# IMPA Commodities Course:
## Introduction

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# Basic Issues

- Commodities are **NOT** like equities, interest rates, or currencies!
- Commodities are real assets: produced, consumed, transported and stored
- Owing a commodity at point $A$ at time $T$ is not the same as owing it at point $B$ at time $S$!
- Customers of commodity derivatives are typically
    - Industrial producers / consumers
    - Governments
- Customers tend to be **very risk averse** due to
    - Required consumption
    - Legal risks

## Basic Issues

- Spot markets per se do not exist
- Instead there are major **futures** market
  - Electricity
    - day-ahead / day-of / hour-ahead
    - balance-of-week /month
    - years ahead
  - Crude Oil, Heating Oil, Nat Gas: months / years
- Contracts
  - May be for physical delivery or financial settlement
  - May require delivery over a period of time
  - May be interruptible
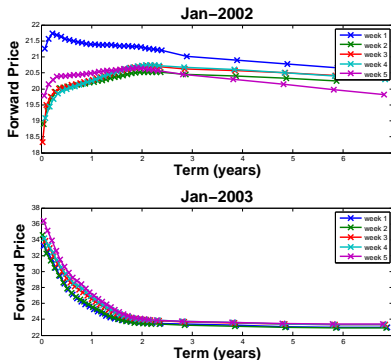  - May vary the amount delivered

# Crude Oil Forward Price Data

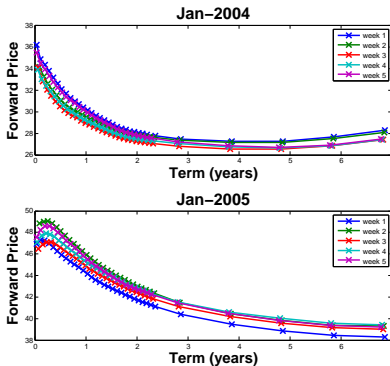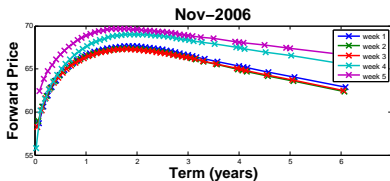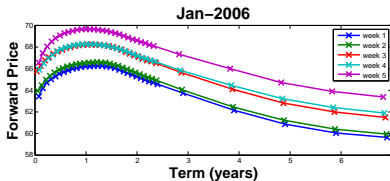Forward price curves from Jan-2002 to Dec-2006
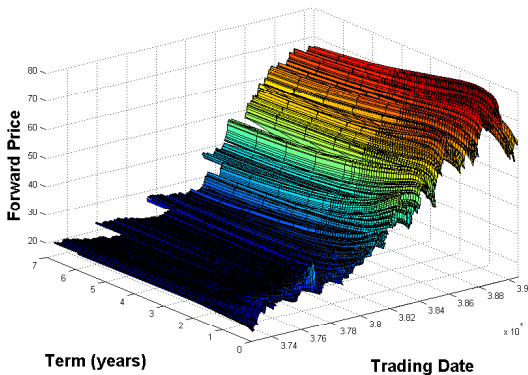
# Crude Oil Forward Price Data

Weekly deviations in Forward price curves Jan-2002 and 2003

# Crude Oil Forward Price Data

Weekly deviations in Forward price curves Jan-2004 and 2005

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**Crude Oil**
Heating Oil
Data Observations

# Crude Oil Forward Price Data

Weekly deviations in Forward price curves Jan-2006 and Dec-2006

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# Crude Oil Forward Price Data

Long end and short end of the Forward price curves from Jan-2002 to Dec-2006



Notice periods of Backwardation and Contango.

# Crude Oil Forward Price Data
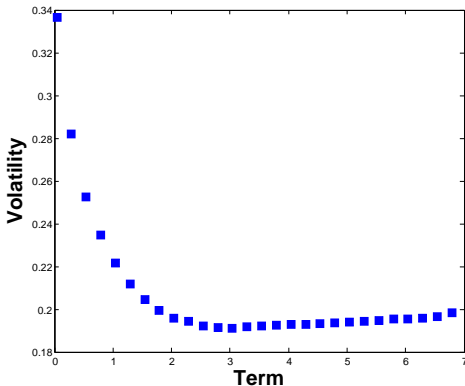
Term Structure of volatilities



Exponentially decaying volatilities

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# Crude Oil Forward Price Data

Correlation of constant maturity forward price returns



As expected, terms further apart less correlated

# Heating Oil Forward Price Data

Forward price curves from Jan-2002 to Dec-2006

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# Heating Oil Forward Price Data

