ACT348 December 2004 Final Exam

The following information applies to questions 1-2.
$q_x = .1$, $q_{x+1} = .2$, $q_{x+2} = .4$, $q_{x+3} = .5$, $i = .2$.

1. A fully discrete 4-year endowment insurance with face amount 1000 has level annual benefit premium $1000P_{x:\overline{4}|} = 233.33$. Which of the following is the 2nd year terminal prospective loss random variable (given that the insured is still alive at age $x+2$)? Amounts are rounded to nearest 1.

A) $\begin{cases} 600 & K(x+2)=0 \\ 267 & K(x+2)=1 \\ -420 & K(x+2)\geq 2 \end{cases}$ B) $\begin{cases} 600 & K(x+2)=0 \\ 267 & K(x+2)=1 \\ 0 & K(x+2)\geq 2 \end{cases}$

C) $\begin{cases} 600 & K(x+2)=0 \\ 267 & K(x+2)\geq 1 \end{cases}$ D) $\begin{cases} 600 & K(x+2)=0 \\ -428 & K(x+2)\geq 1 \end{cases}$

E) $\begin{cases} 600 & K(x+2)=0 \\ 0 & K(x+2)\geq 1 \end{cases}$

2. Find the 2nd year terminal reserve.
A) 200 B) 400 C) 600 D) 800 E) 1000

3. Which of the following are correct expressions for the reserve at the end of m years for a fully discrete h-payment, n-year term insurance, where $h < n$.

I. ${}_hP^{1}_{x:\overline{n}|}\ddot{s}_{x:\overline{m}|} - \dfrac{A^{1}_{x:\overline{m}|}}{v^m\ {}_mp_x}$ for $m < h$

II. $({}_hP^{1}_{x:\overline{n}|} - P^{1}_{x:\overline{m}|}) \cdot \ddot{s}_{x:\overline{m}|}$ for $h \leq m \leq n$

III. ${}_hP^{1}_{x:\overline{n}|}\ddot{s}_{x:\overline{h}|} \cdot \dfrac{1}{v^{m-h}\ {}_{m-h}p_{x+h}}$ for $h \leq m \leq n$

A) I only B) All but II C) All but III D) All E) None

4. A fully discrete whole life insurance of 1 is issued to (25). Premiums are paid annually to age 65. The benefit premium during the first 10 years is P_{25}, followed by a different level annual premium payable during the next 30 years. You are given:

(i) $A_{35} = 0.3$ (ii) $P_{25} = 0.01$ (iii) $d = 0.06$

Calculate the reserve at the end of the tenth year.

A) .123 B) .143 C) .163 D) .183 E) .203

5. You are given ${}_{10}V(\overline{A}_{x:\overline{25}|}) = .405$, ${}_{10}V_{x:\overline{25}|} = .4$, $i = .1$ and U.D.D. is assumed.

A 25 year fully discrete policy is issued at age x with a term insurance benefit of 1 and a pure endowment benefit of 2. Find the 10th year terminal reserve for this policy (nearest .01).

A) .62 B) .64 C) .66 D) .68 E) .70

6. You are given:

(i) ${}_{10}V = 2000$ (ii) $b_{11} = 10,000$ (iii) $\pi_{10} = 15$ (iv) $q_{x+10} = .012$ (v) $i = .06$

The policyholder is given the option of canceling the premium π_{10} and reducing the death benefit for the coming year to $10,000 - C$. The arrangement is set up so that ${}_{11}V$ will be the same either way.

Find C.

A) 1300 B) 1325 C) 1350 D) 1375 E) 1400

7. You are given: $q_x = .1$, $q_{x+1} = .2$, $q_{x+2} = .25$, $i = .10$

A 3-year fully discrete endowment insurance has a death benefit of 1000 and an endowment amount of 2000. The annual benefit premium is 491.59. Find the expected value of the unconditional second year net cash loss C_1 (nearest 100).

