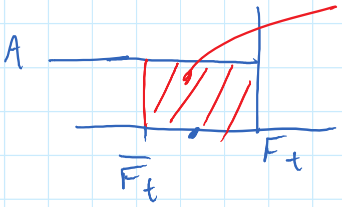
19 March 2014 14:28



$v_t \Delta t = V \cdot ($

$$(F_t - \bar{F}_t) \times A = v_t \Delta t$$

$$\Rightarrow \bar{F}_t = F_t - v_t \frac{\Delta t}{A}$$

$$S_t^{\vee} = \frac{1}{2}(F_t + \bar{F}_t) = F_t - v_t \left(\frac{\Delta t}{2A}\right) = a$$

$$\mathcal{L} f(x) \triangleq \lim_{h \downarrow 0} \frac{\mathbb{E}[\overbrace{f(X_{t+h}) - f(X_t)} \mid X_t = x]}{h}$$

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mean cost / share ?

std cost / share ?

$$\frac{\mathbb{E}[X_T^{v^*}]}{q_0} \quad \& \quad \frac{\mathbb{V}[X_T^{v^*}]}{q_0^2}$$

$$d X_t^{v^*} = S_t^{v^*} \cdot v_t^* \, dt$$

$$= (F_t - a v_t^*) v_t^* \, dt$$

$$X_T^{v^*} = \int_0^T (F_u - a v_u^*) v_u^* \, du$$

$$\mathbb{E}[X_T^{v^*}] = \int_0^T (\underbrace{\mathbb{E}[F_u]}_{F_0} - a v_u^*) v_u^* \, du$$

$$= \int_0^T (F_0 - a v_u^*) v_u^* \, du$$

$$\mathbb{V}[X_T^{v^*}] = \mathbb{V}\left[\int_0^T F_u v_u^* \, du\right]$$

$$\mathbb{E}\left[\left(\int_0^T F_u v_u^* \, du\right)^2\right] = \mathbb{E}\left[\int_0^T \int_0^T F_u v_u^* F_s v_s^* \, du \, ds\right]$$

$$= \int_0^T \int_0^T v_u^* v_s^* \underbrace{\mathbb{E}[F_u F_s]}\, du \, ds$$

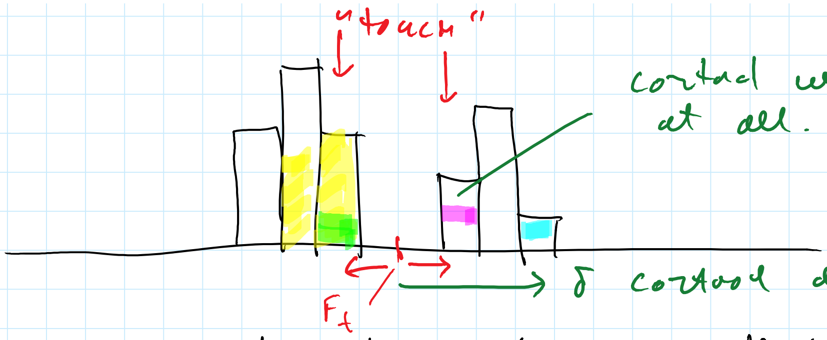
$$\mathbb{E}\Big[\big((F_u - F_0) + F_0\big)\big((F_s - F_0) + F_0\big)\Big]$$

$$= \sigma^2 \min(u,s) + F_0^2$$

$$I = \int_0^T F_u v_u^* \, du \qquad V_t = \int_0^t v_u^* \, du$$

$$d(F_t V_t^*) = dF_t V_t^* + F_t v_t^* dt$$

$$\Rightarrow I = \left(F_T V_T^* - F_0 V_0^*\right) - \int_0^T V_u^* \, dF_u$$



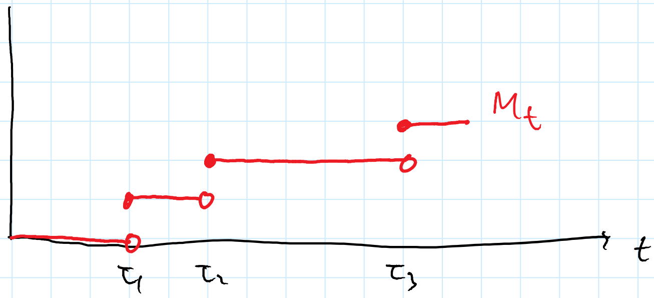
control whether to post at all.

* when to post as • a limit order (pays $\Delta/2$)
  • a market order (costs $\Delta/2$)

Basic Model:

MOs arrive at Poisson times $\tau_1, \tau_2, \ldots$

M_t – counting process (rate λ)



$V_1 \quad V_2 \quad V_3$ volume of MO

$\longrightarrow \quad \mathbb{P}(\Delta N_{\tau_k} = 1) = \rho \longleftarrow$ incorporates volume r.v. & position in queue

· N_t – counting process of your filled LOs

$\Delta N_t \triangleq N_t - N_{t^-}$

N_t is indeed a Poisson process with intensity $\rho\lambda$.

Let $\ell_t \in \{0,1\}$ which represents whether we are posted or not.

Controlled counting process N_t^ℓ has intensity

$$\Lambda_t = \rho\lambda\,\ell_t$$

* F_t is a B.mtn

- Δ - the spread is constant
- X_t - cash process

$$dX_t^l = \left(F_t + \Delta/2\right) dN_t^l \quad = l_t \, dN_t$$

$$\left(F_t - a\nu_t\right) \quad \boxed{\nu_t \, dt}$$

- $$dq_t = - dN_t^l = - l_t \, dN_t$$

$+\infty$? NO! must be $<+\infty$ but can be large

$$H^l(t, x, q, f) = \mathbb{E}_{t,x,q,f}\left[X_T^l + q_T^l\left(F_T - \frac{\Delta}{2} - \alpha q_T^l\right) - \phi \int_t^T (q_s^l)^2 ds \right]$$

N_t^δ - controlled counting process for your filled LOs

intensity $\Lambda_t = \lambda g(\delta_t)$

$\llcorner$ decreasing in δ

e.g. $e^{-\kappa \delta_t}$

$$dX_t^\delta = (F_t + \delta_t) \, dN_t^\delta$$

$$dq_t^\delta = - dN_t^\delta$$