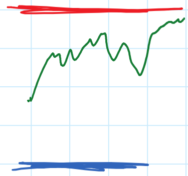
28 March 2014 10:10

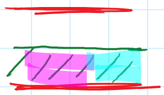


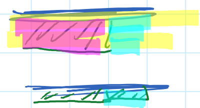
$$P_t = \frac{P_t^A V_t^B + P_t^B V_t^A}{V_t^A + V_t^B}$$

$$= P_t^A \, \omega_t + P_t^B (1 - \omega_t)$$

$$\omega_t = \frac{V_t^B}{V_t^A + V_t^B} \in [0, 1] \qquad , \qquad \frac{V_t^A - V_t^B}{V_t^A + V_t^B} \in [-1, 1]$$

Order Imbalance





$$dx_t = -\kappa x_t \, dt + \sigma \, dW_t$$

$$x_t = x_0 e^{-\kappa t} + \sigma \int_0^t e^{-\kappa u} \, dW_u$$

$$x_t \big|_{\mathcal{F}_0} \sim \mathcal{N}\Big(x_0 e^{-\kappa t} \, , \; \sigma^2 \underbrace{\int_0^t e^{-2\kappa u} \, du}_{\frac{1 - e^{-2\kappa t}}{2\kappa}} \Big)$$

$$x_\infty \big|_{\mathcal{F}_0} \sim \mathcal{N}\Big(0 \, , \; \frac{\sigma^2}{2\kappa} \Big)$$

$$y_t = \frac{e^{x_t}}{1 + e^{x_t}} = \frac{1}{1 + e^{-x_t}}$$

$$z_t = 1 - 2 y_t$$

order imbalance: $\rho_t \in \{ [0, a_1), [a_1, a_2), [a_2, a_3), \ldots, [a_n, 1] \}$

$Z_t \in \{ 1, 2, \ldots, n+1 \}$

↳ markov chain approximation of ρ

rate of arrival of MO conditional on $Z_t \in k$

$$T_k = \int_0^T \mathbb{1}_{Z_s = k} \, ds$$ time spent in regime k

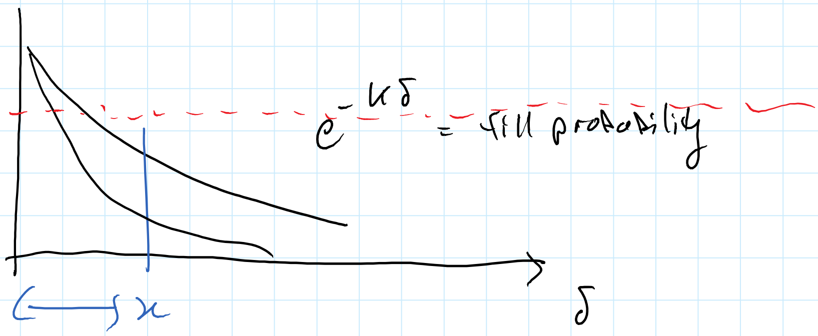
$$\lambda^b_k = \frac{M^b_k}{T_k} \quad , \quad M^b_k = \sum_{m=1}^{M^b_T} \mathbb{1}_{Z_{\tau_m} = k}$$

Z_t is a e.g., Markov chain (continuous time, discrete space)

A - transition rate matrix

recall $\mathbb{P}(Z_{t+\Delta t} = k \mid Z_t = j) = (e^{A \Delta T})_{jk}$

$$\mathcal{L} f(z) = \sum_{k=1}^{K} (f(k) - f(z)) A_{zk}$$



"expected instantaneous profit" $= x\, e^{-\kappa x} \xrightarrow[\text{max}]{} x^* = \frac{1}{\kappa}$!

prob of fill at $x^* = e^{-1}$