A) 600 B) 700 C) 800 D) 900 E) 1000

8. $T(x)$ and $T(y)$ are independent lives, $T(x)$ has constant force of mortality $\mu_x(t) = .1$ and $T(y)$ has constant force of mortality $\mu_y(t) = .2$. Find $\mu_{\overline{xy}}(10)$, the force of failure for the last survivor status at time 10 (10 years after current ages x and y) (nearest .1).
A) 0.0 B) 0.1 C) 0.2 D) 0.3 E) 0.4

Questions 9 and 10 relate to the following information.
$T^*(x)$, $T^*(y)$, and Z make up the usual independent components of a common shock model. $T^*(x)$ has a constant force of mortality of $.2$, $T^*(y)$ has a constant force of mortality of $.3$, and the common shock has a constant force of $\lambda = .1$.

9. Find ${}_{1|1}q_{\overline{xy}}$, the probability that the second death occurs between 1 and 2 years from now (nearest .05).
A) Less than .1 B) At least .1, but less than .2 C) At least .2, but less than .3
D) At least .3, but less than .4 E) At least .4

10. Find the probability $P[T(x) < T(y)]$.
A) $\frac{1}{6}$ B) $\frac{1}{5}$ C) $\frac{1}{4}$ D) $\frac{1}{3}$ E) $\frac{1}{2}$

11. Jim and Mary (both currently age 60) purchase an annuity payable continuously at the rate of:
- 1 per year with certainty for 15 years
- 1 per year after 15 years if both Jim and Mary are alive
- $\frac{3}{4}$ per year after 15 years if Jim is alive and Mary is dead
- $\frac{1}{2}$ per year after 15 years if Mary is alive and Jim is dead

You are given:
- The future lifetimes for Jim and Mary are independent.
- Both are subject to a constant force of mortality, $\mu = 0.06$.
- The force of interest is $\delta = 0.04$.
- A 15-year temporary life annuity of 1 payable continuously on a life age 60 has a single benefit premium of 7.769.
- A 15-year temporary joint life annuity of 1 payable continuously on two lives aged 60 has an actuarial present value of 5.683.

Calculate the actuarial present value of this annuity.
A) 6.25 B) 11.28 C) 13.93 D) 17.53 E) 20.17

12. You are given $q_x = .1$, $q_y = .2$, $i = .1$. $T(x)$ and $T(y)$ are independent, and each of them individually satisfies the Uniform Distribution of Deaths assumption in each year of each. Find the actuarial present value of a one year continuous term insurance that pays 1 at the instant of the second death. You are given that $\int_0^1 tv^t\,dt = .4693$ (answer to nearest .001).

A) .015 B) .017 C) .019 D) .021 E) .023

13. Which of the following is a correct expression for the probability that the second death of independent lives (x) and (y) will occur in year 6?

A) ${}_{5|}q_x \cdot {}_5q_y + {}_{5|}q_y \cdot {}_5q_x - {}_{5|}q_x \cdot {}_{5|}q_y$ B) ${}_{5|}q_x + {}_5q_y - {}_{5|}q_x \cdot {}_{5|}q_y$ C) $1 - {}_5p_{\overline{xy}}$

D) ${}_5p_{\overline{xy}} \cdot {}_{5|}q_{\overline{x+5:y+5}}$ E) ${}_5p_{\overline{xy}} - {}_6p_{\overline{xy}}$

ACT348 December 2004 Final Exam Solutions

1. ${}_2L = \begin{cases} 1000v - P = 600 & K(x+2) = 0 \\ 1000v^2 - P(1+v) = 267 & K(x+2) \geq 1 \end{cases}$ Answer: C

2. ${}_2V = 1000A_{x+2:\overline{2}|} - P\ddot{a}_{x+2:\overline{2}|} = 1000[vq_{x+2} + v^2\ {}_{1|}q_{x+2} + v^2\ {}_2p_{x+2}] - 233.33[1 + vp_{x+2}]$
$= 1000[\frac{.4}{1.2} + \frac{(.6)(.5)}{1.2^2} + \frac{(.6)(.5)}{1.2^2}] - 233.33[1 + \frac{.6}{1.2}] = 400$. Answer: B

