IMPA Commodities Course : Electricity Models

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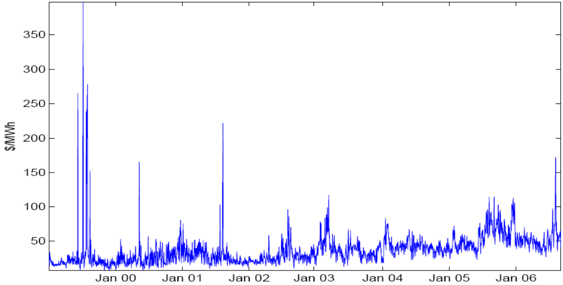
Data

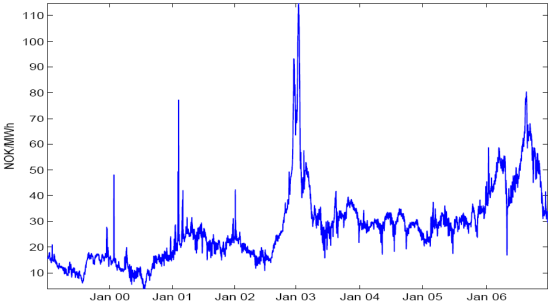
PJM spot prices Jan 1999 – Aug 2006



Thanks to Álvaro Cartea

Data

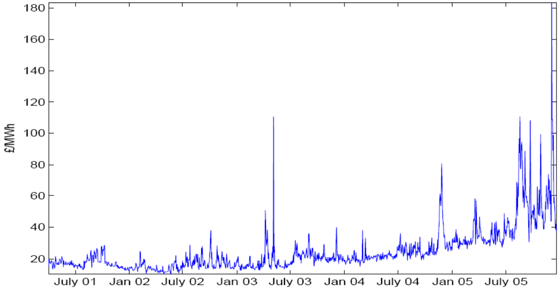
Nord Pool spot prices Jan 1999 – Dec 2006



Thanks to Álvaro Cartea

Data

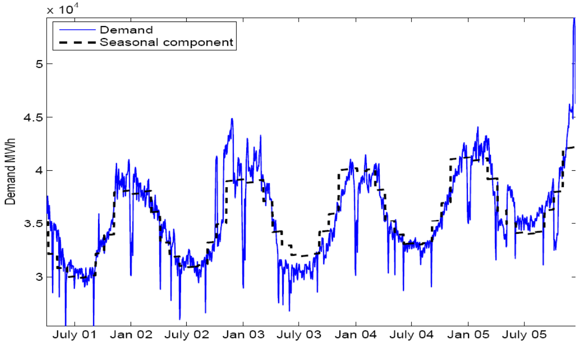
England & Whales spot prices Mar 2001 – Mar 2006



Thanks to Álvaro Cartea

Data

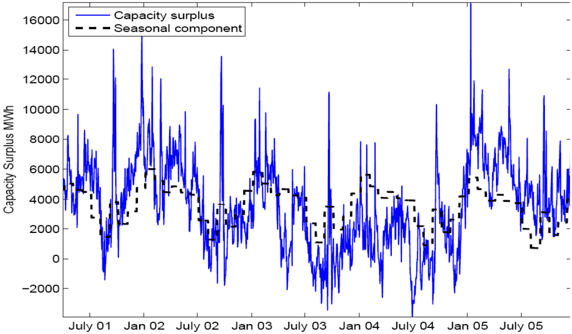
England & Whales demand Mar 2001 – Mar 2006



Thanks to Álvaro Cartea

Data

England & Whales excess capacity Mar 2001 – Mar 2006



Thanks to Álvaro Cartea

One-Factor Model

- Electricity prices contain **large spikes** which revert to normal levels quickly
- **Clewlow & Strickland** (2001) and **Cartea & Figueroa** (2005) simple **extension of Gaussian OU process**:

$$\begin{aligned} S_t &= \exp\{X_t\} \\ dX_t &= \kappa(\theta - X_{t-})\,dt + \sigma\,dW_t + dJ_t \end{aligned}$$

where J_t is a pure jump process – such as **compound Poisson**

$$J_t = \sum_{n=1}^{N_t} j_i$$

with N_t a Poisson process with activity rate λ and $\{j_1, j_2, \dots\}$ i.i.d. random variables.

