$$q_t = \frac{\sinh\left(\sqrt{\frac{\phi}{a}}(T-t)\right)}{\sinh\left(\sqrt{\frac{\phi}{a}}\,T\right)}$$

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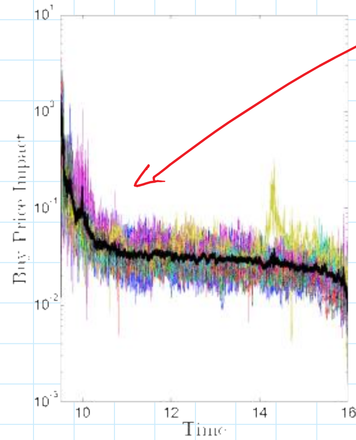
$$\text{micro price} = \frac{p^b V^a + p^a V^b}{V^a + V^b}$$

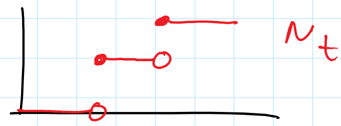
$$a_t = \bar{a}_t \, e^{\varepsilon_t}$$

$$d\varepsilon_t = -\kappa \, \varepsilon_t \, dt + \sigma \, dW_t$$

a_t



$\bar{a}_t$



N_t – Poisson process (λ)

$$X_t = f(N_t) = \sum_k \left(f(N_{t_k}) - f(N_{t_{k-1}}) \right) + f(N_0)$$

$$dX_t = [f(N_t) - f(N_{t-})]\, dN_t \qquad N_{t-} = \lim_{s \nearrow t} N_s$$

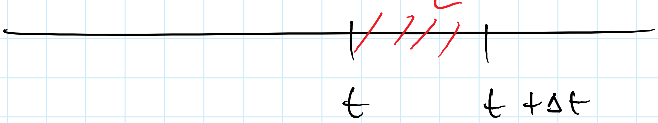
$$X_t - X_0 = \int_0^t \left(f(N_u) - f(N_{u-}) \right) dN_u$$

$$= \sum_{n=1}^{N_t} \Delta f_{\tau_n} \quad \hookrightarrow \quad f(N_{\tau_n}) - f(N_{\tau_n^-})$$

$\hat{N}_t = N_t - \lambda t$ is a mtg.

$\hat{N}_t = N_t - \int_0^t \lambda_s \, ds$ is a mtg.

$\hookrightarrow$ e.g. if λ_t is stochastic



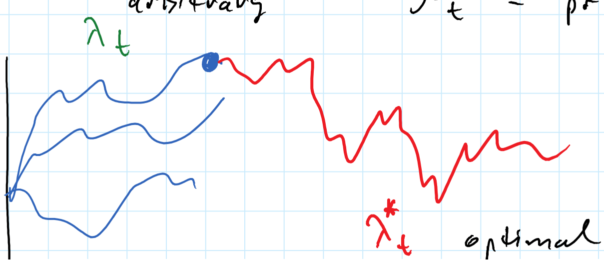
$$\mathbb{P}(N_{t+\Delta t} - N_t = n \mid \mathcal{F}_t) = \begin{cases} 1 - \lambda_t \Delta t + o(\Delta t), & n = 0 \\ \lambda_t \Delta t + o(\Delta t), & n = 1 \\ o(\Delta t), & n > 1 \end{cases}$$

$$H^\lambda(t,n) = \mathbb{E}_{t,n}\left[g(N_T^\lambda) + \int_t^T f(N_u^\lambda)\, du \right]$$

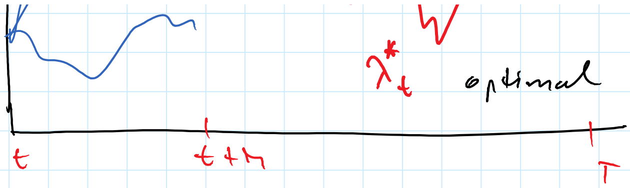
$\hookrightarrow E[\cdot \mid N_{t-} = n]$

