ACT348 December 2003 Final Exam

**Questions 1 to 4 relate to** a fully discrete whole life insurance policy of face amount 1 issued at age 40. A level annual benefit (equivalence principle) premium $P$ is payable for life. Mortality is based on the Illustrative Life Table at $i = .06$ (from the Actuarial Math Text).

1. Find ${}_{25}V_{40}$ .

(A) .29 (B) .31 (C) .33 (D) .35 (E) .37

2. Find $Var[{}_{25}L|K(40) \geq 25]$ (variance of the 25-th year terminal prospective loss given that the insured is still alive).
(A) Less than .060 (B) At least .060, but less than .062
(C) At least .062, but less than .064 (D) At least .064, but less than .066
(E) At least .066

3. How many of the following are correct formulations for ${}_{25}V_{40}$ ?
I. $A_{65} - P_{65} \cdot \ddot{a}_{65}$ II. $1 - \frac{\ddot{a}_{65}}{\ddot{a}_{40}}$ III. $\frac{P_{40} - P_{\substack{1\\40:\overline{25}|}}}{P_{40:\overline{25}|}}$ IV. $[P_{65} - P_{40}] \cdot \ddot{a}_{65}$
(A) None (B) 1 (C) 2 (D) 3 (E) 4

4. Find the conditional expectation of the 25th year Net Cash Loss $C_{24}$ (for the period age 64 to 65) given that insured is still alive at age 64.
(A) .0075 (B) .0085 (C) .0095 (D) .0105 (E) .0115

5. A fully continuous whole life insurance issued at age $x$ has a death benefit of 2 if death occurs within 10 years, and a death benefit of 1 if death occurs after 10 years. The force of mortality is constant at .04 for all ages, and the force of interest is .06. The level benefit premium payable for life is $\overline{P} = .0065285$ . Find the 5th year terminal reserve for this policy.
(A) Less than $-.3$ (B) At least $-.3$, but less than $-.2$
(C) At least $-.2$, but less than $-.1$ (D) At least $-.1$, but less than $0$
(E) At least $0$

**Questions 6 and 7 relate to** two separate 3-year fully discrete insurance policies issued at age $x$ with level benefit premiums for which the death benefit is equal to $1000+$ terminal reserve in the year of death ($b_1 = 1000 + {}_1V$ , $b_2 = 1000 + {}_2V$ and $b_3 = 1000 + {}_3V$).
You are given: $i = .10$ , $q_x = .25$ , $q_{x+1} = .50$ , $q_{x+2} = .80$
Policy 1 is a term insurance and policy 2 is an endowment insurance with an endowment amount of 1000. The annual benefit premium for Policy 1 is $P_1$ and for Policy 2 it is $P_2$ .

6. Find first year terminal benefit reserve for Policy 2 (nearest 10).
(A) 510 (B) 520 (C) 530 (D) 540 (E) 550

7. Which of the following is a correct expression for $P_2 - P_1$ ?
(A) $\frac{1000}{\ddot{s}_{\overline{3}|.1}}$ (B) $\frac{1000}{\ddot{a}_{\overline{3}|.1}}$ (C) $1000\ddot{s}_{\overline{3}|.1}$ (D) $1000\ddot{a}_{\overline{3}|.1}$ (E) $333.33$

**Questions 8 to 10 relate to** the following information. Smith is now 70 years old and Jones is now 80 years old and their future lifetimes are independent. Mortality for both of them is based on the Illustrative Life Table (at the end of this exam paper), and $i = .06$ .

8. Find the probability that the first death of the two will occur between 10 and 20 years from now.
(A) .150 (B) .154 (C) .158 (D) .162 (E) .166

9. A last survivor annuity-due will pay 100,000 per year while both Smith and Jones are alive, it will pay 75,000 per year while Smith is alive if Jones dies first, and it will pay 50,000 per year while Jones is alive if Smith dies first. Find the actuarial present value of the annuity.
(A) Less than 725,000 (B) At least 725,000, but less than 750,000
(C) At least 750,000, but less than 775,000 (D) At least 775,000, but less than 800,000
(E) At least 800,000

10. Suppose it is assumed that the survival distributions of both Smith and Jones separately satisfy the uniform distribution of deaths model within each year of age. Which of the following is a correct representation for the difference between $\overline{A}_{\overset{1}{\overline{70:80}}:\overline{1}|} - \frac{i}{\delta} \cdot A_{\overset{1}{\overline{70:80}}:\overline{1}|}$
(these are one-year term, last-survivor insurances)?
(A) $\left(1 - \frac{2}{\delta} + \frac{2}{i}\right) v\, q_{70}\, q_{80}$ (B) $-\left(1 - \frac{2}{\delta} + \frac{2}{i}\right) v\, q_{70}\, q_{80}$
(C) $\frac{i}{\delta}\left(1 - \frac{2}{\delta} + \frac{2}{i}\right) v\, q_{70}\, q_{80}$ (D) $-\frac{i}{\delta}\left(1 - \frac{2}{\delta} + \frac{2}{i}\right) v\, q_{70}\, q_{80}$
(E) $\frac{i}{\delta}\left(1 - \frac{2}{\delta} + \frac{2}{i}\right) v\, q_{70:80}$

