LIFE CONTINGENCIES - PROBLEM SET 6

Bowers, Chapters 9

1. If for two independent lives, each (x), the prob. that the last survivor status will survive at least n years is seven times the prob. that the joint status will survive at least n years then ${}_nq_x$ =
A) 0 B) .25 C) .50 D) .75 E) .90

2. If the force of mortality is constant with value $\mu(z) = \mu$ for all z, what is $\frac{Var[T(\overline{xy})]}{Var[T(xy)]}$ where $T(x)$ and $T(y)$ are independent?
A) 5 B) 4 C) 3 D) 5μ E) 4μ

3. Given ${}_{20}p_{60} = .478$ and ${}_{20}p_{70} = .160$, the probability that of two independent individuals aged 60 and 70 on Jan. 7, 2000, exactly one will die before Jan. 7, 2020 is
A) .475 B) .485 C) .495 D) .505 E0 .515

4. If $\ell_{75+t} = \ell_{75} - t$ for $0 \leq t \leq 5$ and ${}_5q_{75} = .35$, then for two independent lives, $e_{\overline{75:75}:\overline{5|}}$ =
A) 4.21 B) 4.39 C) .456 D) 4.73 E) 4.90

5. The actuarial present value of an annuity-immediate that pays \$1000 per annum while (65) and (70) are both alive, \$800 per annum while only (65) is alive and \$500 per annum while only (70) is alive is \$10,390. For independent lives, if $a_{65} = 10.553$ and $a_{70} = 8.780$, then $a_{65:70}$ =
A) 8.10 B) 8.14 C) 8.18 D) 8.22 E) 8.26

6. A continuous annuity of 1 per year is to start 20 years from now and be payable so long as either (40) or (50) is alive, provided that both had lived to age 55 . What is the net single premium for this annuity? Assume that (40) and (50) are independent lives.

A) ${}_{20|}\bar{a}_{\overline{40:50}}$ B) ${}_5p_{50} \cdot {}_{20|}\bar{a}_{40} + {}_{20|}\bar{a}_{55} - {}_{20|}\bar{a}_{40}$ C) ${}_{20|}\bar{a}_{40} + {}_{15}p_{40} \cdot {}_{20|}\bar{a}_{55} - {}_{20|}\bar{a}_{40:50}$

D) ${}_5p_{50} \cdot {}_{20|}\bar{a}_{40} + {}_{15}p_{40} \cdot {}_{20|}\bar{a}_{50} - {}_{20|}\bar{a}_{40:50}$

E) ${}_5p_{50} \cdot {}_{20|}\bar{a}_{40} + {}_{15}p_{40} \cdot {}_{20|}\bar{a}_{50} - {}_5p_{50} \cdot {}_{15}p_{40} \cdot {}_{20|}\bar{a}_{40:50}$

7. The joint distribution function of T(x) and T(y) is $F_{T(x)T(y)}(s,t) = .000125(2s^2t + st^2)$ on the rectangle $0 \leq s \leq 10$ and $0 \leq t \leq 20$, and the density function for the joint distribution is 0 outside the rectangle. Find ${}_5p_{\overline{xy}} - {}_5p_{xy}$.

A) 1/32 B) 3/16 C) 5/16 D) 3/8 E) 7/16

8. A joint distribution of survival time for two lives incorporates a common shock. Two lives are being considered, and it is assumed that in the absence of the independent common shock, each one of the pair of independent lives follows a DeMoivre survival law with $\omega = 100$. The individuals are currently at ages $x = 40$ and $y = 50$. It is assumed that the time until common shock has an exponential distribution with $\lambda = .01$. Find ${}_{10|10}q_{40:50}$.

A) .210 B) .232 C) .254 D) .276 E) .298

9. If ${}_nq_x = .2$, ${}_nq_y = .3$ and ${}_nq^{\,1}_{xy} = .18$ then ${}_nq^{\,2}_{xy} =$

A) .12 B) .10 C) .08 D) .06 E) .04

10. The probability that (60) will die after (70) is .7 and the probability that (60)'s age at death will exceed (70)'s age at death is .4 . What is the probability that two lives aged 60 will die within 10 years of each other?

A) .65 B) .52 C) .39 D) .26 E) .13

11. A husband and wife are both (50). They set up a one year term insurance policy with the following benefits payable at the end of the year:
(i) if both die within the year, the benefit is $1,000 if the husband dies first and $500 if the wife dies first
(ii) if only one of them dies within the year, the benefit is $1000 if it was the wife and $500 if it was the husband who dies. Find the single benefit premium (within $1) for this policy if the wife and husband have the same mortality characteristics, $q_{50} = .09$, and $i = .10$.
A) 111 B) 113 C) 115 D) 117 E) 119

12. If $q^{\;1}_{40:41} = .0945$ and $q^{\;\;\;2}_{40:41} = .0055$, what is $q_{41} - q_{40}$, assuming U.D.D. for single lives?
A) .005 B) .010 C) .015 D) .020 E) .025

13. Actuary A assumes that $T(x)$ and $T(y)$ are independent lives with constant forces of mortality of $\mu_x = .02$ and $\mu_y = .03$. Based on a force of interest of 5%. Actuary A calculates the Single benefit premium for an insurance of 1000 paid at the moment of (y)'s death, if (y) dies after (x) (no benefit is paid if (y) dies before (x).
Actuary B uses a common shock model with $T^*(x)$ having constant force .01, $T^*(y)$ having constant force .02, and common shock having force .01. Using the same force of interest of 5%, Actuary B calculates the single benefit premium for the same insurance policy considered by Actuary A. Find the absolute difference between the two premiums.
A) 11.11 B) 22.22 C) 33.33 D) 44.44 E) 55.55

OLD EXAM QUESTIONS

14. (SOA) Which of the following is a correct expression for the probability that the second death of independent lives (x) and (y) will occur in year 6?
A) ${}_{5|}q_x \cdot {}_5q_y + {}_{5|}q_y \cdot {}_5q_x - {}_{5|}q_x \cdot {}_{5|}q_y$ B) ${}_{5|}q_x + {}_5q_y - {}_{5|}q_x \cdot {}_{5|}q_y$ C) $1 - {}_5p_{\overline{xy}}$
D) ${}_5p_{\overline{xy}} \cdot {}_{5|}q_{\overline{x+5:y+5}}$ E) ${}_5p_{\overline{xy}} - {}_6p_{\overline{xy}}$

