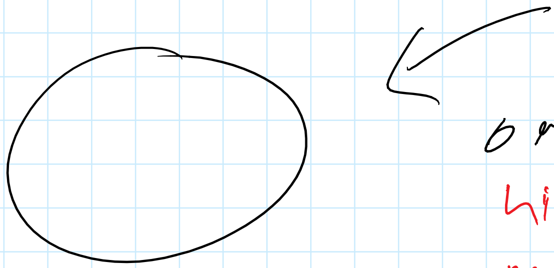
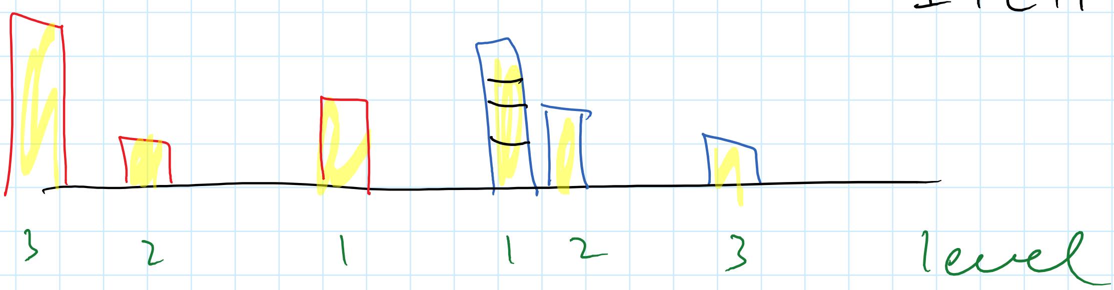
26 February 2014 14:15

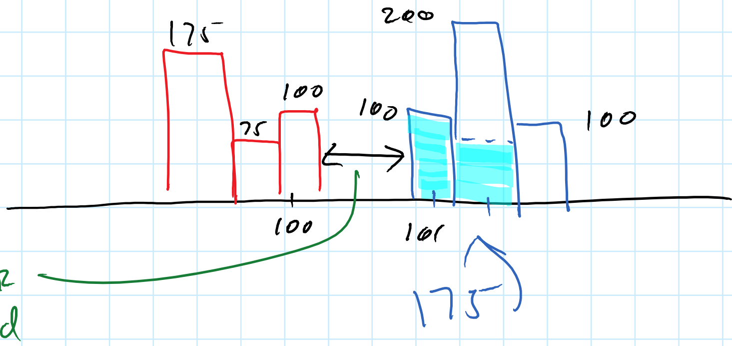


orders
Limit — Limit order Book (LOB)
Market

ITCH




bid ask / offer




bid-ask spread

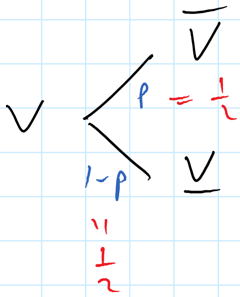
walk the book

M.O. buy lift offer

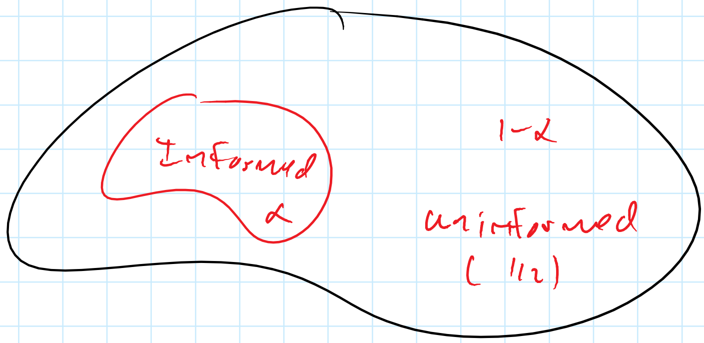
M.O. sell hit bid

Limit orders
→ patient
→ price sensitive.

Market orders
→ impatient (immediate execution)
→ "best" price




$$\mathbb{E}[V] \overset{?}{=} \text{bid} = \text{ask}$$




bid $= \mathbb{E}[V \mid \text{Buy order arriving}] <$

ask $= \mathbb{E}[V \mid \text{sell order arriving}]$

$$\text{bid} = \mathbb{E}[V \mid B] = \bar{V}\, \overbrace{\mathbb{P}(V=\bar{V} \mid B)}^{\frac{1+\alpha}{2}} + \underline{V}\, \overbrace{\mathbb{P}(V=\underline{V} \mid B)}^{\frac{1-\alpha}{2}}$$

$$\mathbb{P}(V=\bar{V} \mid B) = \frac{\mathbb{P}(V=\bar{V}, B)}{\mathbb{P}(B)} = \frac{\frac{1}{2}\left(\alpha + (1-\alpha)\frac{1}{2}\right)}{1/2} = \frac{1+\alpha}{2}$$

$$\mathbb{P}(V=\bar{V}, B) = \mathbb{P}(B \mid V=\bar{V}) \cdot \mathbb{P}(V=\bar{V})$$
$$= \left(\alpha \cdot 1 + (1-\alpha)\frac{1}{2}\right)\frac{1}{2}$$

$$\mathbb{P}(B) = \mathbb{P}(V=\bar{V}, B) + \mathbb{P}(V=\underline{V}, B)$$
$$= \mathbb{P}(B \mid V=\bar{V})\,\mathbb{P}(V=\bar{V}) + \mathbb{P}(B \mid V=\underline{V})\,\mathbb{P}(V=\underline{V})$$
$$= \left(\alpha \cdot 1 + (1-\alpha)\frac{1}{2}\right)\frac{1}{2} + \left(\alpha \cdot 0 + (1-\alpha)\frac{1}{2}\right)\frac{1}{2}$$
$$= \frac{\alpha}{2} + \frac{(1-\alpha)}{2} = \frac{1}{2}$$