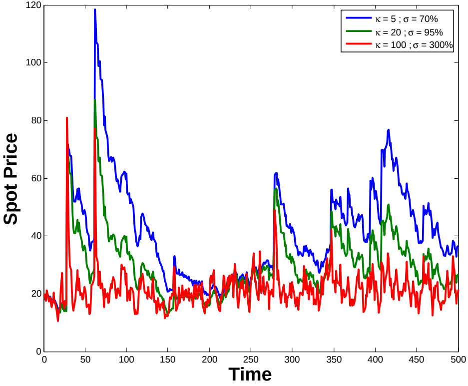
- Can easily solve the SDE to find

$$S_T = \exp\left\{\theta + (\ln S_t - \theta)e^{-\kappa(T-t)} + \int_t^T e^{-\kappa(T-u)}dW_u + \sum_{i=1}^{N_t} e^{-\kappa(T-t_i)} j_i\right\}$$

  Here, $\{t_1, t_2, \dots\}$ are the arrival times of the Poisson process.
- Notice that both diffusions and jumps decay at rate of κ

One-factor Model

Sample path from simple model



Notice that as reversion rate increases, need to increase volatility to compensate – otherwise diffusion will be washed out

Two-factor Model

- **Hikspoors & Jaimungal** (2007) propose instead

$$
\begin{aligned}
S_t &= \exp\{X_t + Y_t\} \\
dX_t &= \kappa(\theta - X_t)\, dt + \sigma\, dW_t \\
dY_t &= -\alpha\, Y_{t-}\, dt + dJ_t
\end{aligned}
$$

where J_t is a pure jump process – such as compound Poisson

$$
J_t = \sum_{n=1}^{N_t} j_i
$$

with N_t a Poisson process with activity rate λ and $\{j_1, j_2, \dots\}$ i.i.d. random variables.

- Can easily solve the SDE to find

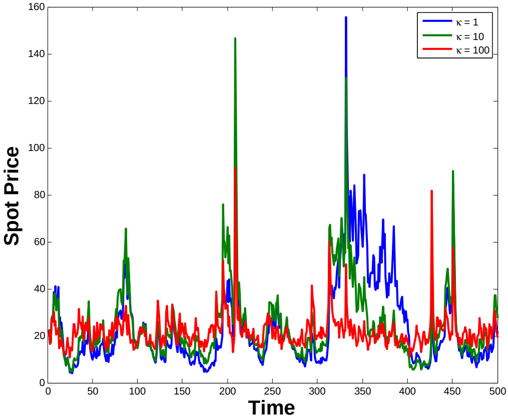
$$S_T = \exp\left\{\theta + (\ln S_t - \theta)e^{-\kappa(T-t)} + \int_t^T e^{-\kappa(T-u)}dW_u + \sum_{i=1}^{N_t} e^{-\alpha(T-t_i)} j_i\right\}$$

Here, $\{t_1, t_2, \ldots\}$ are the arrival times of the Poisson process.

- Notice that diffusions and jumps have two separate rates of decay

Two-factor Model

Sample path from HJ model



Notice that diffusion and jumps are uncoupled

Two-factor Model

- Can also incorporate **frequent small jumps**

$$
\begin{aligned}
S_t &= \exp\{X_t + Y_t\} \\
dX_t &= \kappa(\theta - X_{t-})\,dt + \sigma\,dW_t + dQ_t \\
dY_t &= -\alpha\,Y_{t-}\,dt + dJ_t
\end{aligned}
$$

- Q_t contains the small frequent jumps reverting at same rate as diffusion
- J_t contains the large infrequent fast mean-reverting jumps

Forward Prices

- Single factor model Forward prices $F_t(T)$ satisfy the PDE

$$\begin{cases} (\partial_t + \mathcal{L})F & = 0 \\ F_T(T) & = e^x \end{cases}$$

where $\mathcal{L}$ is the infinitesimal generator of the process X_t:

$$\mathcal{L}\, g = \kappa(\theta - x)\partial_x g + \tfrac{1}{2}\sigma^2 \partial_{xx} g + \lambda \int_{-\infty}^{\infty} [g(x+j) - g(x)] dF(j)$$

and $F(j)$ is the cdf of the iid jumps.