15. (SOA) You are given:
(i) Mortality follows DeMoivre's law with $\omega = 100$.
(ii) x and y are independent lives both age 90.
Calculate the probability that the last survivor of x and y will die between ages 95 and 96.
A) 0.05 B) 0.06 C) 0.10 D) 0.11 E) 0.20

16. (SOA) A fully discrete last-survivor insurance of 1 is issued to two independent lives each age x. Annual benefit premiums are reduced by 25% after the first death. You are given:
(i) $A_x = 0.4$ (ii) $A_{xx} = 0.55$ (iii) $\ddot{a}_x = 10.0$
Calculate the initial annual benefit premium.
A) 0.019 B) 0.020 C) 0.022 D) 0.024 D) 0.025

17. (SOA) A fully continuous insurance policy is issued to (x) and (y). A death benefit of 10,000 is payable upon the second death. The premium is payable continuously until the last death. The rate of annual premium is K while (x) is alive and reduces to $0.5K$ upon the death of (x) if (x) dies before (y). You are given:
(i) $\delta = 0.05$ (ii) $\overline{a}_x = 12$ (iii) $\overline{a}_y = 15$ (iv) $\overline{a}_{xy} = 10$
Calculate K.
A) 79.61 B) 86.19 C) 88.24 D) 93.75 E) 103.45

18. (SOA) S is the actuarial present value of a continuous annuity of 1 per annum payable while at least one of (30) and (45) is living, but not if (30) is alive and under age 40. Which of the following is equal to S?
I. $\overline{a}_{45} + \overline{a}_{30} - \overline{a}_{30:45:\overline{10|}}$ II. $\overline{a}_{45} + {}_{10|}\overline{a}_{30} - \overline{a}_{30:45}$
III. $\overline{a}_{45} + {}_{10|}\overline{a}_{30} - {}_{10|}\overline{a}_{30:45}$ IV. $\overline{a}_{45} + {}_{10|}\overline{a}_{30} - \overline{a}_{30:45:\overline{10|}}$
A) None B) I only C) II only D) III only E) IV only

19. (SOA) A temporary life annuity-due of 1 is payable to (x) as long as (x) lives jointly with (y), and for 10 years after the death of (y), provided (x) is still alive. In no event will payments be made after 20 years. You are given:

(i) $\ddot{a}_x = 15$ (ii) ${}_{10|}\ddot{a}_x = 8$ (iii) $\ddot{a}_y = 16$

(iv) $\ddot{a}_{y:\overline{10|}} = 7$ (v) $\ddot{a}_{\overline{y:y:\overline{10|}}} = 8$ (vi) (y) is 10 years older than (x)

(vii) (x) and (y) are independent lives.

Calculate the actuarial present value of this annuity.

A) 10 B) 11 C) 12 D) 13 E) 14

20. (SOA) (x) and (y) purchase a joint-and-survivor annuity-due with an initial monthly benefit amount equal to 500. You are given:

(i) If (x) predeceases (y), the benefit amount changes to 300 per month.

(ii) If (y) predeceases (x), the benefit amount changes to B per month.

(iii) The annuity is actuarially equivalent to a single life annuity-due on (x) with a monthly benefit amount equal to B.

(iv) $\ddot{a}_x^{(12)} = 10$ (v) $\ddot{a}_y^{(12)} = 14$ (iii) $\ddot{a}_{xy}^{(12)} = 8$

Calculate B.

A) 520 B) 680 C) 725 D) 800 E) 1025

21. (SOA) Tom, Dick and Harry have the same birthday, and their current ages are 30,31 and 32, respectively. What is the probability that they will not all be alive in their 40's simultaneously? Assume all lives are independent and have the same mortality function ${}_tp_x = 1 - \frac{t}{100-x}$.

A) Less than .34 B) At least .34, but less than .35 C) At least .35, but less than .36

D) At least .36, but less than .37 E) At least .37

22. (SOA) For two independent lives (x) and (y), you are given:

(i) $\mu_x = 0.1$ (ii) $\mu_y = 0.15$ (iii) $\delta = 0.05$

Calculate $\overline{P}_{\overline{xy}}$, the annual benefit premium for a life insurance payable at the moment of the second death, and with premiums payable continuously while either (x) or (y) is alive. Assume that the lives are independent

A) Less than .08 B) At least .08, but less than .12 C) At least .12, but less than .16

D) At least .16, but less than .20 E) At least .20

23. (CAS) Given:

$_3p_{40} = .990$, $_6p_{40} = .980$, $_9p_{40} = .965$, $_{12}p_{40} = .945$, $_{15}p_{40} = .920$, $_{18}p_{40} = .890$

For two independent lives aged 40, calculate the probability that the first death occurs after 6 years, but before 12 years.

A) Less than 0.050 B) At least 0.050, but less than 0.055

C) At least 0.055, but less than 0.060 D) At least 0.060, but less than 0.065

E) At least 0.065

24. (CAS) For a special fully-continuous last-survivor insurance of 1 on (x) and (y), you are given:

i) $\delta = 0.05$ ii) $T(x)$ and $T(y)$ are independent

iii) $\mu(x+t) = \mu(y+t) = 0.07$, for $t > 0$

iv) Premiums are payable until the first death.

Calculate the level annual benefit premium.

A) Less than 0.050 B) At least 0.050, but less than 0.075

C) At least 0.075, but less than 0.100 D) At least 0.100, but less than 0.125

E) At least 0.150

25. (CAS) Twins age (30) purchase a fully continuous joint life annuity along with a provision for joint life insurance. Their future lifetimes are independent and identically distributed.

Given: $\delta = 0.05$

$\mu_x(t) = 0.04$ for all x and t

The special annuity (with insurance provision) says:

1,000 per year while both are alive,

1,000 at the moment of the first death,

600 per year after the first death until the second death, and

800 at the moment of the second death.

Calculate the actuarial present value of this specially annuity (with insurance provision).

