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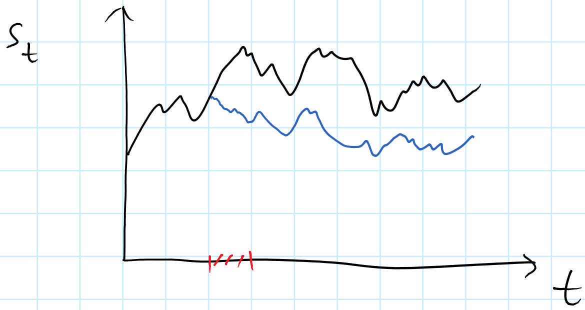
Optimal Liquidation



mid-price

$$S_t = S_0 + \sigma W_t - \int_0^t h(v_s)\, ds$$

$$dS_t = \sigma\, dW_t - h(v_t)\, dt$$

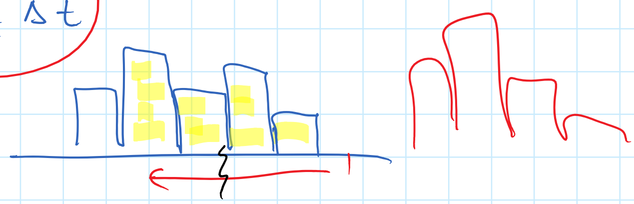
permanent impact function

$h(v_t) = \zeta v_t$

$(\zeta > 0)$

v_t - rate of trading

$v_t \Delta t$

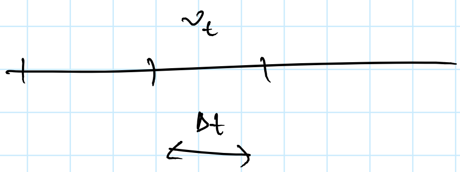


temporary impact

$$\hat{S}_t = S_t - g(v_t) \qquad \text{(execution price)}$$

$\hookrightarrow a v_t - \varepsilon$ transaction cost

(essentially same as assuming LOB is flat)



$\left.\begin{array}{l} \# \; v_t \Delta t \\ \hat{S}_t \text{ price executed} \end{array}\right\} \Rightarrow \Delta X = \hat{S}_t v_t \Delta t$

$$dX_t = \hat{S}_t v_t\, dt$$

$$X_t = X_0 + \int_0^t \hat{S}_u v_u\, du \qquad \text{wealth process}$$

$H = \sup_{v \in \mathcal{A}} \mathbb{E}[X_T]$ is the most basic question to pose.

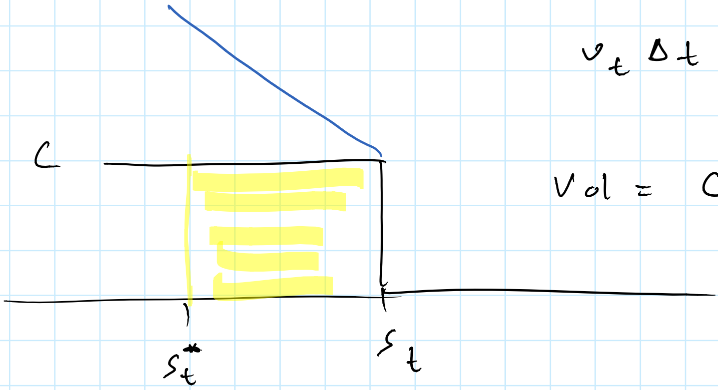
$\hookrightarrow$ admissible strategies $\mathcal{F}_t$ - predictable this does not guarantee that $Q_T = 0$.

$$Q_t = Q_0 - \int_0^t \nu_u \, du$$

(unless we put that into the set $\mathcal{A}$)

$$H^\phi = \sup_{\nu \in \mathcal{A}} \mathbb{E}\left[X_T - \phi \, Q_T^2 \right]$$

$\lim_{\phi \uparrow +\infty} H^\phi$ will have optimal strategy s.t. $Q_T = 0$.



$v_t \Delta t$

$$Vol = c(S_t - S_t^*) = v_t \Delta t$$

$$\Rightarrow S_t^* = S_t - \frac{v_t \Delta t}{c}$$

$$\hat{S}_t = \frac{S_t + S_t^*}{2}$$

$$= S_t - \frac{v_t \Delta t}{2c}$$

$$= S_t - \alpha v_t$$

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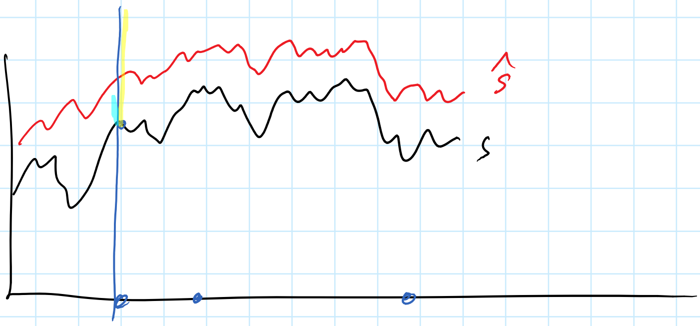
Optimal liquidation with LOs.

