<u>ACT348F - TERM TEST 1 - OCTOBER 18, 2003</u>

<u>Print your name and student number clearly on each page</u>

Do all work on this question paper, with work for question 1 on this page (use both sides if necessary), work for question 2 on the next page, etc. All calculations should be done to <u>at least 6 digit accuracy</u>. Answers are expected to have at least 4 digit accuracy.
<u>THIS IS A CLOSED BOOK TEST - NO BOOKS OR NOTES ARE ALLOWED.</u>
A calculator may be used

1. A 4-year fully discrete term insurance with face amount 1000 is issued at age x (premiums are scheduled for the lifetime of the policy). The effective annual interest rate is $i=25\%$, and the mortality probabilities are
$q_x = .20$, $q_{x+1} = .25$, $q_{x+2} = .40$, $q_{x+3} = .50$.
The equivalence principle annual premium is $1000P^{\,1}_{x:\overline{4}|} = 219.45$.

3 (a) Formulate the 2nd year terminal prospective loss random variable (conditional distribution).

2 (b) Find the 2nd year terminal benefit reserve as the expected value of the loss random variable in part (a).

4 (c) Write the prospective form of the 2nd year terminal benefit reserve $1000\,{}_2V^{\,1}_{x:\overline{4}|}$, and calculate it by calculating each of the factors in the expression (the premium is given above).

4 (d) Write the retrospective form of the 2nd year terminal reserve , and calculate it by calculating each of the factors in the expression.

Solution: (a)

$${}_2L = \begin{cases} 1000v - P = 580.55 & \text{Prob. } q_{x+2} = .4 \\ 1000v^2 - P(1+v) = 245.00 & \text{Prob. } {}_{1|}q_{x+2} = (.6)(.5) = .3 \\ -P(1+v) = -395.00 & \text{Prob. } {}_2p_{x+2} = (.60(.5) = .3 \end{cases}$$

(b) ${}_2V = E[{}_2L] = (580.55)(.4) + (245.00)(.3) + (-395.00)(.3) = 187.22$.

(c) $1000\,{}_2V^{\,1}_{x:\overline{4}|} = 1000A^{\,1}_{x+2:\overline{2}|} - 1000P^{\,1}_{x:\overline{4}|} \cdot \ddot{a}_{x+2:\overline{2}|}$.
$A^{\,1}_{x+2:\overline{2}|} = vq_{x+2} + v^2\,{}_{1|}q_{x+2} = \frac{.4}{1.25} + \frac{(.6)(.5)}{(1.25)^2} = .512$, $\ddot{a}_{x+2:\overline{2}|} = 1 + vp_{x+2} = 1.48$.
$1000\,{}_2V^{\,1}_{x:\overline{4}|} = 1000(.512) - (219.45)(1.48) = 187.21$.

(d) $1000\,{}_2V^{\,1}_{x:\overline{4}|} = 1000P^{\,1}_{x:\overline{4}|} \cdot \ddot{s}_{x:\overline{2}|} - 1000 \cdot \frac{A^{\,1}_{x:\overline{2}|}}{v^2\,{}_2p_x}$.
$v^2\,{}_2p_x = \frac{(.8)(.75)}{(1.25)^2} = .384$, $\ddot{s}_{x:\overline{2}|} = \frac{\ddot{a}_{x:\overline{2}|}}{v^2\,{}_2p_x} = \frac{1+vp_x}{v^2\,{}_2p_x} = \frac{1+\frac{.8}{1.25}}{.384} = 4.2708$,
$A^{\,1}_{x:\overline{2}|} = vq_x + v^2\,{}_{1|}q_x = \frac{.2}{1.25} + \frac{(.8)(.25)}{(1.25)^2} = .288$.
$1000\,{}_2V^{\,1}_{x:\overline{4}|} = (219.45)(4.2708) - 1000 \cdot \frac{.288}{.384} = 187.23$.

6 2. The following derivation of the algebraic equivalence between the t-th year terminal prospective and restrospective reserves for a fully discrete whole life insurance of 1 was given in class:

Step 1: $A_x = P_x \cdot \ddot{a}_x$

Step 2: $A^{1}_{x:\overline{t}|} + v^t\, {}_tp_x\, A_{x+t} = P_x[\, \ddot{a}_{x:\overline{t}|} + v^t\, {}_tp_x\, \ddot{a}_{x+t}]$

Step 3: $v^t\, {}_tp_x\, [A_{x+t} - \, P_x\, \ddot{a}_{x+t}] = P_x\, \ddot{a}_{x:\overline{t}|} - A^{1}_{x:\overline{t}|}$

Step 4: $A_{x+t} - \, P_x\, \ddot{a}_{x+t} = P_x\, \ddot{s}_{x:\overline{t}|} - \frac{A^{1}_{x:\overline{t}|}}{v^t\, {}_tp_x}$ (prospective $=$ retrospective)

Apply the same sequence of steps to derive the equivalence of the prospective and restrospective reserve for a fully discrete n-year deferred annuity-due of 1 per year whole life insurance of 1, $({}_tV({}_{n|}\ddot{a}_x)$) . This requires two separate derivations, one for $t < n$ and one for $t \geq n$. An alternative valid derivation is acceptable.