A) Less than 12,700 B) At least 12,700, but less than 14,200

C) At least 14,200, but less than 15,700 D) At least 15,700, but less than 17,200

E) At least 17,200

26. (CAS) For John, currently 30 years old, the force of mortality is $\mu_x = \frac{1}{100-x}$.
For Bob, an independent life also 30 years old, it is also known that ${}_{10}p_{30} = .94$ and ${}_{5}p_{35} = .96$.
Calculate the probability that at least one of John or Bob will die within 5 years.
A) Less than 0.0895 B) At least 0.0895, but less than 0.0905
C) At least 0.0905, but less than 0.0915 D) At least 0.0915, but less than 0.0925
E) At least 0.0925

27. (CAS) The force of mortality is $\mu_x = \frac{1}{100-x}$ for $0 \leq x < 100$.
For independent lives 40 and 50 calculate ${}_{10}p_{\overline{40:50}}$, the probability that at least one survives 10 years.
A) Less than 0.66 B) At least 0.66, but less than 0.76
C) At least 0.76, but less than 0.86 D) At least 0.86, but less than 0.96
E) At least 0.96

28. (CAS May 05) John, age 40, and Mary, age 50, are independent lives following the same mortality, as follows.

Age (x)	${}_{10}q_x$
40	0.039
50	0.085
60	0.192

Calculate the probability that John and Mary both live at least 10 years and then both die during the following 10 years.
A) Less than 0.015 B) At least .015, but less than 0.095
C) At least 0.095, but less than 0.175 D) At least 0.175, but less than 0.255
E) 0.255 or more

29. (CAS May 05) Assuming that the time-until-death random variables for (x) and (y) are independent, which of the following are true?
1. $T(xy) + T(\overline{xy}) = T(x) + T(y)$ 2. ${}_tp_{xy} + {}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y$ 3. $A_{xy} + A_{\overline{xy}} = A_x + A_y$
A) 1. only B) 3. only C) 1. and 2. only D) 2. and 3. only E) 1., 2. and 3.

30. (CAS Nov 05) You are given:

- $\mu_x = \frac{1}{100-x}$ for $x < 100$
- The future lifetimes of (45) and (50) are independent.

Calculate ${}_5p_{\overline{45:50}}$, the probability that the last-survivor status of (45) and (50) survives 5 years.

A) Less than .80 B) At least .80, but less than .85 C) At least .85, but less than .90

D) At least .90, but less than .95 E) At least .95

31. (CAS Nov 05) A new communications device uses 2 independent, non-rechargeable batteries. Failure of either of the batteries results in device failure. The device has a 15-year warranty against failure. Each battery has the following probability of survival to age t, where t is measured in years:

- ${}_tp_0 = 1 - 0.02t$ for $t < 50$
- ${}_tp_0 = 0$ elsewhere

Calculate the probability that the device fails before the warranty runs out.

A) Less than .10 B) At least .10, but less than .30 C) At least .30, but less than .50

D) At least .50, but less than .70 E) At least .70

32. (CAS Nov 05) For two independent lives, both age x, you are given

$${}_tp_x = (1 - \tfrac{t}{20})^2 \text{ for } t < 20.$$

Calculate the difference in the expected time between the first and second death.

A) Less than 5 B) At least 5, but less than 7 C) At least 7, but less than 9

D) At least 9, but less than 11 E) At least 11

33. (CAS May 06) For two independent lives, (x) and (y), the probability density function for death at time t is defined as: $f(t) = c(5+t)$ for $0 \le t \le 20$ and constant c.

Calculate the probability that (x) and (y) are both alive at $t = 5$.

A) Less than 0.725 B) At least 0.725, but less than 0.750

C) At least 0.750, but less than 0.775 D) At least 0.775, but less than 0.800

E) At least 0.800

34. (CAS May 06) Gadgets, Inc. manufactures a device that uses two non-rechargeable, non-replaceable batteries. The company backs the product with a guarantee that if the device fails in the first three years after purchase, Gadgets, Inc. will pay the consumer \$100 at the end of the year in which the device failed. The device will fail if either battery fails. Each battery has a probability of failure according to the following mortality table:

Age (x)	q_x
0	0.03
1	0.09
2	0.14
3	0.18

The interest rate of 5%. Calculate the actuarial present value of the warranty.

A) Less than \$20 B) At least \$20, but less than \$30

C) At least \$30, but less than \$40 D) At least \$40, but less than \$50

E) At least \$50

35. (CAS May 06) The force of mortality is given as: $\mu(x) = \frac{1}{100-x}$ for $0 \leq x < 100$.
Calculate the probability that exactly one of the lives (40) and (50) will survive 10 years.

A) 9/30 B) 10/30 C) 19/30 D) 20/30 E) 29/30

36. (CAS May 07) A joint annuity-due on independent lives (x) and (y) pays 1 per month with certainty for 5 years. After 5 years, it pays 1 per year as long as both (x) and (y) are alive, and it pays $\frac{1}{2}$ per year as long as either (x) is alive and (y) is dead or (y) is alive and (x) is dead.

You are given:

- $i = 6\%$
- $\ddot{a}_x = 14.817$
- $\ddot{a}_{x:\overline{5}|} = 4.440$
- $\ddot{a}_y = 13.267$
- $\ddot{a}_{y:\overline{5}|} = 4.411$
- $\ddot{a}_{xy} = 12.478$
- $\ddot{a}_{xy:\overline{5}|} = 4.387$

Calculate the actuarial present value of this annuity.

A) Less than 10.0 B) At least 10.0, but less than 11.5 C) At least 11.5, but less than 13.0

D) At least 13.0, but less than 14.5 E) At least 14.5

37. (CAS May 07) For independent lives (x) and (y), ${}_tp_x = {}_tp_y = 1 - \frac{t}{100}$ for $0 \leq t \leq 100$.
Calculate $Var[T(xy)]$.
A) Less than 560 B) At least 560, but less than 570
C) At least 570, but less than 580
D) At least 580, but less than 590 E) At least 590

38. (CAS May 07) For two independent lives (x) and (y), you are given:
- ${}_tp_x = e^{-\mu t}$ for $t > 0$
- ${}_tp_y = e^{-\alpha t}$ for $t > 0$

Calculate the covariance of $T(xy)$ and $T(\overline{xy})$.
A) $\frac{1}{\alpha+\mu}$ B) $\frac{\alpha\mu}{\alpha+\mu}$ C) $\frac{1}{(\alpha+\mu)^2}$ D) $\frac{\alpha\mu}{(\alpha+\mu)^2}$ E) $\frac{\alpha\mu-(\alpha+\mu)}{\alpha\mu(\alpha+\mu)}$

LIFE CONTINGENCIES - PROBLEM SET 6 SOLUTIONS

1. ${}_np_{\overline{xy}} = 7 \cdot {}_np_{xy} \rightarrow 2 \cdot {}_np_x - {}_np_x^2 = 7 \cdot {}_np_x^2 \rightarrow 8 \cdot {}_np_x^2 - 2 \cdot {}_np_x = 0 \rightarrow {}_np_x = 0$ or $\frac{1}{4}$
$\rightarrow {}_nq_x = \frac{3}{4}$. Answer: D.