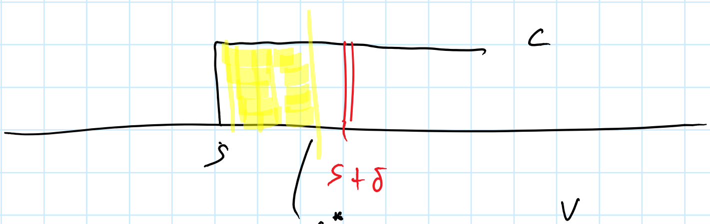
$S_t = S_0 + \sigma W_t$ mid-price

$\hat{S}_t = S_t + \delta_t$

our spread

M_t - counting process for M.O. assume Poisson (λ)





$S^* = S + \frac{V}{c}$

assume exponential (A)

$$\mathbb{P}(\text{Fill} \mid \text{M.O.}) = \mathbb{P}\left(\frac{V}{c} > \delta_t\right) = \mathbb{P}(V > c\,\delta_t)$$

$$= e^{-\underbrace{Ac}_{\kappa}\delta_t}$$

so $\lambda e^{-\kappa\delta_t}$ is the rate of arrival of your filled LOs.

N_t is the associated counting process.

(it is a dbly stochastic Poisson process)

$$dX_t = (S_t \underset{+\varepsilon}{+} \delta_t)\, dN_t$$

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$$dX_t^u = a u_t\,dt + \sigma u_t\,dW_t, \qquad X_0 = x$$

$$H(x) = \inf_{u \in \mathcal{A}} \mathbb{E}\left[(X_T^u)^2 \right]$$

$$H^u(t,x) = \mathbb{E}\left[(X_T^u)^2 \mid X_t^u = x \right], \quad H(t,x) = \inf_{u \in \mathcal{A}} H^u(t,x)$$

$$\begin{cases} \inf_u (\partial_t + \mathcal{L}^u) H(t,x) + 0 = 0 \\ H(T,x) = x^2 \end{cases}$$

$$\mathcal{L}^u = u a\,\partial_x + \tfrac{1}{2}\sigma^2 u^2\,\partial_{xx}$$

$$\partial_t H + a\,\partial_x H + \inf_u \left(\tfrac{1}{2}\sigma^2 u^2\,\partial_{xx} H \right) = 0$$

if $\partial_{xx} H > 0$, then $u^* = 0$

$$\Rightarrow \begin{cases} \partial_t H + a\,\partial_x H = 0 \\ H(T,x) = x^2 \end{cases}$$

$$H = \alpha(t) + \beta(t)\,x + x^2$$

$$\alpha(T) = \beta(T) = 0$$

$$\dot\alpha + \dot\beta x + a(\beta + 2x) = 0$$

$$\underbrace{(\dot\alpha + a\beta)}_{0} + \underbrace{(\dot\beta + 2a)}_{0}\,x = 0$$

$$\Rightarrow \beta = 2a(T-t)$$

$$\overset{0}{\overset{\shortparallel}{\alpha(T)}} - \alpha(t) + a\int_t^T \beta(s)\,ds = 0$$

$$\alpha(t) = a^2 (T-t)^2$$

$$\partial_t M + \inf_u \left(a u \, \partial_x M + \tfrac{1}{2} \sigma^2 u^2 \, \partial_{xx} M \right) = 0$$

$$a \, \partial_x M + \sigma^2 u \, \partial_{xx} M = 0$$

$$\Rightarrow \quad u = - \frac{a}{\sigma^2} \, \frac{\partial_x M}{\partial_{xx} M}$$

$$\tfrac{1}{2} \sigma^2 \partial_{xx} M \left(u^2 + \frac{a \, \partial_x M}{\tfrac{1}{2}\sigma^2 \partial_{xx} M} u \right)$$

$$= \tfrac{1}{2} \sigma^2 \partial_{xx} M \left[\left(u + \frac{a}{\sigma^2} \frac{\partial_x M}{\partial_{xx} M} \right)^2 - \frac{a^2}{\sigma^4} \left(\frac{\partial_x M}{\partial_{xx} M} \right)^2 \right]$$

$$= - \frac{a^2}{\sigma^2} \frac{(\partial_x M)^2}{\partial_{xx} M}$$

$$\begin{cases} \partial_t M - \dfrac{a^2}{\sigma^2} \dfrac{(\partial_x M)^2}{\partial_{xx} M} = 0 \\ M(T, x) = x^2 \end{cases}$$

$$\mathbb{E}\left[G(X) + \int F(X)\, ds \right]$$

$$d Q_t = - v_t \, dt$$

$$\mathcal{L} = - v \, \partial_q$$

$$\sup_{v} \left(\partial_t + \mathcal{L}^{v} \right) + F = 0$$

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$$v^* = \frac{q}{T-t}$$

$$dq = -v_t\,dt = -\frac{q_t}{T-t}\,dt$$

$$\ln(q_t / q_0) = + \ln(T-s)\Big|_{s=0}^{t}$$

$$= \ln\left(\frac{T-t}{T}\right)$$

$$\Rightarrow \quad q_t = q_0\left(\frac{T-t}{T}\right) = q_0\left(1 - \frac{t}{T}\right)$$