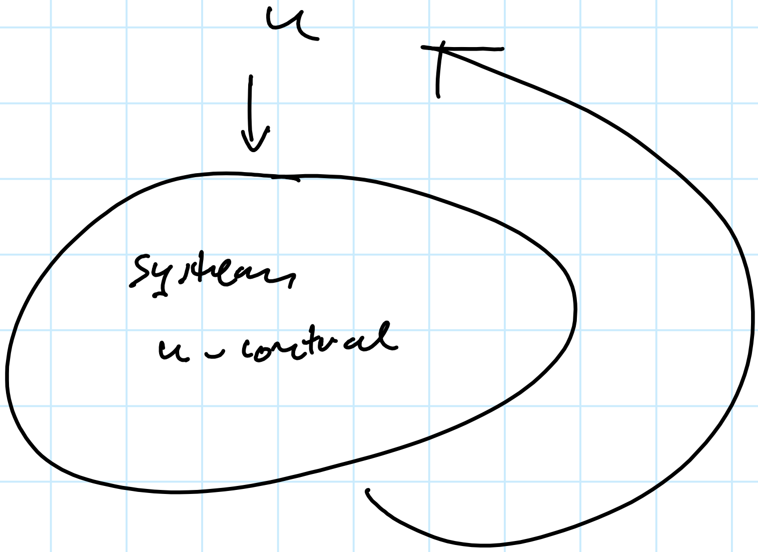
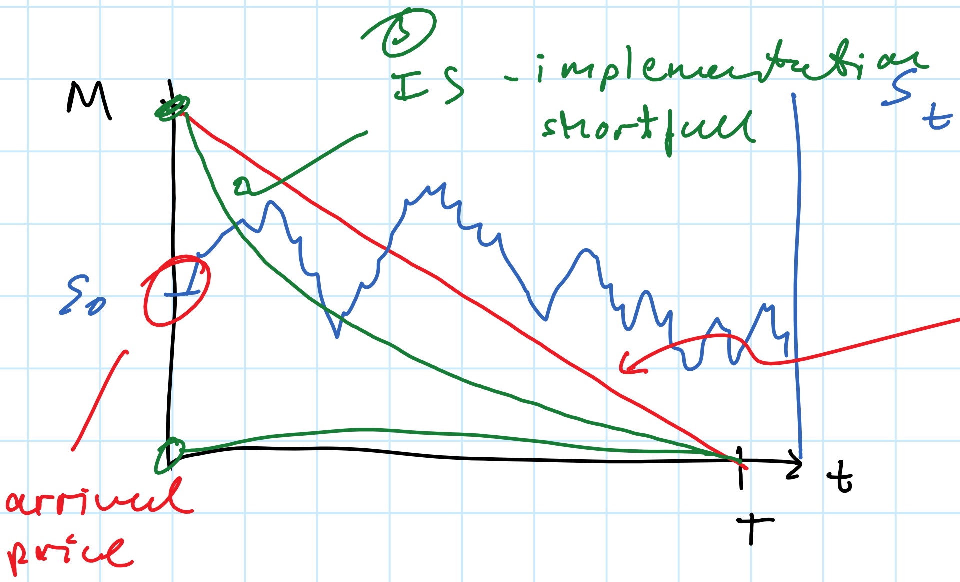


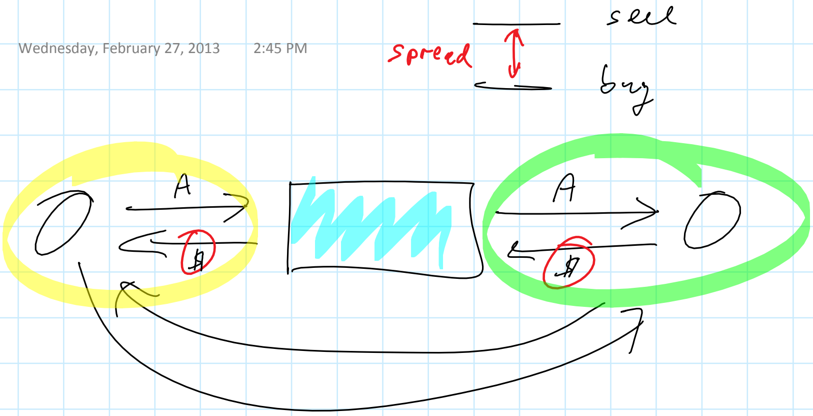
$$\sup_{u \in \mathcal{A}} \mathbb{E}[X_T^u]$$

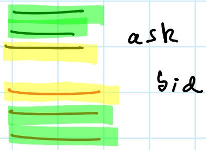


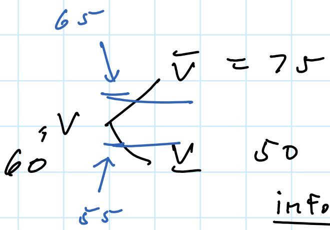
TWAP ①
time weighted average price.