3. I only. Answer: A

4. For the first 10 years, the death benefit is 1 and the premium is P_{25}. For the first 10 years this policy is identical to an ordinary whole life insurance with level premiums for life, and therefore, for the first 10 years this policy has k-th terminal reserve equal to ${}_kV_{25}$. The 10-th year terminal reserve is equal to ${}_{10}V_{25}$. We can use the information given to find (prospectively) ${}_{10}V_{25} = A_{35} - P_{25}\ddot{a}_{35} = A_{35} - P_{25}(\frac{1-A_{35}}{d}) = .183$.
This does not mean that ${}_kV$ for this policy equal to ${}_kV_{25}$ for all k, but only for $k \leq 10$.
The fact that ${}_{10}V$ for this policy is equal to ${}_{10}V_{25}$ says that we can use any methods available to find ${}_{10}V_{25}$ and then we have ${}_{10}V$, the reserve we are looking for. Answer: D

5. Under UDD, ${}_{10}V(\overline{A}_{x:\overline{25}|}) = \frac{i}{\delta}\,{}_{10}VA^1_{x:\overline{25}|} + {}_{10}V_{x:\overline{25}|}^{\ \ 1}$

${}_{10}V_{x:\overline{25}|} = {}_{10}VA^1_{x:\overline{25}|} + {}_{10}V_{x:\overline{25}|}^{\ \ 1}$

$\rightarrow\ {}_{10}V(\overline{A}_{x:\overline{25}|}) - {}_{10}V_{x:\overline{25}|} = .405 - .4 = (\frac{i}{\delta} - 1)\,{}_{10}VA^1_{x:\overline{25}|} \rightarrow {}_{10}VA^1_{x:\overline{25}|} = .101616$

$\rightarrow {}_{10}V_{x:\overline{25}|}^{\ \ 1} = .4 - .101614 = .298386 \rightarrow {}_{10}VA^1_{x:\overline{25}|} + 2 \cdot {}_{10}V_{x:\overline{25}|}^{\ \ 1} = .6984$. Answer: E

6. $(2000 + 15)(1.06) - 10,000(.012) = .988\,{}_{11}V$
$(2000)(1.06) - (10,000 - C)(.012) = .988\,{}_{11}V$.
$\rightarrow 15(1.06) - C(.012) = 0 \rightarrow C = 1,325$. Answer: B

7. $E[C_h] = (0)({}_2q_x) + (1000v - 491.59)({}_{2|}q_x) + (2000v - 491.59)({}_3p_x)$
$= (417.5)(.9)(.8)(.25) + (1326.59)(.9)(.8)(.75) = 791.51$. Answer: C

8. $\mu_{\overline{xy}}(t) = \frac{-\frac{d}{dt}\,{}_tp_{\overline{xy}}}{{}_tp_{\overline{xy}}}$.

${}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y - {}_tp_{xy} = e^{-.1t} + e^{-.2t} - e^{-.3t}$

$\frac{d}{dt}\,{}_tp_{\overline{xy}} = \frac{d}{dt}(e^{-.1t} + e^{-.2t} - e^{-.3t}) = -.1e^{-.1t} - .2e^{-.2t} + .3e^{-.3t}$

$\mu_{\overline{xy}}(10) = \frac{.1e^{-1}+.2e^{-2}-.3e^{-3}}{e^{-1}+e^{-2}-e^{-3}} = .1079$. Answer: B

9. ${}_{1|1}q_{\overline{xy}} = {}_{1|1}q_x + {}_{1|1}q_y - {}_{1|1}q_{xy} = p_x - {}_2p_x + p_y - {}_2p_y - p_{xy} + {}_2p_{xy}$

$= {}_1p_x^* \cdot e^{-.1} - {}_2p_x^* \cdot e^{-.1(2)} + {}_1p_y^* \cdot \cdot e^{-.1} - {}_2p_y^* \cdot e^{-.1(2)} - {}_1p_x^* \cdot {}_1p_y^* \cdot e^{-.1} + {}_2p_x^* \cdot {}_2p_y^* \cdot e^{-.1(2)}$

