \tau = \inf(t : q_t = 0) \wedge T$ then

we add the condition $H(t,x,S,0) = x$

so for $q = 1, \ldots$

$$(\partial_t + \tfrac{1}{2}\sigma^2 \partial_{ss}) H + \sup_{\delta} \left[\lambda e^{-\kappa\delta} \left(x + S + \delta - H(t,x,S,1) \right) \right] = \phi \sigma^2 1^2$$

$$\partial_\delta \left(e^{-\kappa\delta} (\delta + A) \right)$$

$$= -\kappa e^{-\kappa\delta} (\delta + A) + e^{-\kappa\delta}(1) = 0$$

$$\Rightarrow \boxed{\delta = \frac{1}{\kappa} - A}$$

$$A = S + x - H(t,x,S,1)$$

$$H(T,x,S,q) = x + q(S - \Delta - \alpha q)$$

$$H(t,x,S,q) = x + q(S - \Delta) + h(t,q)$$

$$\longrightarrow \quad H(t, x + (S+\delta), S, q-1) - H(t,x,S,q)$$

$$= x + S + \delta + (q-1)(S - \Delta) + h(t, q-1)$$

$$- \left[x + q(S - \Delta) + h(t,q) \right]$$

$$= \delta - (q-1)\Delta + q\Delta + h(t,q-1) - h(t,q)$$

$$= \delta + \Delta + h(t,q-1) - h(t,q)$$

$$h(t,0) = 0$$

$$M(T, q) = -\alpha q^2$$

20 March 2013 15:46

M_t – Poisson processes

$$\mathbb{E}_t[M_{t+\Delta t} - M_t] = \lambda \Delta t$$

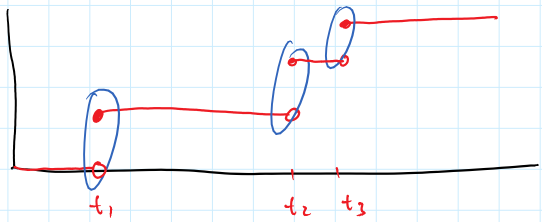
$$\mathbb{E}_t[\, M_{t+\Delta t} - M_t - \lambda(t - (t+\Delta t))] = 0$$

$\hat{M}_t = M_t - \lambda t$ is a mtg.

$$\int_0^t a_s \, dN_s = \sum_{n=1}^{N_t} a_{t_n}$$

↳ deterministic

↳ jump times of N_t.



$$g(N_{t+\Delta t}) - g(N_{t-}) \quad , \quad g_t = g(N_t)$$

$$\underset{g_{t+\Delta t}}{\shortparallel} \qquad \qquad \underset{g_t}{\shortparallel}$$

$$= \big(g(N_{t-}+1) - g(N_{t-})\big) \, \Delta N_t$$

$$\boxed{dg_t = \big(g(N_{t-}+1) - g(N_{t-})\big) \, dN_t}$$

$$\boxed{dg(t, N_{t^-}) = \partial_t g(N_{t-}) \, dt + \big(g(N_{t-}+1) - g(N_{t^-})\big) \, dN_t}$$

$$g(t, N_t, W_t)$$

$$\boxed{dg = (\partial_t + \mathcal{L}) g \, dt + \partial_w g \, dW_t + \big(g_{t}(N_{t-}+1, W_t) - g_{t}(N_{t-}, W_t)\big) dN_t}$$

N_t, N_{t-} – doesn't matter!

$$\int_0^t a_{s-}\, dl_s = \int_0^t a_s\, dl_s$$

suppose g is a mtg .... $\quad 0 = \mathbb{E}_t[dg]$

$$\Rightarrow \quad 0 = (\partial_t + \mathcal{L})\, g\, dt + \left(g(t, N_{t-}+1, W_t) - g(t, N_{t-}, W_t)\right) \underbrace{\mathbb{E}_t[dN_t]}_{\lambda\, dt}$$

Partial Integro-Differential Eqn (PIDE)

$$\boxed{(\partial_t + \mathcal{L})\, g(t,n,w) + \lambda\left(g(t,n+1,w) - g(t,n,w)\right) = 0}$$

$$\hookrightarrow \int \left(g(t, n+m, w) - g(t,n,w)\right) D_m(1)$$

doubly stochastic Poisson process. $\quad \lambda_t \quad$ then $\quad \mathbb{E}_t[dN_t] = \lambda_t\, dt$

$\hookrightarrow \lambda e^{-\kappa \delta_t}$