11. A fully discrete life insurance policy is issued on a pair of lives whose ages are $x$ and $y$. The benefit of 1 is paid at the end of the year of the second death. A level annual benefit premium $P$ (based on the equivalence principle) is paid as long as both are alive.
You are given $A_x = \frac{1}{7}$ , $A_y = \frac{1}{4}$ , $A_{xy} = \frac{1}{3}$ , and $\ddot{a}_{xy} = 9$ . Find $P$ (nearest .0001).
(A) .0044 (B) .0055 (C) .0066 (D) .0077 (E) .0088

12. A joint distribution of survival time for two lives incorporates a common shock. $T^*(x)$ has a constant force of mortality of .02 and $T^*(y)$ has constant force of mortality .04. The common shock parameter is $\lambda = .01$ . Find the probability that $x$ and $y$ both die before the occurrence of the common shock.
(A) Less than .60 (B) At least .6, but less than .65 (C) At least .65, but less than .70
(D) At least .70, but less than .75 (E) At least .75

13. Which of the following statements is false?
Assume that all forces of mortality and interest are $> 0$.
(A) $A_{xy} > A_x$ (B) $\ddot{a}_{xy} < \ddot{a}_x$ (C) $P_{xy} < P_x$ (D) $P_{\overline{xy}} < P_x$ (E) $q_{\overline{xy}} < q_x$

SOLUTIONS

1. ${}_{25}V_{40} = 1 - \frac{\ddot{a}_{65}}{\ddot{a}_{40}} = 1 - \frac{9.89693}{14.81661} = .332$ . Answer: C

2. ${}_{25}L = Z' - P_{40} \cdot Y' = [1 + \frac{P_{40}}{d}]Z' - \frac{P_{40}}{d}$ .
$Var[{}_{25}L|K(40) \geq 25] = [1 + \frac{P_{40}}{d}]^2\, Var[Z'] = [1 + \frac{P_{40}}{d}]^2\, [\,{}^2A_{65} - (A_{65})^2]$
$P_{40} = \frac{A_{40}}{\ddot{a}_{40}} = .010888\,,\ d = \frac{.06}{1.06} = .056604$ .
$Var[{}_{25}L|K(40) \geq 25] = [1 + \frac{.010888}{.056604}]^2\, [.2360299 - .4397965^2] = .0606$ .
Answer: B

3. I. $A_{65} - P_{65} \cdot \ddot{a}_{65}$ WRONG. Should be $A_{65} - P_{40} \cdot \ddot{a}_{65}$
II. $1 - - \frac{\ddot{a}_{65}}{\ddot{a}_{40}}$ CORRECT
III. $\dfrac{P_{40} - P^{\ 1}_{40:\overline{25}|}}{P_{40:\overline{25}|}}$ WRONG. Should be $\dfrac{P_{40} - P^{\ 1}_{40:\overline{25}|}}{P_{40:\overline{25}|}^{\ \ \ \ 1}}$
IV. $[P_{65} - P_{40}] \cdot \ddot{a}_{65}$ CORRECT Answer: C

4. The conditional distribution of the 25-th year net cash loss
$C_{24} = \begin{cases} v - P_{40} = .93251 & \text{prob. } q_{64} = .0195231 \\ -P_{40} = -.010888 & \text{prob. } p_{64} = .9804769 \end{cases}$ .
Conditional expectation is $(.93251)(.0195231) + (-.010888)(.9804769) = .00753$ .
Answer: A

5. $\int_0^\infty e^{-.06t} \cdot e^{-.04t} \cdot (.04)\, dt + \int_0^{10} e^{-.06t} \cdot e^{-.04t} \cdot (.04)\, dt = \overline{P} \int_0^\infty e^{-.06t} \cdot e^{-.04t}\, dt$
$\rightarrow \frac{.04}{.06+.04} + \frac{(.04)(1-e^{-10(.06+.04)})}{.06+.04} = \overline{P}\,[\frac{1}{.06+.04}] \rightarrow \overline{P} = .065285$ .
${}_5\overline{V} = (.065285)\, \frac{\int_0^5 e^{-.06t} \cdot e^{-.04t}\, dt}{e^{-5(.06)} \cdot e^{-5(.04)}} - \frac{2\int_0^5 e^{-.06t} \cdot e^{-.04t} \cdot (.04)\, dt}{e^{-5(.06)} \cdot e^{-5(.04)}} = (.065285) \cdot \frac{3.9347}{.60653} - 2 \times \frac{.15739}{.60653} = -.095$ .
Answer: D