2. $T(xy) = \min[T(x), T(y)]$ has an exponential distribution with mean $\frac{1}{\mu+\mu} = \frac{1}{2\cdot\mu}$
$\rightarrow Var[T(xy)] = \frac{1}{(2\cdot\mu)^2} = \frac{1}{4\cdot\mu^2}$. $Var[T(\overline{xy})] = 2 \cdot \int_0^\infty t \cdot {}_tp_{\overline{xy}}\, dt - (\mathring{e}_x + \mathring{e}_y - \mathring{e}_{xy})^2$
$= 2 \cdot \int_0^\infty t \cdot (e^{-\mu t} + e^{-\mu t} - e^{-2\mu t})\, dt - (\frac{1}{\mu} + \frac{1}{\mu} - \frac{1}{2\cdot\mu})^2 = 2 \cdot (\frac{1}{\mu^2} + \frac{1}{\mu^2} - \frac{1}{4\cdot\mu^2}) - \frac{9}{4\cdot\mu^2} = \frac{5}{4\cdot\mu^2}$.
Answer: A.

3. ${}_{20}p_{60} \cdot {}_{20}q_{70} + {}_{20}q_{60} \cdot {}_{20}p_{70} = .485$. Answer: B.

4. $e_{\overline{75:75}:\overline{5}|} = \sum_{k=1}^{5} {}_kp_{\overline{75:75}} = 2 \cdot e_{75:\overline{5}|} - e_{75:75:\overline{5}|} = 2 \cdot \sum_{k=1}^{5} {}_kp_{75} - \sum_{k=1}^{5} {}_kp_{75}^2$.
But $\frac{\ell_{75} - \ell_{80}}{\ell_{75}} = \frac{5}{\ell_{75}} = .35 \rightarrow {}_kp_{75} = \frac{\ell_{75+k}}{\ell_{75}} = 1 - \frac{k}{\ell_{75}} = 1 - \frac{.35k}{5}$
$\rightarrow \sum_{k=1}^{5} {}_kp_{75} = .93 + .86 + .79 + .72 + .65 = 3.95$ and $\sum_{k=1}^{5} {}_kp_{75}^2 = 3.1695 \rightarrow e_{\overline{75:75}:\overline{5}|} = 4.73$.
Answer: D.

5. $10{,}390 = 1000 \cdot a_{65:70} + 800 \cdot \sum_{k=1}^{\infty} v^k \cdot {}_kp_{65} \cdot {}_kq_{70} + 500 \cdot \sum_{k=1}^{\infty} v^k \cdot {}_kp_{70} \cdot {}_kq_{65}$
$= 1000 \cdot a_{65:70} + 800 \cdot (a_{65} - a_{65:70}) + 500 \cdot (a_{70} - a_{65:70}) = 800 \cdot a_{65} + 500 \cdot a_{70} - 300 \cdot a_{65:70}$
$\rightarrow a_{65:70} = \frac{-10{,}390 + 800\cdot a_{65} + 500\cdot a_{70}}{300} = 8.14$. Answer: B.

6. $SBP = \int_{20}^\infty v^t \cdot {}_tp_{40:50}\, dt + \int_{20}^\infty v^t \cdot {}_tp_{40} \cdot [{}_5p_{50} \cdot {}_{t-5}q_{55}]\, dt + \int_{20}^\infty v^t \cdot {}_tp_{50} \cdot [{}_{15}p_{40} \cdot {}_{t-15}q_{55}]\, dt$
$= {}_{20|}\bar{a}_{40:50} + \int_{20}^\infty v^t \cdot {}_tp_{40} \cdot [{}_5p_{50} \cdot (1 - {}_{t-5}p_{55})]\, dt + \int_{20}^\infty v^t \cdot {}_tp_{50} \cdot [{}_{15}p_{40} \cdot (1 - {}_{t-15}p_{55})]\, dt$
$= {}_{20|}\bar{a}_{40:50} + {}_5p_{50} \cdot {}_{20|}\bar{a}_{40} - {}_{20|}\bar{a}_{40:50} + {}_{15}p_{40} \cdot {}_{20|}\bar{a}_{50} - {}_{20|}\bar{a}_{40:50}$. Answer: D.

7. ${}_5p_{\overline{xy}} = 1 - {}_5q_{\overline{xy}} = 1 - F_{T(x)T(y)}(5,5) = .953125$, ${}_5p_{xy} = {}_5p_x + {}_5p_y - {}_5p_{\overline{xy}}$
$\rightarrow {}_5p_{\overline{xy}} - {}_5p_{xy} = 2{}_5p_{\overline{xy}} - {}_5p_x - {}_5p_y$, but ${}_5p_x = 1 - {}_5q_x$
$= 1 - F_{T(x)T(y)}(5,\infty) = 1 - F_{T(x)T(y)}(5,20)$, so that ${}_5p_x = 1 - .375 = .625$,
and similarly, ${}_5p_y = 1 - F_{T(x)T(y)}(\infty,5) = 1 - F_{T(x)T(y)}(10,5) = .84375$.
Then ${}_5p_{\overline{xy}} - {}_5p_{xy} = 2(.953125) - .625 - .84375 = .4375 = 7/16$. Answer: E.

8. In the common shock model,
${}_tp_{xy} = s_{T^*(x)}(t) \cdot s_{T^*(y)}(t) \cdot e^{-\lambda t}$. ${}_{10|10}q_{40:50} = {}_{10}p_{40:50} - {}_{20}p_{40:50}$
$= s_{T^*(40)}(10) \cdot s_{T^*(50)}(10) \cdot e^{-10(.01)} - s_{T^*(40)}(20) \cdot s_{T^*(50)}(20) \cdot e^{-20(.01)}$
In this case, with DeMoivre's law and $\omega = 100$, we have $s_{T^*(x)}(t) = \frac{\omega-x-t}{\omega-x}$, so that
${}_{10|10}q_{40:50} = \frac{50}{60} \cdot \frac{40}{50} \cdot e^{-10(.01)} - \frac{40}{60} \cdot \frac{30}{50} \cdot e^{-20(.01)} = .2757$. Answer: D.