Weekly deviations in Forward price curves Jan-2002 and 2003

# Heating Oil Forward Price Data

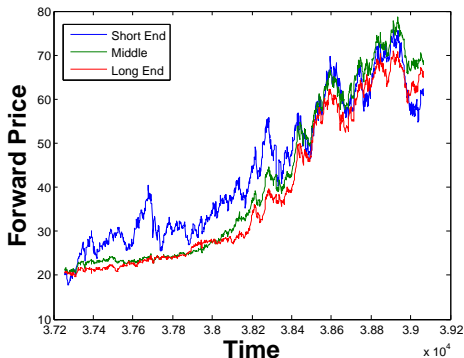Weekly deviations in Forward price curves Jan-2004 and 2005

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# Heating Oil Forward Price Data

Weekly deviations in Forward price curves Jan-2006 and Dec-2006

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# Heating Oil Forward Price Data

Long end and short end of the Forward price curves from Jan-2002 to Dec-2006
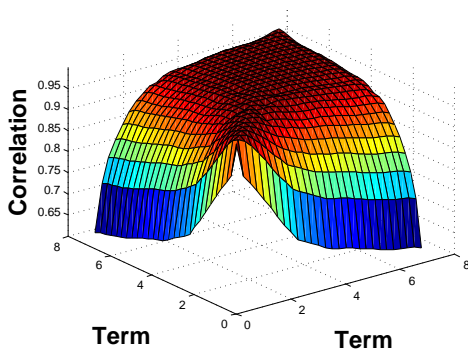


Notice periods of Backwardation and Contango.

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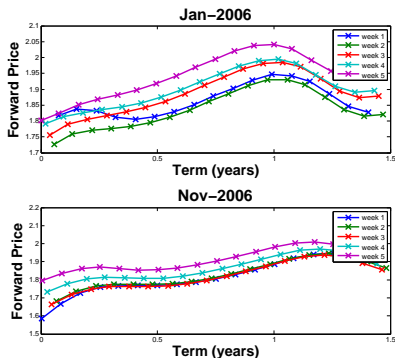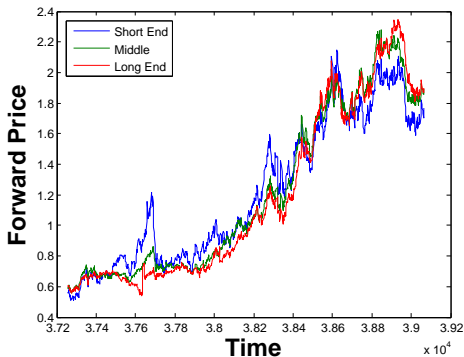
# Heating Oil Forward Price Data

Term Structure of volatilities
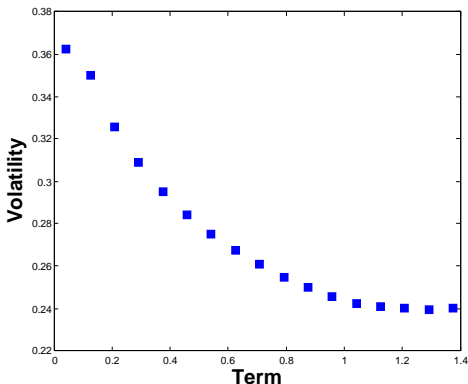


Exponentially decaying volatilities

# Heating Oil Forward Price Data

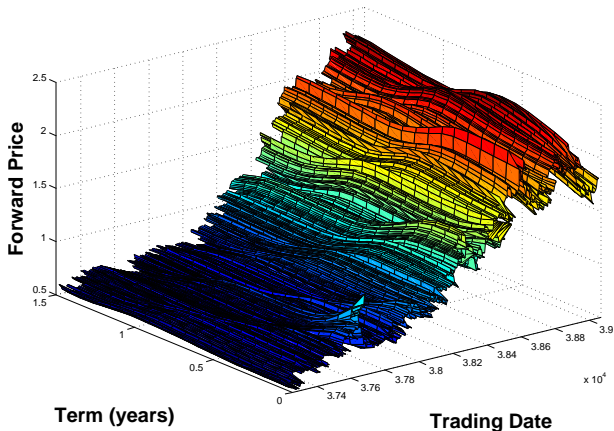Correlation of constant maturity forward price returns
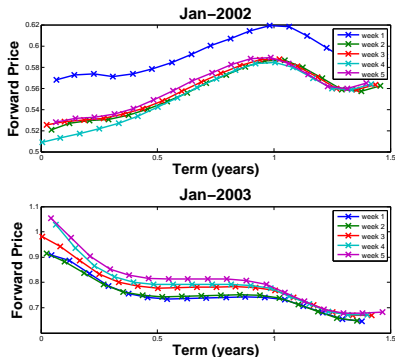


As expected, terms further apart less correlated

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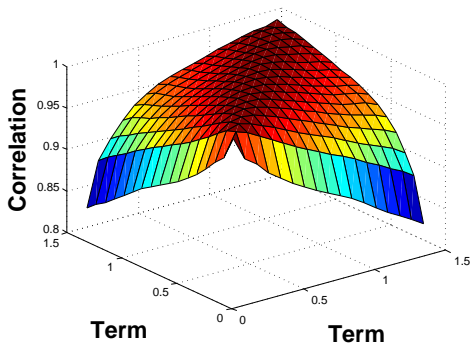
# Observations: Arbitrage Opportunity?