$$\text{bid} = \mathbb{E}[V] + \frac{\alpha}{2}(\bar{V} - \underline{V}) \quad \Big\} \quad \text{spread} = \alpha(\bar{V} - \underline{V})$$

$$\text{bid} = \mathbb{E}[V] + \frac{\alpha}{2}(\bar{V} - \underline{V})$$

$$\text{ask} = \mathbb{E}[V] - \frac{\alpha}{2}(\bar{V} - \underline{V})$$

$\text{spread} = \alpha(\bar{V} - \underline{V})$

protects MM from adverse selection

# Optimal Liquidation / AC

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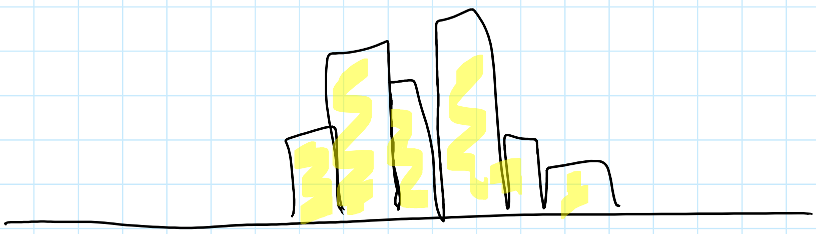

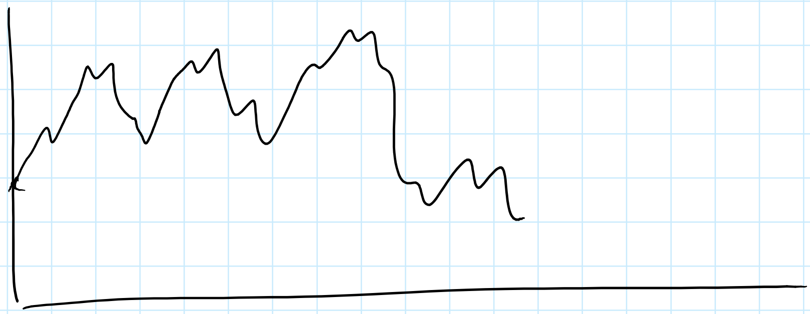

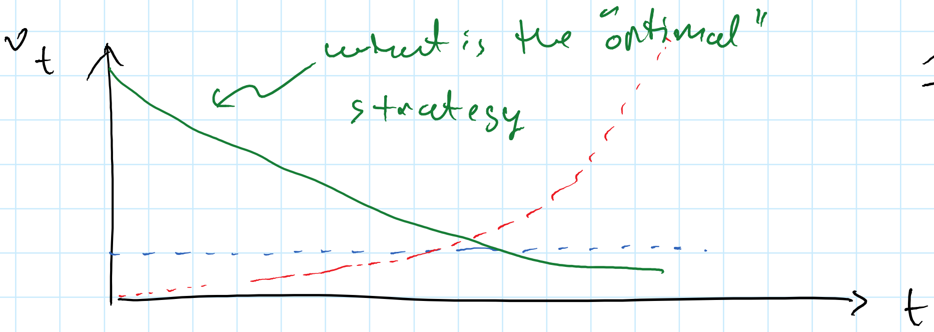


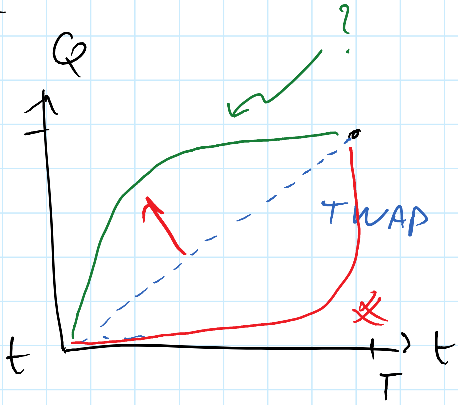


TWAP - time weighted average price

$$= \frac{1}{T}\int_0^T S_u \, du$$

$$dF_t^\nu = \sigma \, dW_t + \underbrace{b \, \nu_t \, dt}_{\text{permanent price impact}}$$

Fundamental or mid price

$$S_t^\nu = F_t^\nu + \underbrace{a \, \nu_t}_{\text{temporary impact}}$$

execution price

$$dX_t^\nu = - S_t^\nu \, \nu_t \, dt$$

cash process

performance criteria:

given $\nu$ :

"penalize variance"

$$\tilde{H}(t,x,F,q) = \mathbb{E}_{t,x,F,q}\left[X_T^{\nu} + ??\right]$$

$\mathbb{E}_{t,x,F,q} \to \mathbb{E}[\,\cdot \mid X_t = x, F_t = F, q_t = q]$

Value function:

$$H(t,x,F,q) = \sup_{\nu \in \mathcal{A}} H^{\nu}(t,x,F,q)$$

$\mathcal{A}$: $\mathcal{F}_t$ - predictable bounded processes

$$\mathcal{F}_t = \sigma\left((W_s)_{0 \le s \le t}\right)$$

penalize variance:

$$-\phi \, \mathbb{V}[X_T^{\nu}]$$

quadratic variation

$[X^{\nu}]_T$ $\qquad dX_t^{\nu} = -S_t \, \nu_t \, dt$

$$dy_t = g(t,W_t)\,dt + h(t,W_t)\,dW_t$$

$$[y]_T = \int_0^T h^2(s,W_s)\,ds$$

$$\boxed{\phi \int_t^T (q_u - Q)^2 \, du}$$ is

inventory at $u$ $\qquad$ target inventory