9. ${}_nq_{\overset{\ \ 2}{xy}} = \int_0^n {}_tp_y \cdot \mu_{y+t} \cdot {}_tq_x \, dt = {}_nq_y - {}_nq_{x\overset{1}{y}}$. But ${}_nq_{\overset{1}{x}y} + {}_nq_{x\overset{1}{y}} = {}_nq_{xy}$

$= 1 - (1 - {}_nq_x)(1 - {}_nq_y) = .44 \rightarrow {}_nq_{x\overset{1}{y}} = .26 \rightarrow {}_nq_{x\overset{2}{y}} = .04$.
Alternatively, ${}_nq_{x\overset{2}{y}} = {}_nq_{\overset{1}{x}y} - {}_nq_x \cdot {}_np_y$. Answer: E.

10. We are given ${}_\infty q_{\overset{\ \ 1}{60:70}} = .7$ and $\int_0^\infty {}_tp_{70} \cdot \mu_{70+t} \cdot {}_{t+10}p_{60} \, dt = {}_{10}p_{60} \cdot {}_\infty q_{\overset{1}{70}:70} = \frac{1}{2} \cdot {}_{10}p_{60} = .4$

$\rightarrow {}_{10}p_{60} = .8 \rightarrow p = \int_0^\infty {}_tp_{60} \cdot \mu_{60+t} \cdot ({}_tp_{60} - {}_{t+10}p_{60}) \, dt = {}_\infty q_{\overset{1}{60}:60} - {}_{10}p_{60} \cdot {}_\infty q_{\overset{1}{60}:70}$

$= \frac{1}{2} - (.8) \cdot (1 - .7) = .26$. The probability in question is $2 \cdot p = .52$. Answer: B.

11. APV = $v \cdot q_{x:\overset{2}{x}} \cdot (1000 + 500) + v \cdot q_x \cdot p_x \cdot (1000 + 500)$
$= \frac{1}{2} \cdot v \cdot q_{\overline{x:x}} \cdot (1500) + v \cdot q_x \cdot (1 - q_x)(1500)$

$= \frac{1}{2} \cdot v \cdot q_x^2 \cdot (1500) + v \cdot q_x \cdot (1 - q_x)(1500) = 117.2$. Answer: D.

12. $q_{\overset{1}{40}:41} = \int_0^1 {}_tp_{40} \cdot \mu_{40+t} \cdot (1 - t \cdot q_{41}) \, dt = q_{40} \cdot (1 - \frac{1}{2} \cdot q_{41}) = .0945$ and
$q_{40:\overset{2}{41}} = \int_0^1 t \cdot q_{40} \cdot {}_tp_{41} \cdot \mu_{41+t} \, dt = \frac{1}{2} \cdot q_{40} \cdot q_{41} = .0055 \rightarrow q_{40} = .10$ and $q_{41} = .11$.
Answer: B.

13. Actuary A's premium is
$1000\int_0^{\infty} e^{-.05t}\,(1-e^{-.02t})\,e^{-.03t}\,(.03)\,dt = 1000[\,\frac{3}{8} - \frac{3}{10}\,] = 75$.
In order for (y) to die second, it must be true that (x) has already died by the time of (y)'s death, but the common shock has not yet occurred.
Actuary B's premium is
$1000\int_0^{\infty} e^{-.05t}\,(1-e^{-.01t})\,e^{-.01t}\,e^{-.02t}\,(.03)\,dt = 1000[\,\frac{3}{8} - \frac{3}{9}\,] = 41.67$.
The absolute difference is 33.33 . Answer: C

14. $P[5 < T(\overline{xy}) \le 6] = P[K(\overline{xy}) = 5] = {}_{5|}q_{\overline{xy}} = {}_5p_{\overline{xy}} - {}_6p_{\overline{xy}}$. Answer: E

15. ${}_{5|}q_{\overline{90:90}} = {}_6q_{\overline{90:90}} - {}_5q_{\overline{90:90}} = {}_6q_{90}\cdot{}_6q_{90} - {}_5q_{90}\cdot{}_5q_{90}$
$= \frac{6}{100-90}\cdot\frac{6}{100-90} - \frac{5}{100-90}\cdot\frac{5}{100-90} = .11$. Answer: D

16. $P\ddot{a}_{xx} + .75P(\ddot{a}_x - \ddot{a}_{xx}) + .75P(\ddot{a}_x - \ddot{a}_{xx}) = A_{\overline{xx}} = A_x + A_x - A_{xx}$.
$A_x = 1 - d\ddot{a}_x \rightarrow d = \frac{1-.4}{10} = .06 \rightarrow \ddot{a}_{xx} = \frac{1-A_{xx}}{d} = 7.5$.
Then, $P = \frac{2(.4)-.55}{7.5+.75(10-7.5)+.75(10-7.5)} = .0222$. Answer: C

17. $K\overline{a}_{xy} + K(\overline{a}_x - \overline{a}_{xy}) + .5K(\overline{a}_y - \overline{a}_{xy}) = 10,000\overline{A}_{\overline{xy}} = 10,000(\overline{A}_x + \overline{A}_y - \overline{A}_{xy})$.
$\overline{A}_x = 1 - \delta\overline{a}_x = .4$, $\overline{A}_y = 1 - \delta\overline{a}_y = .25$, $\overline{A}_{xy} = 1 - \delta\overline{a}_{xy} = .5$.
Then $K = \frac{.4+.25-.5}{10+(12-10)+(.5)(15-10)} = .010345$. Answer: E