- There are no simple "calendar spread" arbitrage opportunities
  e.g. if curve is upward sloping: purchase short-term contract; sell
  long-term contract $\rightarrow$ what's wrong?
- The contract with the shortest maturity is called the **first nearby**
- The contract with the next shortest maturity is called the **second nearby**, etc..
- When the first nearby matures, it is said to **roll off**
- Many exotics have nearbys as underliers – as opposed to a specific contract

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**Data**    Heating Oil
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## Observations: Shape

- Most commodities have **volatile short-ends**, and **quasi-stable long-ends**
- Long-end is "determined" by marginal cost of production
- Short-end governed by short termed supply/demand
  - When there are excesses of commodity :
    curve is upward sloping (**contango**)
  - When there are shortages of commodity :
    curve is downward sloping (**backwardation**)

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## Observations: Seasonality

- **Seasonality** occurs in many commodities (crude oil is the main exception)
- If storage capacity exceeds the wavelength, then no humps
- Natural gas has large hump in winter, small hump in summer
- Gasoline has large hump in summer
- Electricity has humps in winter and summer, negative hump on weekends, and intra-day structure

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## Observations: Bias

- Trading volume is clumped in the long-end of the curve
- Short-end used to cover unexpected demand
- Short-end is positively biased to avoid large negative impact if demand is not met – risk averse investors
- Hedge funds becoming larger players in this market

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Single Name Options
Two-Name Options
Exotic Options

## Swap

**Swaps** entail exchanging a fixed payment stream for a floating payment stream

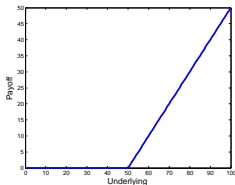– Floating typically linked to a commodity spot price or price index
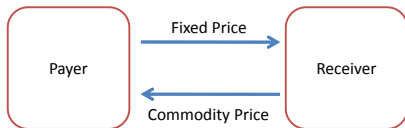


- The floating-leg can be viewed as series of forward contracts
- The fixed-leg can be viewed as a series of "coupon" payments
- fixed-leg payments determined such that both legs have equal value

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# Call Option

**Call options** give the holder the right (but not obligation) to purchase a commodity (or a forward on commodity) at a specified future date for a specified price

$$\varphi = \max(F_T(T_1) - K)_+$$



- Stochastic models must be built to obtain the evolution and valuation

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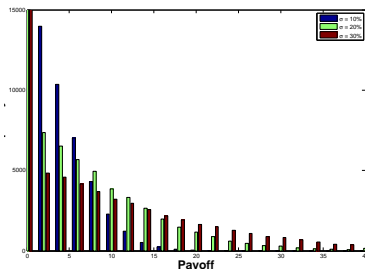Swap
**Single Name Options**
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# Call Option

- Simplest model is to assume forward price at maturity is log-normal :

$$F_T(T_1) = F_T(0) \exp\{-\frac{1}{2}\sigma^2 T + \sigma\sqrt{T} Z\} \quad \text{and} \quad Z \sim \mathcal{N}(0,1)$$
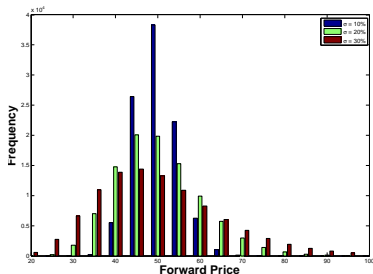
(a) Forward Price

(b) Call Payoff

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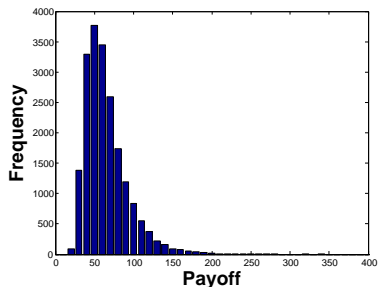Two-Name Options
Exotic Options

## Asian Call Option

**Asian call options** are call options on the average of the commodity (forward) price on several days
$A(t_1, t_2; T) \triangleq \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} F_T(u) du$

- Two main classes:
    - Average Price: $\varphi = (A(t_1, t_2, T) - K)_+$
    - Average Strike: $\varphi = (F_T(t_1) - A(t_1, t_2, T))_+$
- Tend to be cheaper than a regular option – averaged out the volatility
- Moment match approximations are sometimes used to obtain "closed-form" results: $\sim$ log-normal or $\sim$ inverse-gaussian
- Geometric averaging used to analytically compute price and corrected via control variate
- PDE / transform methods