Forward Prices

- This can be solved by assuming the ansatz

$$F_t(T) = \exp\{a_t(T) + b_t(T)\,x\}$$

where a and b are deterministic functions of time only

- Then solve the system of ODEs:

$$\begin{cases} \partial_t b - \kappa b &= 0 \\ \partial_t a + \theta\kappa\, b + \frac{1}{2}\sigma^2\, b^2 + \lambda \int_{-\infty}^{\infty} \left[e^{b\,j} - 1\right]\, dF(j) &= 0 \\ a_T(T) &= 0 \\ b_T(T) &= 1 \end{cases}$$

- These are exactly integrable

Forward Prices

- Two factor model Forward prices $F_t(T)$ satisfy the PDE

$$\begin{cases} (\partial_t + \mathcal{L})F &= 0 \\ F_T(T) &= e^{x+y} \end{cases}$$

where $\mathcal{L}$ is the infinitesimal generator of the process (X_t, Y_t):

$$\begin{aligned} \mathcal{L}\, g \;=\; & \kappa(\theta - x)\partial_x g + \tfrac{1}{2}\sigma^2 \partial_{xx} g - \alpha y \partial_y g \\ & + \lambda \int_{-\infty}^{\infty} [g(x, y+j) - g(x,y)]dF(j) \end{aligned}$$

and $F(j)$ is the cdf of the iid jumps.

Forward Prices

- This can be solved by assuming the ansatz

$$F_t(T) = \exp\{a_t(T) + b_t(T)\,x + c_t(T)\,y\}$$

where a, b and c are deterministic functions of time only

- Then solve the system of ODEs:

$$\begin{cases} \partial_t b - \kappa b &= 0\,, \\ \partial_t c - \alpha c &= 0\,, \\ \partial_t a + \theta\kappa\, b + \frac{1}{2}\sigma^2\, b^2 + \lambda \int_{-\infty}^{\infty} \left[e^{c\,j} - 1\right]\, dF(j) &= 0\,, \\ a_T(T) &= 0\,, \\ b_T(T) &= 1\,, \\ c_T(T) &= 1 \end{cases}$$

- These are exactly integrable as well

- European option prices can be determined using **transform methods**
- In two-factor model, since jumps decay extremely quickly and are uncoupled, can ignore them in European option pricing
- For path-dependent options, jumps are important – use Monte Carlo or **Fourier Space Time-Stepping** method

Regime Switching Models

- **Regime switching models** take into account the structural change which occurs during a price spike
  - outages
  - excess demand – unpredicted weather
  - poor rain fall to meet regular base demand
- Basic idea two (or more regimes) exists in which
  - regime I : normal price levels – diffusive behavior
  - regime II : high price levels – spiky behavior

Regime Switching Models

- Write as usual $S_t = e^{g_t + X_t}$, g_t contains predictable effects
- $Z_t \in \{0, 1\}$ denotes the "world-state"
- Z_t evolves according to a continuous time Markov chain with generator matrix $\mathbf{A}$, i.e.

$$\mathbb{P}(Z_T = i | Z_t = j) = (\exp\{\mathbf{A}(T - t)\})_{ji}$$

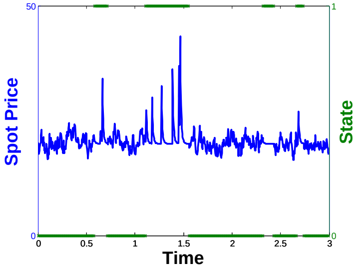
- X_t satisfies SDE:

$$\begin{aligned} dX_t &= (1 - Z_t)\left[-\kappa\, X_t\, dt + \sigma\, dW_t\right] \\ &\quad + Z_t\left[-\kappa\, X_{t-}\, dt + dJ_t\right] \end{aligned}$$