18. For the first 10 years, payment is made only of (45) is alive and (30) is not. The probability of this at time $t \le 10$ is ${}_tp_{45} - {}_tp_{30:45}$. The APV of payment made during the first 10 years is $\int_0^{10} v^t({}_tp_{45} - {}_tp_{30:45})\,dt = \overline{a}_{45:\overline{10}|} - \overline{a}_{30:45:\overline{10}|}$.
After 10 years, payment is made if at least on of (30) or (45) is alive. This is a 10-year deferred last-survivor annuity to (30) and (45), with APV ${}_{10|}\overline{a}_{\overline{30:45}} = {}_{10|}\overline{a}_{30} + {}_{10|}\overline{a}_{45} - {}_{10|}\overline{a}_{30:45}$.
The total APV is $\overline{a}_{45:\overline{10}|} - \overline{a}_{30:45:\overline{10}|} + {}_{10|}\overline{a}_{30} + {}_{10|}\overline{a}_{45} - {}_{10|}\overline{a}_{30:45} = \overline{a}_{45} + {}_{10|}\overline{a}_{30} - \overline{a}_{30:45}$
(since $\overline{a}_{45:\overline{10}|} + {}_{10|}\overline{a}_{45} = \overline{a}_{45}$ and $\overline{a}_{30:45:\overline{10}|} + {}_{10|}\overline{a}_{45} = \overline{a}_{30:45}$). Answer: C

19. During the first 10 years, payment is made if (x) is alive, since it is within 10 years of (y)'s death (even if (y) dies immediately after the annuity begins). The APV for the first 10 years of payment is $\ddot{a}_{x:\overline{10}|}$. For the second 10 years (no payments are made after 20 years), payment is made at time k if (x) is alive and if (y) was alive 10 years earlier (this guarantees that k is within 10 years after the death of (y)). The probability of a payment being made at time k, where $10 \leq k \leq 19$ is ${}_kp_x \cdot {}_{k-10}p_y$. The APV of the second 10 years of payments is $\sum_{k=10}^{19} v^k \cdot {}_kp_x \cdot {}_{k-10}p_y$. Using the change of variable $j = k - 10$, this sum becomes

$$\sum_{j=0}^{9} v^{10+j} \cdot {}_{10+j}p_x \cdot {}_jp_y = v^{10} \cdot {}_{10}p_x \cdot \sum_{j=0}^{9} v^j \cdot {}_jp_{x+10} \cdot {}_jp_y$$

$$= v^{10} \cdot {}_{10}p_x \cdot \sum_{j=0}^{9} v^j \cdot {}_jp_y \cdot {}_jp_y = v^{10} \cdot {}_{10}p_x \cdot \ddot{a}_{y:y:\overline{10}|} \text{ (since } y = x+10).$$

The total APV is $\ddot{a}_{x:\overline{10}|} + v^{10} \cdot {}_{10}p_x \cdot \ddot{a}_{y:y:\overline{10}|}$.

From the given information, we have $\ddot{a}_{x:\overline{10}|} = \ddot{a}_x - {}_{10|}\ddot{a}_x = 7$; and

${}_{10|}\ddot{a}_x = v^{10}\,{}_{10}p_x \cdot \ddot{a}_{x+10} = v^{10}\,{}_{10}p_x \cdot \ddot{a}_y$ so that $v^{10}\,{}_{10}p_x = .5$; and

$\ddot{a}_{\overline{y:y}:\overline{10}|} = \ddot{a}_{y:\overline{10}|} + \ddot{a}_{y:\overline{10}|} - \ddot{a}_{y:y:\overline{10}|}$, so that $\ddot{a}_{y:y:\overline{10}|} = 6$.

Then the APV of the annuity is $7 + (.5)(6) = 10$. Answer: A

20. $12B\,\ddot{a}_x^{(12)} = 6000\,\ddot{a}_{xy}^{(12)} + 3600(\ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)}) + 12B(\ddot{a}_x^{(12)} - \ddot{a}_{xy}^{(12)})$

$\rightarrow 12B\,\ddot{a}_{xy}^{(12)} = 6000\,\ddot{a}_{xy}^{(12)} + 3600(\ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)}) \rightarrow B = \frac{6000(8)+3600(14-8)}{12(8)} = 725$.

Answer: C

21. In order for all three to be alive simultaneously in their 40's, they must all be alive as soon as the youngest turns 40, which is in 10 years. This is the event that all three of them survive at least 10 years. The probability of this event is

${}_{10}p_{32} \cdot {}_{10}p_{31} \cdot {}_{10}p_{30} = \frac{100-32-10}{100-32} \cdot \frac{100-31-10}{100-31} \cdot \frac{100-30-10}{100-30} = .625$.

The probability that they will not all be alive simultaneously in their 40's is $1 - .625 = .375$.

Answer: E

22. This premium is denoted $\overline{P}(\overline{A}_{\overline{xy}})$ in the Bowers book. $\overline{P}(\overline{A}_{\overline{xy}}) = \frac{\overline{A}_{\overline{xy}}}{\overline{a}_{\overline{xy}}}$.

$\overline{a}_{\overline{xy}} = \overline{a}_x + \overline{a}_y - \overline{a}_{xy}$. $\overline{a}_x = \int_0^\infty e^{-.05t} \cdot e^{-.1t}\,dt = \frac{1}{.15}$, $\overline{a}_y = \frac{1}{.2}$,

$\overline{a}_{xy} = \int_0^\infty e^{-.05t} \cdot e^{-.1t} \cdot e^{-.15t}\,dt = \frac{1}{.3} \rightarrow \overline{a}_{\overline{xy}} = 8.33 \rightarrow \overline{A}_{\overline{xy}} = 1 - \delta\overline{a}_{\overline{xy}} = .583$

$\rightarrow \overline{P}(\overline{A}_{\overline{xy}}) = .07$. Answer: A

23. The first death corresponds to failure of the joint life status. We are asked to find ${}_{6|6}q_{40:40}$.
We use the relationship ${}_{t|u}q_{xy} = {}_tp_{xy} - {}_{t+u}p_{xy}$ to get ${}_{6|6}q_{40:40} = {}_6p_{40:40} - {}_{12}p_{40:40}$.
Since the two lives are independent, we use the rule ${}_tp_{xy} = {}_tp_x \cdot {}_tp_y$, which is valid for independent lives aged x and y. Then,
${}_{6|6}q_{40:40} = {}_6p_{40:40} - {}_{12}p_{40:40} = {}_6p_{40} \cdot {}_6p_{40} - {}_{12}p_{40} \cdot {}_{12}p_{40} = (.98)^2 - (.945)^2 = .067375$.
Answer: E