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# Asian Call Option



(c) Average years $0-5$

(d) Year 5

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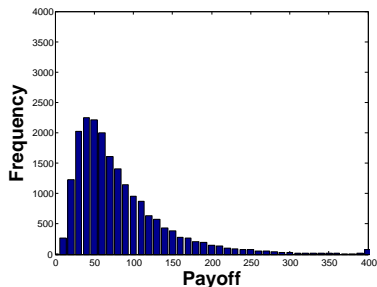
## Floating Strike Call

**Floating Strike Call Options** are call options with the strike level set at a future date based on some price index

$$\varphi = (F_T(T_1) - I_T)_+$$

- There are two sources of uncertainty here – the commodity price and the index price

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# Calendar Spread Options

A **calendar spread option** is an option to **exchange** a $T_2$-**maturity forward** contract for a $T_1$-**maturity forward** contract at a **cost of** $K$ at time $T$:

$$\varphi = (F_T(T_1) - F_T(T_2) - K)_+ \ .$$

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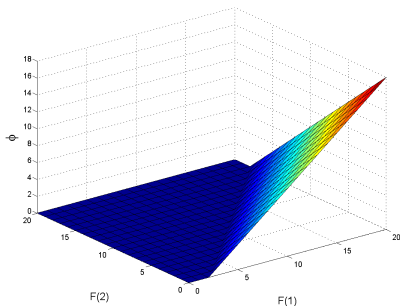
## Calendar Spread Options

Payoff profile ( $F_0(T_1) = 10$, $F_0(T_2) = 9$, $K = 1$, $T = 1$yr.,
$\sigma_1 = \sigma_2 = 50\%$)

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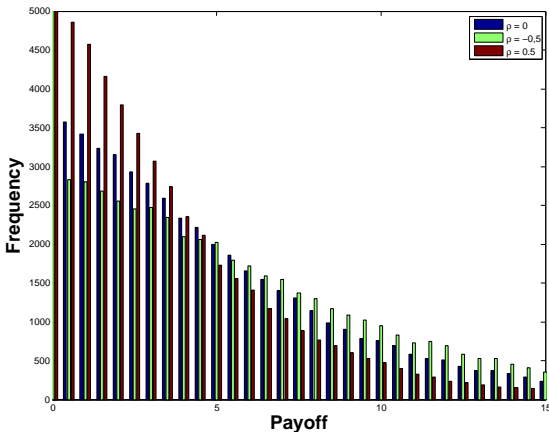
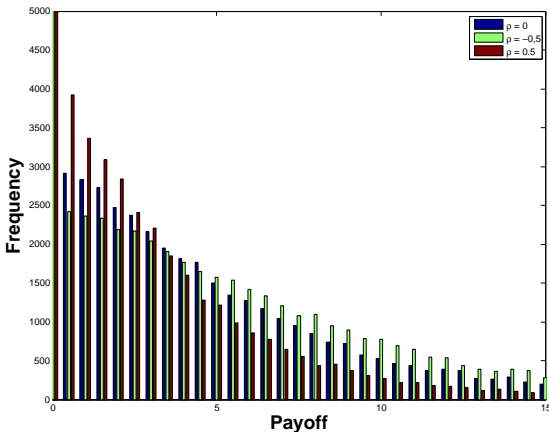# Calendar Spread Options

Payoff profile ( $F_0(T_1) = 10$, $F_0(T_2) = 11$, $K = 1$, $T = 1$yr., $\sigma_1 = \sigma_2 = 50\%$)

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## Intercommodity Spread

**Intercommodity Spread Options** are options based on the difference in two commodities

$$\varphi = \left( F_T^{(1)}(T_1) - \alpha \, F_T^{(2)}(T_1) \right)_+$$

- **Crack Spread** is the option on the spread between crude oil and refined products (such as heating oil)
- **Spark Spread** is the option on the spread between electricity and fuel (such as coal/gas)
  – $\alpha$ is then referred to as the heat rate (BTUs needed to create 1 kWh of electricity)

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## Barrier Option

**Barrier options** are like other options except they are turned on (knock-in) or turned off (knock-out) when the commodity price enters a given price $\mathcal{D}$

- Typically, the region corresponds to the asset dropping above or below a critical price level
- The barrier may be single sided or double sided
- Hitting one barrier may turn on additional barriers – **onion options**

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# Swing Option

**Swing options** are options on contract volume. They give the holder the right to change the volume of commodity delivered to them.

- Delivery occurs over several days
- The holder is allowed to vary volume within a specified range
- The holder has $K < N$ "swing" opportunities ($N$ is number of days during which delivery occurs)
- These are complex options with embedded American like features
- Tree implementations require solving a forest – not a single tree