$S_0 - \lambda$ $\qquad \lambda \gg 1$

②
VWAP - volume weighted average price

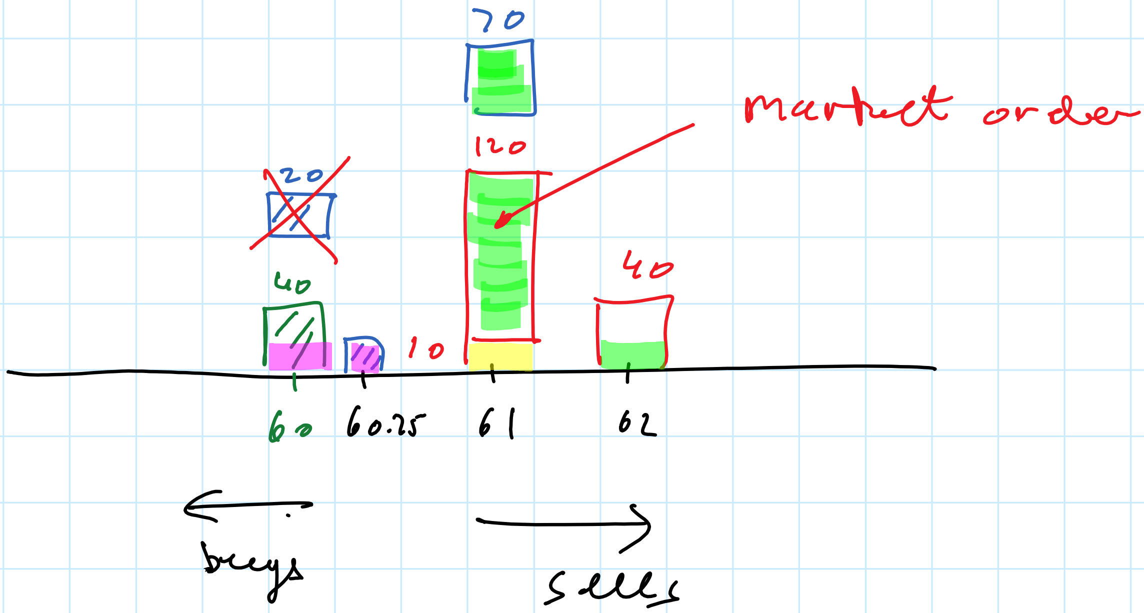


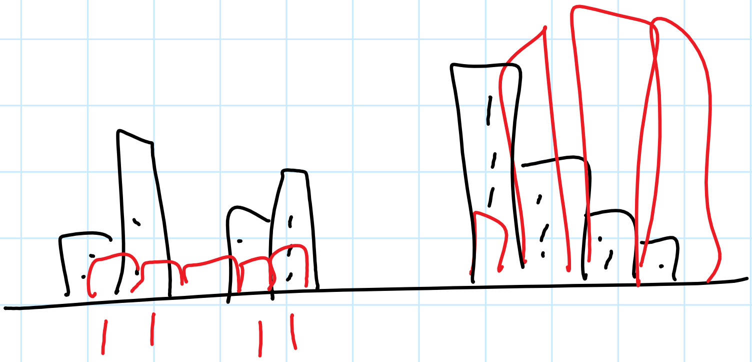




informed,	uniformed
buy if $V_1 = \bar{V}$	buy 1/2
sell if $V_1 = \underline{V}$	

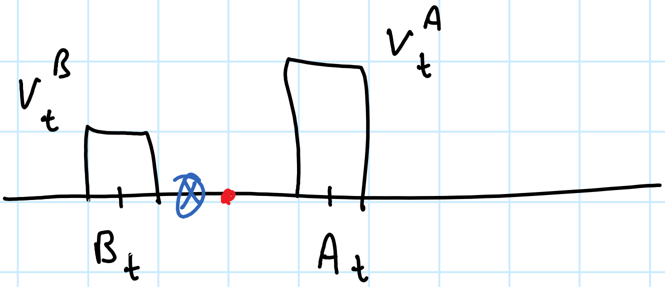
Limit order book (LOB)





mid-price $$P_t = \frac{A_t + B_t}{2}$$

micro-price $$\boxed{P_t = \frac{V_t^B A_t + V_t^A B_t}{V_t^B + V_t^A}}$$



1st-level "touch"