24. The actuarial present value of the benefit is $\overline{A}_{\overline{xy}} = \overline{A}_x + \overline{A}_y - \overline{A}_{xy}$.
With constant force of interest δ and constant force of mortality μ we have
$\overline{A}_x = \frac{\mu}{\mu+\delta} = \frac{.07}{.07+.05} = \frac{7}{12}$, and similarly, $\overline{A}_y = \frac{7}{12}$.
Since $T(x)$ and $T(y)$ are independent, it follows that $\mu_{xy}(t) = \mu(x+t) + \mu(y+t) = .14$.
Since the joint life status $T(xy)$ has constant force of failure $.14$, it follows that
$\overline{A}_{xy} = \frac{.14}{.14+.05} = \frac{14}{19}$. Alternatively, we see that ${}_tp_{xy} = {}_tp_x \cdot {}_tp_y = e^{-.07t} \cdot e^{-.07t} = e^{-.14t}$,
so that $\overline{a}_{xy} = \int_0^\infty e^{-\delta t}\, {}_tp_{xy}\, dt = \int_0^\infty e^{-.05t} \cdot e^{-.14t}\, dt = \frac{1}{.19} = 5.263158$;
then $\overline{A}_{xy} = 1 - \delta \cdot \overline{a}_{xy} = 1 - (.05)(\frac{1}{.19}) = \frac{14}{19}$.
The APV of the benefit is $\overline{A}_{\overline{xy}} = \frac{7}{12} + \frac{7}{12} - \frac{14}{19} = .429825$.
The premium is payable continuously at a rate of Q per year until the first death. The APV of the premium is $Q \cdot \overline{a}_{xy} = 5.263158Q$. The benefit premium is found by setting APV of benefit equal to APV of premium: $5.263158Q = .429825$. Solving for Q results in .0817 .
Answer: C

25. When the force of mortality is constant at μ per person, and with force of interest δ, we have
$\overline{A}_x = \frac{\mu}{\delta+\mu}$, $\overline{a}_x = \frac{1}{\delta+\mu}$, $\overline{A}_{xy} = \frac{2\mu}{\delta+2\mu}$, $\overline{a}_{xy} = \frac{1}{\delta+2\mu}$.
The actuarial present value of the annuity part of the benefit is
$1000\,\overline{a}_{30:30} + 600(\overline{a}_{30} - \overline{a}_{30:30}) + 600(\overline{a}_{30} - \overline{a}_{30:30})$
$= 1000\,\frac{1}{.05+2(.04)} + 2 \times 600(\frac{1}{.05+.04} - \frac{1}{.05+2(.04)}) = 11,795$.
The actuarial present value of the insurance part of the benefit is
$1000\overline{A}_{30:30} + 800\overline{A}_{\overline{30:30}} = 1000\overline{A}_{30:30} + 800(A_{30} + A_{30} - \overline{A}_{30:30})$
$= 1000\,\frac{2(.04)}{.05+2(.04)} + 800(2 \times \frac{.04}{.05+.04} - \frac{2(.04)}{.05+2(.04)}) = 832$.
Total actuarial present value is $11,795 + 832 = 12,629$. Answer: A

26. The probability that both will survive at least 5 years is the joint-life probability ${}_5p_{30J:30B} = {}_5p_{30J} \cdot {}_5p_{30B}$. The probability that at least one will die within 5 years is the complement $1 - {}_5p_{30J} \cdot {}_5p_{30B}$.
John's force of mortality indicates DeMoivre's law with $\omega = 100$, so ${}_5p_{30J} = \frac{100-30-5}{100-30} = \frac{65}{70}$.
We use the relationship ${}_{10}p_{30} = {}_5p_{30} \cdot {}_5p_{35}$, to get $.94 = {}_5p_{30} \cdot (.96)$ for Bob, so that ${}_5p_{30B} = \frac{.94}{.96}$. The probability of at least one of them dying within 5 year is
$1 - (\frac{65}{70})(\frac{.94}{.96}) = .0908$. Answer: C

27. As in problem 10, the force of mortality indicates DeMoivre's Law with $\omega = 100$.
We write ${}_{10}p_{\overline{40:50}} = 1 - {}_{10}q_{\overline{40:50}}$. It follows from independence that ${}_{10}q_{\overline{40:50}} = {}_{10}q_{40} \cdot {}_{10}q_{50}$.
Using DeMoivre's Law, we have ${}_{10}q_{\overline{40:50}} = {}_{10}q_{40} \cdot {}_{10}q_{50} = \frac{10}{100-40} \cdot \frac{10}{100-50} = .0333$.
Then ${}_{10}p_{\overline{40:50}} = 1 - .0333 = .9667$. Answer: E

28. This probability is
${}_{10}p_{40} \cdot {}_{10}p_{50} \cdot {}_{10}q_{50} \cdot {}_{10}q_{60} = (1-.039)(1-.085)(.085)(.192) = .01435$. Answer: A

29. All are true even without the assumption of independence of $T(x)$ and $T(y)$. Answer: E

30. The force of mortality for DeMoivre's Law with upper age ω is $\mu_x + \frac{1}{\omega - x}$, so the survival model is DeMoivre's law with $\omega = 100$. Then ${}_tq_x = \frac{t}{100-x}$.
${}_5p_{\overline{45:50}} = 1 - {}_5q_{\overline{45:50}} = 1 - ({}_5q_{45})({}_5q_{50}) = 1 - (\frac{5}{100-45})(\frac{5}{100-50}) = .9909$. Answer: E

31. ${}_tp_0 = 1 - .02t$ is the same as $s(t) = 1 - \frac{t}{50}$ (survival from birth). Mortality (battery failure) follows DeMoivres law with $\omega = 50$. The device fails on the first battery failure. This is a joint-life status for the two batteries. The probability of the device not failing within 15 years is the joint-life status probability ${}_{15}p_{0:0} = {}_{15}p_0 \cdot {}_{15}p_0 = (1-.3)(1-.3) = .49$, so the probability that the device fails within the 15 year warranty period is .51 . Answer: D