N_t is a counting process intensity λ_t

arbitrary $\quad \mathcal{F}_t^N$ – predictable bounded by $\bar{\lambda}$



$$H(t,n) = \sup_{\lambda \in \mathcal{A}} H^\lambda(t,n)$$



$$H^\lambda(t,n) = \mathbb{E}_{t,n}\Big[g(N^\lambda_T) + \int_t^{t+h} F(N^\lambda_u)\,du + \underbrace{\int_{t+h}^{T} F(N^\lambda_u)\,du}_{\text{recall } \lambda_u = \lambda^*_u,} \Big]$$

$$= \mathbb{E}_{t,n}\Big[\int_t^{t+h} F(N^\lambda_u)\,du + \underbrace{\mathbb{E}_{t+h, N^\lambda_{t+h}}\Big[g(N^\lambda_T) + \int_{t+h}^{T} F(N^\lambda_u)\,du \Big]}_{H(t+h,\, N^\lambda_{t+h})} \Big]$$

but we have that:

$$H(t+h, N^\lambda_{t+h}) = H(t,n) + \int_t^{t+h} \partial_t H(u, N^\lambda_u)\,du$$
$$+ \int_t^{t+h} \big(H(u, N^\lambda_u) - H(u, N^\lambda_{u^-}) \big)\, d\hat{N}_u$$
$$+ \int_t^{t+h} \big(H(u, N^\lambda_u) - H(u, N^\lambda_{u^-}) \big)\, \lambda_u\,du$$

$$\Rightarrow H^\lambda(t,n) = H(t,n) + \mathbb{E}_{t,n}\Big[\int_t^{t+h} \Big(\partial_t H_u + \underbrace{\Delta H_u}_{H(u,N^\lambda_u) - H(u,N^\lambda_{u^-})} \lambda_u + F_u \Big)\,du \Big]$$

$$\lim_{h \downarrow 0} \sup_\lambda \frac{1}{h} \times \Big(- \quad = \quad \Big)$$

$$\boxed{0 = \partial_t H(t,n) + \sup_{\lambda \in A_t} \Big\{ \big(H(t,n+1) - H(t,n) \big) \lambda \Big\} + F(n)} \quad \text{DPE}$$

$$H(T,n) = g(n)$$

Let $l_t \in \{0,1\}$ which represents whether we are posted or not.

Controlled counting process N_t^l has intensity

$$\Lambda_t = \rho\lambda\, l_t$$

* F_t is a B.mtn
* Δ - the spread is constant
* X_t - cash process

$$dX_t^l = (F_t + \Delta/2)\, dN_t^l \qquad = l_t\, dN_t$$

$$(F_t - a\nu_t)\ \nu_t\, dt$$

* $$dq_t = -dN_t^l = -l_t\, dN_t$$

$$H(t,x,q,f) = \mathbb{E}_{t,x,q,f}\left[X_T^l + q_T^l\left(F_T - \frac{\Delta}{2} - \alpha q_T^l\right) - \phi\int_t^T (q_s^l)^2\, ds \right]$$

$+\infty$? NO! must be $<+\infty$ but can be large

DPE:

$$\boxed{\begin{aligned} &\partial_t H + \sup_{l\in\{0,1\}} \Big\{ \underbrace{\big(H(t, x + (f+\Delta/2), q-1, f) - H(t,x,q,f) \big)}_{\Delta H} \rho\lambda l \Big\} \\ &+ \frac{1}{2}\sigma^2 \partial_{ff} H - \phi q^2 = 0 \\ &H(T,x,q,f) = x + q\left(f - \frac{\Delta}{2} - \alpha q\right) \end{aligned}}$$

$$H = x + qf + h(t,q)$$

s.t. $h(T,q) = -q\left(\frac{\Delta}{2} + \alpha q\right)$

$$\begin{aligned} \Delta H &= (x + f + \Delta/2) + (q-1)f + h(t,q-1) \\ &\quad - [x + qf + h(t,q)] \\ &= \frac{\Delta}{2} + h(t,q-1) - h(t,q) \end{aligned}$$

$$\Rightarrow \boxed{\begin{aligned} &\partial_t h(t,q) + \lambda p \left(\frac{\Delta}{2} + h(t,q-1) - h(t,q) \right)_+ - \phi q^2 = 0 \\ &h(T,q) = -q\left(\frac{\Delta}{2} + \alpha q\right) \end{aligned}}$$

$$h(t,0) = 0$$

no trading once $q_t = 0$.