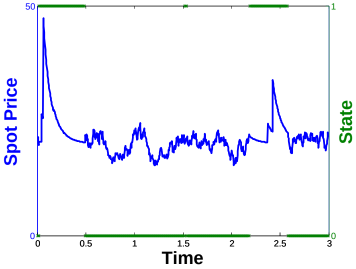
where W_t is a Brownian motion and J_t is a compound Poisson process

Regime Switching Models

Sample paths from basic regime switching model



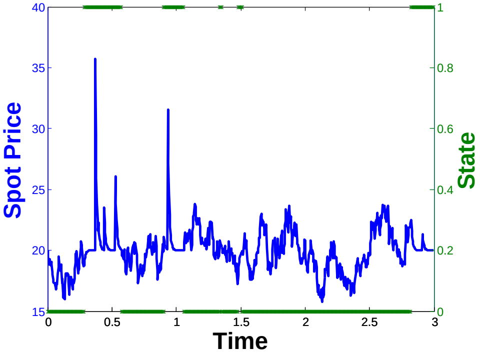
(a) $\kappa = 100$



(b) $\kappa = 10$

Regime Switching Models

Sample paths from regime switching model with $\kappa_1 = 100$ and $\kappa_0 = 10$:



Regime Switching Models

De Jong & Huisman (2003) model

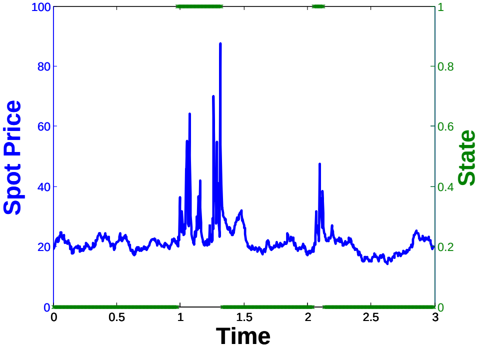
- In this model, the price switches between two independent processes $X_t = (1 - Z_t) X_t^D + Z_t\, X_t^J$

$$\begin{aligned} dX_t^D &= -\kappa_D\, X_t^D\, dt + \sigma\, dW_t \\ dX_t^J &= -\kappa_J X_{t-}^J + dJ_t \end{aligned}$$

- Both processes are always evolving; however, only one is observed at any point in time
- Price spikes may be generated by plant going down and upon recovery prices return to normal

Regime Switching Models

De Jong & Huisman (2003) model



Threshold Models

Geman & Roncoroni (2006) model – not a regime model

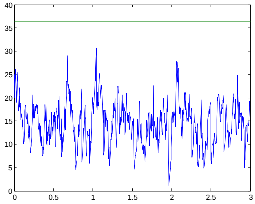
- In this model, spikes are affected when prices cross a threshold

$$dX_t = -\kappa\, X_{t-}\, dt + \sigma\, dW_t + h_t\, dJ_t$$

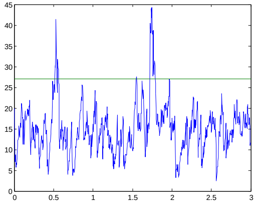
$$h_t = \begin{cases} +1 & , X_t \geq \overline{X} \\ -1 & , X_t < \overline{X} \end{cases}$$

- When prices cross from below, spikes are positive
- When prices cross from above, spikes are negative
- Mean-reversion is calibrated from data to be high – around 50.

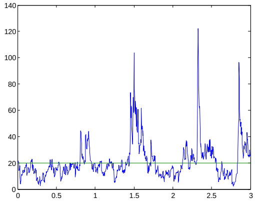
Threshold Models



(c) high threshold



(d) moderate threshold



(e) low threshold