32. The expected time to the first death is
$\overset{\circ}{e}_{xx} = \int_0^{20} {}_tp_{xx}\,dt = \int_0^{20} {}_tp_x \cdot {}_tp_x\,dt = \int_0^{20} (\frac{20-t}{20})^2 (\frac{20-t}{20})^2\,dt$
$= \frac{1}{16{,}000} \int_0^{20} (20-t)^4\,dt = 4$.
The expected time to the second death is $\overset{\circ}{e}_{\overline{xx}} = \overset{\circ}{e}_x + \overset{\circ}{e}_x - \overset{\circ}{e}_{xx} = 2\overset{\circ}{e}_x - 4$.
$\overset{\circ}{e}_x = \int_0^{20} {}_tp_x\,dt = \int_0^{20} (\frac{20-t}{20})^2\,dt = \frac{1}{400} \int_0^{20} (20-t)^2\,dt = 6.67$.
Then $\overset{\circ}{e}_{\overline{xx}} = 2(6.67) - 4 = 9.34$.
The difference is $9.34 - 4 = 5.34$. Answer: B

33. We first note that $\int_0^{20} f(t)\,dt$ must be equal 1 , so that $\int_0^{20} c(5+t)\,dt = 300c = 1$,
and therefore, $c = \frac{1}{300}$. The probability that both (x) and (y) are alive at time 5 is
${}_5p_{xy} = {}_5p_x \cdot {}_5p_y$. We see that ${}_5p_x = 1 - {}_5q_x = 1 - \int_0^5 f(t)\,dt = 1 - \int_0^5 \frac{5+t}{300}\,dt = .875$.
Then, ${}_5p_{xy} = (.875)(.875) = .766$. Answer: C

34. This is a three year term insurance on the joint life status of two new batteries. The APV is
$100A^{\,1}_{0:0:\overline{3|}} = 100[v\,q_{0:0} + v^2\,{}_{1|}q_{0:0} + v^3\,{}_{2|}q_{0:0}]$.
From the given data, we have $q_{0:0} = 1 - p_0 \cdot p_0 = 1 - (.97)^2 = .0591$,
${}_{1|}q_{0:0} = p_{0:0} \cdot (1 - p_{1:1}) = (.97)^2[1 - (.91)^2] = .1617$, and
${}_{2|}q_{0:0} = {}_2p_{0:0} \cdot (1 - p_{2:2}) = (.97)^2(.91)^2[1 - (.86)^2] = .2029$.
The APV is $100[\frac{.0591}{1.05} + \frac{.1617}{(1.05)^2} + \frac{.2029}{(1.05)^3}] = 37.82$. Answer: C

35. The probability is ${}_{10}p_{40} \cdot {}_{10}q_{50} + {}_{10}q_{40} \cdot {}_{10}p_{50}$.
Mortality follows DeMoivre's Law with $\omega = 100$, so ${}_{10}q_{40} = \frac{10}{100-40} = \frac{1}{6}$ and
${}_{10}q_{50} = \frac{10}{100-50} = \frac{1}{5}$. The probability is $(\frac{5}{6})(\frac{1}{5}) + (\frac{1}{6})(\frac{4}{5}) = \frac{9}{30}$. Answer: A

36. The actuarial present value is
$\ddot{a}_{\overline{5|}} + (\ddot{a}_{xy} - \ddot{a}_{xy:\overline{5|}}) + \frac{1}{2}[(\ddot{a}_x - \ddot{a}_{x:\overline{5|}}) - (\ddot{a}_{xy} - \ddot{a}_{xy:\overline{5|}})] + \frac{1}{2}[(\ddot{a}_x - \ddot{a}_{x:\overline{5|}}) - (\ddot{a}_{xy} - \ddot{a}_{xy:\overline{5|}})]$
$= 4.465 + (12.478 - 4.387)) + \frac{1}{2}[(14.817 - 4.440) - (12.478 - 4.387)]$
$+ \frac{1}{2}[(13.267 - 4.411) - (12.478 - 4.387)] = 14.1$. Answer: D

37. $T(xy)$ is the time until failure of the joint life status. The joint status has survival probability ${}_tp_{xy} = {}_tp_x \cdot {}_tp_y = (1-.01t)^2$. Then
$E[T(xy)] = \mathring{e}_{xy} = \int_0^{100} {}_tp_{xy}\,dt = \int_0^{100} (1-.01t)^2\,dt = \frac{(1-.01t)^3}{-.03}\Big|_{t=0}^{t=100} = 33.33$.
The second moment of $T(xy)$ is
$E[T(xy)] = \int_0^{100} 2t \cdot {}_tp_{xy}\,dt = \int_0^{100} 2t(1-.01t)^2\,dt$
$= \int_0^{100} 2t(1-.02t+.0001t^2)\,dt = 2\int_0^{100}(t-.02t^2+.0001t^3)\,dt = 1666.67$.
The variance of $T(xy)$ is $1666.67-(33.33)^2 = 556$. Answer: A

38. $Cov[T(xy), T(\overline{xy})] = E[T(xy)\cdot T(\overline{xy})] - E\{T(xy)] \cdot E[T(\overline{xy})]$.
We know that $E[T(x)] = \frac{1}{\mu}$ and $E[T(y)] = \frac{1}{\alpha}$, and $E\{T(xy)] = \frac{1}{\alpha+\mu}$, since ${}_tp_{xy} = {}_tp_x \cdot {}_tp_y = e^{-\mu t} \cdot e^{-\alpha t} = e^{-(\alpha+\mu)t}$.
$E[T(xy)\cdot T(\overline{xy})] = E\{T(x)T(y)] = E[T(x)] \cdot E[T(y)] = \frac{1}{\mu}\cdot\frac{1}{\alpha}$
(this is true since one of $T(xy)$ and $T(\overline{xy})$ is $T(x)$ and the other is $T(y)$).
Also, $E[T(\overline{xy})] = E[T(x)] + E[T(y)] - E[T(xy)] = \frac{1}{\mu} + \frac{1}{\alpha} - \frac{1}{\alpha+\mu}$.
Then,
$Cov[T(xy), T(\overline{xy})] = E[T(xy)\cdot T(\overline{xy})] - E\{T(xy)] \cdot E[T(\overline{xy})]$
$= \frac{1}{\mu}\cdot\frac{1}{\alpha} - \left(\frac{1}{\alpha+\mu}\right)\left(\frac{1}{\mu} + \frac{1}{\alpha} - \frac{1}{\alpha+\mu}\right) = \frac{1}{\alpha\mu} - \frac{(\alpha+\mu)^2-\alpha\mu}{\alpha\mu(\alpha+\mu)^2} = \frac{1}{(\alpha+\mu)^2}$. Answer: C