ACT348F - TERM TEST 2 - NOVEMBER 21, 2001

Print your name and student number clearly on each page

This is a closed book test. Do all work on this question paper, with work for question 1 on this page (use both sides if necessary), work for question 2 on the next page, etc. All calculations should be done to at least 6 digit accuracy. Answers are expected to have at least 4 digit accuracy.

1. A fully discrete 2-year endowment insurance issued at age x has a death benefit equal to $1000+$ Terminal Reserve: $b_1 = 1000 + {}_1V$, $b_2 = 1000 + {}_2V$. The endowment benefit paid to survivors at age $x+2$ is 1000. You are given that the annual effective interest rate is $i = .1$, and $q_x = .2$, $q_{x+1} = .3$.

(a) Show that the annual benefit premium for the policy is 658 .

(b) Find ${}_1V$ and ${}_2V$.

(c) Formulate the unconditional 2nd year net cash loss random variable, C_1 .

BONUS: Formulate C_0 and show that the unconditional expectation $E[C_0 + vC_1] = 0$.

Solution: (a) $\pi \ddot{a}_{\overline{n|}} = B \cdot \sum_{h=0}^{n-1} v^{h+1} q_{x+h} + v^n \, {}_nV$

$\rightarrow \pi(1+v) = 1000(vq_x + v^2 q_{x+1}) + 1000v^2 \rightarrow 1.9091\pi = 1,256.20 \rightarrow \pi = 658$.

Or, $658(1+v) = 1256.2$ and $1000(vq_x + v^2 q_{x+1}) + 1000v^2 = 1256.2$;
since they are equal $\pi = 658$ is the benefit premium.

(b) $\pi(1+i) - (b_1 - {}_1V)q_x = {}_1V$,
$\rightarrow 658(1.1) - (1000 + {}_1V - {}_1V)(.2) = {}_1V$,
$\rightarrow 523.8 = {}_1V$.
$(\pi + {}_1V)(1+i) - (b_2 - {}_2V)q_{x+1} = {}_2V$,
$\rightarrow (523.8 + 658)(1.1) - (1000 + {}_2V - {}_2V)q_{x+1} = {}_2V$
$\rightarrow {}_2V = 1000$ (the endowment amount).

(c) $$C_1 = \begin{cases} 0 & K(x) = 0\,,\ q_x = .2 \\ b_2 \cdot v - \pi = 2000v - 658 = 1160.2 & K(x) = 1\,,\ {}_{1|}q_x = (.8)(.3) = .24 \\ 1000v - \pi = 251.2 & K(x) \geq 2\,,\ {}_2p_x = (.8)(.7) = .56 \end{cases}$$

BONUS: $$C_0 = \begin{cases} b_1 v - \pi = 1523.8v - 658 = 727.27 & K(x) = 0\,,\ q_x = .2 \\ -\pi = -658 & K(x) \geq 1\,,\ p_x = .8 \end{cases}$$

$E[C_0] = (727.3)(.2) + (-658)(.8) = -381$
$E[C_1] = (0)(.2) + (1160.2)(.24) + (251.1)(.56) = 419$
$E[C_0 + vC_1] = -381 + 419v = 0$.

2. The survival function for the joint distribution of $T(x)$ and $T(y)$ is
$s_{T(x),T(y)}(s,t) = P[(T(x) > s) \cap (T(y) > t)] = e^{-.04s}(1 - .01t)$
for $s > 0$, $0 < t \leq 100$.

(a) For the pair of lives x and y, find ${}_{1|1}q_{xy}$.

(b) Find the force of mortality for x at age $x+s$, $\mu_x(s)$.

(c) Find $q_{\overline{xy}}$.

(d) Find $Cov[T(x), T(y)]$.

Solution:

(a) ${}_{1|1}q_{xy} = p_{xy} - {}_2p_{xy} = s_{T(x),T(y)}(1,1) - S_{T(x),T(y)}(2,2)$
$= e^{-.04(1)}(1 - .01(1)) - e^{-.04(2)}(1 - .01(2)) = .9512 - .9047 = .0465$ .15.

(b) $s_{T(x)}(s) = s_{T(x),T(y)}(s,0) = e^{-.04s}$
$\rightarrow \mu_x(s) = -\frac{d}{ds} ln[s_{T(x)}(s)] = -\frac{d}{ds}(-.04s) = .04$.

(c) $q_{\overline{xy}} = q_x + q_y - q_{xy} = [1 - p_x] + [1 - p_y] - [1 - p_{xy}]$
$= [1 - s_{T(x),T(y)}(1,0)] + [1 - s_{T(x),T(y)}(0,1)] - [1 - s_{T(x),T(y)}(1,1)]$
$= [1 - e^{-.04}] + [1 - .99] - [1 - e^{-.04}(.99)] = .00039$.

(d) $S_{T(y)}(t) = s_{T(x),T(y)}(0,t) = 1 - .01t$
$\rightarrow$ $s_{T(x),T(y)}(s,t) = S_{T(x)}(s) \cdot S_{T(y)}(t) \rightarrow T(x)$ and $T(y)$ are independent

$\rightarrow$ $Cov[T(x), T(y)] = 0$.

3. Smith will be retiring at age 65 and is eligible to receive a pension annuity starting at age 65. The annuity will pay $50,000 at the beginning of each year as long as Smith survives (first payment at age 65). Smith is married and his wife is 5 years younger than he is. Smith considers some alternative arrangements for his pension benefit.
You are given that $i = .04$, $\ddot{a}_{60} = 12$, $\ddot{a}_{65} = 10$, $\ddot{a}_{60:65} = 8$.
The following assumes that Smith and his wife are both alive when Smith turns 65 when his retirement benefit is to start. Equivalent benefits are based on actuarial equivalence.
(a) Smith considers a retirement annuity-due with payment of K per year as long as either he or his wife is alive. Find K.
(b) Smith considers a retirement annuity-due with payment of C per year while both he and his wife are alive, and with the payment reducing to $.75C$ after the first death, and continuing at that level until the second death. Find C.
(c) Smith considers the following combination of benefits:
(i) annuity-due of R per year while both he and his wife are alive, reducing to $\frac{1}{2}R$ after the first death, and continuing at that level until the second death, combined with
(ii) insurance payment of $100,000 at the time of the first death.
Find R.
Solution: APV of original benefit is $50,000\ddot{a}_{65} = 500,000$
(a) $500,000 = K\ddot{a}_{\overline{60:65}} = K[\ddot{a}_{60} + \ddot{a}_{65} - \ddot{a}_{60:65}] = K[12 + 10 - 8] \rightarrow K = 35,714$.

(b) $500,000 = C\ddot{a}_{60:65} + .75C(\ddot{a}_{60} - \ddot{a}_{60:65}) + .75C(\ddot{a}_{65} - \ddot{a}_{60:65})$
$= 8C + .75C(12 - 8) + .75C(10 - 8) \rightarrow 12.5C = 500,000 \rightarrow C = 40,000$.

(c) $500,000 = 100,000A_{60:65} + R\ddot{a}_{60:65} + .5R(\ddot{a}_{60} - \ddot{a}_{60:65}) + .5R(\ddot{a}_{65} - \ddot{a}_{60:65})$
$A_{60:65} = 1 - d\ddot{a}_{60:65} = .69231$
$\rightarrow 500,000 = 69,231 + 8R + .5R(12 - 8) + .5R(10 - 8) \rightarrow R = 39,161$.