Solution: Case 1: $t < n$

Step 1: ${}_{n|}\ddot{a}_x = P({}_{n|}\ddot{a}_x) \cdot \ddot{a}_{x:\overline{n}|}$

Step 2: $v^t\, {}_tp_x\, {}_{n-t|}\, \ddot{a}_{x+t} = P \cdot [\, \ddot{a}_{x:\overline{t}|} + v^t\, {}_tp_x\, \ddot{a}_{x+t:\overline{n-t}|}]$

Step3: $v^t\, {}_tp_x\, [\, {}_{n-t|}\, \ddot{a}_{x+t} - P \cdot \ddot{a}_{x+t:\overline{n-t}|}] = P \cdot\, \ddot{a}_{x:\overline{t}|}$

Step 4: ${}_{n-t|}\, \ddot{a}_{x+t} - P \cdot \ddot{a}_{x+t:\overline{n-t}|} = P \cdot\, \ddot{s}_{x:\overline{t}|}$ (prospective $=$ retrospective)

Case 2: $t \geq n$

Step 1: ${}_{n|}\ddot{a}_x = P({}_{n|}\ddot{a}_x) \cdot \ddot{a}_{x:\overline{n}|}$

Step 2: $v^n\, {}_np_x\, \ddot{a}_{x+n:\overline{t-n}|} + v^t\, {}_tp_x\, \ddot{a}_{x+t} = P({}_{n|}\ddot{a}_x) \cdot \ddot{a}_{x:\overline{n}|}$

Step 3: $v^t\, {}_tp_x\, \ddot{a}_{x+t} = P({}_{n|}\ddot{a}_x) \cdot \ddot{a}_{x:\overline{n}|} - v^n\, {}_np_x\, \ddot{a}_{x+n:\overline{t-n}|}$

Step 4: $\ddot{a}_{x+t} = P({}_{n|}\ddot{a}_x) \cdot\, \frac{\ddot{a}_{x:\overline{n}|}}{v^t\, {}_tp_x} - \, \frac{v^n\, {}_np_x}{v^t\, {}_tp_x} \cdot \ddot{a}_{x+n:\overline{t-n}|}$

The left-hand side is the prospective reserve at $t \geq n$, and the right hand-side can be written as

$P({}_{n|}\ddot{a}_x) \cdot\, \frac{\ddot{a}_{x:\overline{n}|}}{v^n\, v^{t-n}\, {}_np_x \cdot {}_{t-n}p_{x+n}} - \, \frac{v^n\, {}_np_x}{v^n\, v^{t-n}\, {}_np_x \cdot {}_{t-n}p_{x+n}} \cdot \ddot{a}_{x+n:\overline{t-n}|}$,

which then becomes $P({}_{n|}\ddot{a}_x) \cdot\, \frac{\ddot{s}_{x:\overline{n}|}}{v^{t-n} \cdot {}_{t-n}p_{x+n}} - \ddot{s}_{x+n:\overline{t-n}|}$, the retrospective form.

3. Mortality follows DeMoivre's Law with $\omega = 100$.
Effective annual interest is at a rate of 10% .
Find the specified terminal benefit reserves for each of the following policies, all issued at age 50 with face amount 1000.

3 (i) 20-th year reserve for fully discrete whole life insurance of $1000 .

3 (ii) 20-th year reserve for fully continuous whole life insurance of $1000.

(Hint: Recall that under DeMoivre's Law $A_x = \frac{1}{\omega-x} \cdot a_{\overline{\omega-x}|}$, and $A^{1}_{x:\overline{n}|} = \frac{1}{\omega-x} \cdot a_{\overline{n}|}$, and you may use other relationships that are valid under DeMoivre's Law).

Solution: (i) $1000\,{}_{20}V_{50} = 1000 \cdot \frac{A_{70}-A_{50}}{1-A_{50}}$.
$A_{50} = \frac{1}{50} \cdot a_{\overline{50}|} = \frac{1}{50}(\frac{1-v^{50}}{i}) - .1983$, $A_{70} = \frac{1}{30} \cdot a_{\overline{30}|} = \frac{1}{30}(\frac{1-v^{30}}{i}) - .3142$.
$1000\,{}_{20}V_{50} = 144.57$.

(ii) Under DeMoivre's Law $\overline{A}_x = \frac{1}{\omega-x} \cdot \overline{a}_{\overline{\omega-x}|}$, so that
$\overline{A}_{50} = \frac{1}{50} \cdot \overline{a}_{\overline{50}|} = \frac{1}{50}(\frac{1-v^{50}}{\delta}) - .2081$, $\overline{A}_{70} = \frac{1}{30} \cdot \overline{a}_{\overline{30}|} = \frac{1}{30}(\frac{1-v^{30}}{\delta}) - .3297$.
Then $1000\,{}_{20}\overline{V}(\overline{A}_{50}) = 1000 \cdot \frac{\overline{A}_{70}-\overline{A}_{50}}{1-\overline{A}_{50}} = 153.55$.