Wednesday, February 27, 2013 3:58 PM

Explaining bid–ask spread ... Glosten + Milgrom

$$V \begin{cases} \bar{V} & 1/2 \\ \underline{V} & \end{cases}$$

traders arrive, you want to make markets

(α) informed uninformed (post sell & buy price)

$$\text{informed}: \quad V_1 = \bar{V} \ \text{buy}, \qquad V_1 = \underline{V} \ \text{sell}$$

$$\text{uninformed}: \quad \tfrac{1}{2} \ \text{buy}, \qquad \tfrac{1}{2} \ \text{sell}$$

$$\left[\; V_0 \begin{cases} \bar{V} & 1/2 \\ \underline{V} & \end{cases} \qquad V_0 = \mathbb{E}[V_1] \; \right]$$

$$\left[\begin{aligned} V_A &= \alpha \bar{V} + (1-\alpha) V_0 \\ V_B &= \alpha \underline{V} + (1-\alpha) V_0 \end{aligned} \right]$$

$$\left\{ \begin{aligned} V_A &= \mathbb{E}[V_1 \mid B] \\ V_B &= \mathbb{E}[V_1 \mid S] \end{aligned} \right.$$

market buy

rationale expectations

market sell

$u(x) = x$ risk-neutral, $\quad u(x) = \dfrac{1 - e^{-\gamma x}}{\gamma}$ ~~CRRA~~ CARA

$u(x)$ do nothing

$$U(x) = \mathbb{E}[\, u(x + V_A - V_1)\, \mathbb{1}_B + u(x)\, \mathbb{1}_S \,]$$

post sell LO

$$U(x) = \mathbb{E}[\, u(x + V_A \underset{-V_1}{}) \mathbb{1}_B + u(x) \mathbb{1}_S \,]$$ post sell Lo

$$u(x) = \mathbb{E}[\, u(x + V_A \underset{-V_1}{}) \mathbb{1}_B + u(x) \mathbb{1}_S \,]$$

$$\Rightarrow \quad \mathbb{E}[u(x) \mathbb{1}_B] = \mathbb{E}[u(x + V_A \underset{-V_1}{}) \mathbb{1}_B]$$

$$x \, \mathbb{E}[\mathbb{1}_B] = (x + V_A) \mathbb{E}[\mathbb{1}_B] - \mathbb{E}[V_1 \mathbb{1}_B]$$

$$\Rightarrow \quad V_A = \frac{\mathbb{E}[V_1 \mathbb{1}_B]}{\mathbb{E}[\mathbb{1}_B]} = \mathbb{E}[V_1 \mid B]$$

$$¿ \left(-\frac{1}{\gamma} \ln \mathbb{E}[\, e^{-\gamma V_1} \mid B] \right) ?$$

$$V_B = \mathbb{E}[V_1 \mid S]$$

$$U(x) = \mathbb{E}[\, u(x + V_A - V_1) \mathbb{1}_B$$
$$+ u(x - V_B + V_1) \mathbb{1}_S \,] = u(x)$$

¿ how to uniquely determine V_A & V_B?
¿ does a unique answer exist?

$$V^A = \mathbb{E}[V_1 \mid B]$$
$$= \bar{V} \, \mathbb{P}(V_1 = \bar{V} \mid B) + \underline{V} \, \mathbb{P}(V_1 = \underline{V} \mid B)$$

$$\mathbb{P}(B \mid V_1 = \bar{V}) = \alpha \cdot 1 + (1-\alpha)\tfrac{1}{2} = \tfrac{1}{2}(1+\alpha)$$

$$\mathbb{P}(V_1 = \bar{V} \mid B) = \frac{\mathbb{P}(V_1 = \bar{V}, B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(V_1 = \bar{V}, B)}{\mathbb{P}(V_1 = \bar{V})} \cdot \frac{\mathbb{P}(V_1 = \bar{V})}{\mathbb{P}(B)}$$

$$= \mathbb{P}(B \mid V_1 = \bar{V}) \cdot \mathbb{P}(V_1 = \bar{V}) \rightsquigarrow 1/2$$

$\mathbb{P}(B)$

$\hookrightarrow \ \mathbb{P}(B \mid v_1 = \bar{v}) \cdot \mathbb{P}(v_1 = \bar{v}) \rightarrow \frac{1}{2}$

$+ \ \mathbb{P}(B \mid v_1 = \underline{v}) \cdot \mathbb{P}(v_1 = \underline{v}) \rightarrow \frac{1}{2}$

$\hookrightarrow \ \alpha \cdot 0 + (1-\alpha)\frac{1}{2}$

$$= \frac{\frac{1}{2}(1+\alpha)\ \frac{1}{2}}{\frac{1}{2}(1+\alpha)\frac{1}{2} + (1-\alpha)\frac{1}{2}\ \frac{1}{2}} = \frac{1+\alpha}{2}$$

informed traders

$$V^A = \bar{V}\,\frac{1+\alpha}{2} + \underline{V}\,\frac{1-\alpha}{2} = \frac{1}{2}(\bar{V} + \underline{V}) + \underbrace{\frac{\alpha}{2}(\bar{V} - \underline{V})}_{\text{adverse selection "wedge"}}$$

$\hookrightarrow \mathbb{E}[v_1]$

$$V^B = \frac{1}{2}(\bar{V} + \underline{V}) - \frac{\alpha}{2}(\bar{V} - \underline{V})$$

$$\boxed{\text{spread} = \alpha(\bar{V} - \underline{V})}$$