$= e^{-.2}e^{-.1} - e^{-.4}e^{-.2} + e^{-.3}e^{-.1} - e^{-.6}e^{-.2} - e^{-.2}e^{-.3}e^{-.1} + e^{-.4}e^{-.6}e^{-.2}$

$= e^{-.3} - e^{-.6} + e^{-.4} - e^{-.8} - e^{-.6} + e^{-1.2} = .166$. Answer: B

10. $P[T(x) < T(y)] = \int_0^\infty {}_tp_x^* \cdot \mu_x(t) \cdot {}_tp_y^* \cdot e^{-\lambda t}\,dt = \int_0^\infty e^{-.2t}(.2)e^{\cdot 3t}e^{-.1t}dt = \frac{.2}{.6} = \frac{1}{3}$.
Answer: D

11. We are given $\overline{a}_{60:\overline{15|}} = 7.769$ and $\overline{a}_{60:60:\overline{15|}} = 5.683$.

We wish to find $\overline{a}_{\overline{15|}} + {}_{15|}\overline{a}_{60:60} + .75({}_{15|}\overline{a}_{60} - {}_{15|}\overline{a}_{60:60}) + .5({}_{15|}\overline{a}_{60} - {}_{15|}\overline{a}_{60:60})$.

$\overline{a}_{\overline{15|}} = \frac{1-e^{-15\delta}}{\delta} = = \frac{1-e^{-15(.04)}}{.04} = 11.280$.

$\overline{a}_{60} = \frac{1}{\mu+\delta}$ when the force of mortality is constant, so that $\overline{a}_{60} = 10$.

Also, $\overline{a}_{60:60} = \int_0^\infty e^{-\delta t}\,{}_tp_{60:60}\,dt = \int_0^\infty e^{-\delta t}\,{}_tp_{60} \cdot {}_tp_{60}\,dt$ (since the lives are independent, we have ${}_tp_{60:60}\,dt = {}_tp_{60} \cdot {}_tp_{60} = e^{-.06t} \cdot e^{-.06t} = e^{-.12t}$.

Therefore, $\overline{a}_{60:60} = \int_0^\infty e^{-.04t} \cdot e^{-.06t} \cdot e^{-.06t}\,dt = \frac{1}{.16} = 6.25$

Then ${}_{15|}\overline{a}_{60} = \overline{a}_{60} - \overline{a}_{60:\overline{15|}} = 10 - 7.769 = 2.231$, and

${}_{15|}\overline{a}_{60:60} = \overline{a}_{60:60} - \overline{a}_{60:60:\overline{15|}} = 6.25 - 5.683 = .567$.

The annuity actuarial present value is

$11.280 + .567 + .75(2.231 - .567) + .5(2.231 - .567) = 13.93$. Answer: C

12. $\overline{A}_{\overset{1}{x}:\overline{1|}} + \overline{A}_{\overset{1}{y}:\overline{1|}} - \overline{A}_{\overset{1}{xy}:\overline{1|}} = \frac{i}{\delta}\,vq_x + \frac{i}{\delta}\,vq_y - \int_0^1 v^t\,{}_tp_x\,{}_tp_y\,(\mu_x(t) + \mu_y(t))\,dt$

$= \frac{i}{\delta}\,vq_x + \frac{i}{\delta}\,vq_y - [q_x\int_0^1 v^t(1 - tq_y)\,dt + q_y\int_0^1 v^t(1 - tq_x)\,dt]$

$= \frac{i}{\delta}\,vq_x + \frac{i}{\delta}\,vq_y - [(q_x + q_y)\int_0^1 v^t\,dt - 2q_xq_y\int_0^1 tv^t\,dt]$

$= \frac{.1}{ln\,1.1}\,(\frac{.1+.2}{1.1}) - [(.3)\frac{1-v}{ln\,1.1} - 2(.1)(.2)(.4693)] = .018772$ Answer: C

13. ${}_{5|}q_{\overline{xy}} = {}_5p_{\overline{xy}} - {}_6p_{\overline{xy}}$. Answer: E