6. $P_2 \cdot \ddot{a}_{\overline{3}|.1} = 1000[vq_x + v^2\, q_{x+1} + v^3\, q_{x+2}] + 1000v^3$ . $P_2 = 728.51$ .
$P_2(1+i) - (b_1 - {}_1V)\, q_x = {}_1V \rightarrow 728.51(1.1) - 1000(.25) = 551.36$ . Answer: E

7. $P_1 \cdot \ddot{a}_{\overline{3}|.1} = 1000[vq_x + v^2\, q_{x+1} + v^3\, q_{x+2}]$ and
$P_2 \cdot \ddot{a}_{\overline{3}|.1} = 1000[vq_x + v^2\, q_{x+1} + v^3\, q_{x+2}] + 1000v^3$ .
Therefore, $(P_2 - P_1) \cdot \ddot{a}_{\overline{3}|.1} = 1000v^3$ , so that $P_2 - P_1 = \frac{1000}{\ddot{s}_{\overline{3}|.1}}$ . Answer: A

8. ${}_{10|10}q_{70:80} = {}_{10}p_{70:80} - {}_{20}p_{70:80} = ({}_{10}p_{70})({}_{10}p_{80}) - ({}_{20}p_{70})({}_{20}p_{80})$
$= \frac{\ell_{80}}{\ell_{70}} \cdot \frac{\ell_{90}}{\ell_{80}} - \frac{\ell_{90}}{\ell_{70}} \cdot \frac{\ell_{100}}{\ell_{80}} = \frac{10{,}584.91}{66{,}161.54} - \frac{10{,}584.91}{66{,}161.54} \cdot \frac{400.49}{39{,}143.64} = .15835$ . Answer: C

9. APV $= 100,000\ddot{a}_{70:80} + 75,000(\ddot{a}_{70} - \ddot{a}_{70:80}) + 50,000(\ddot{a}_{80} - \ddot{a}_{70:80})$
$= 100,000(5.00138) + 75,000(8.56925 - 5.00138) + 50,000(5.90503 - 5.00138)$
$= 812,910.75.$ Answer: E

10. $\overline{A}_{\overset{\;1}{\overline{70:80}}:\overline{1}|} - \frac{i}{\delta} \cdot A_{\overset{\;1}{\overline{70:80}}:\overline{1}|} = \overline{A}_{\overset{1}{70}:\overline{1}|} + \overline{A}_{\overset{1}{80}:\overline{1}|} - \overline{A}_{\overset{\;1}{70:80}:\overline{1}|} - \frac{i}{\delta} \cdot [A_{\overset{1}{70}:\overline{1}|} + A_{\overset{1}{80}:\overline{1}|} - A_{\overset{\;1}{70:80}:\overline{1}|}]$

Under UDD, $\overline{A}_{\overset{1}{70}:\overline{1}|} = \frac{i}{\delta} \cdot A_{\overset{1}{70}:\overline{1}|}$ and $\overline{A}_{\overset{1}{80}:\overline{1}|} = \frac{i}{\delta} \cdot A_{\overset{1}{80}:\overline{1}|}$ , so that

$\overline{A}_{\overset{\;1}{\overline{70:80}}:\overline{1}|} - \frac{i}{\delta} \cdot A_{\overset{\;1}{\overline{70:80}}:\overline{1}|} = \frac{i}{\delta} \cdot A_{\overset{\;1}{70:80}:\overline{1}|} - \overline{A}_{\overset{\;1}{70:80}:\overline{1}|}$ .

Under UDD we have $\overline{A}_{\overset{\;1}{xy}:\overline{n}|} = \frac{i}{\delta} A_{\overset{\;1}{xy}:\overline{n}|} + \frac{i}{\delta}\left(1 - \frac{2}{\delta} + \frac{2}{i}\right)\sum_{k=0}^{n-1} v^{k+1}\, {}_kp_{xy}\, q_{x+k}\, q_{y+k}$ ,

so that $\overline{A}_{\overset{\;1}{xy}:\overline{1}|} = \frac{i}{\delta} A_{\overset{\;1}{xy}:\overline{1}|} + \frac{i}{\delta}\left(1 - \frac{2}{\delta} + \frac{2}{i}\right) v\, q_x\, q_y$ .

Therefore, $\frac{i}{\delta} \cdot A_{\overset{\;1}{70:80}:\overline{1}|} - \overline{A}_{\overset{\;1}{70:80}:\overline{1}|} = -\frac{i}{\delta}\left(1 - \frac{2}{\delta} + \frac{2}{i}\right) v\, q_{70}\, q_{80}$ Answer: D

11. $P\ddot{a}_{xy} = A_{\overline{xy}} = A_x + A_y - A_{xy} \rightarrow P = \frac{\frac{1}{7}+\frac{1}{4}-\frac{1}{3}}{9} = .00661$ . Answer: C

12. $P[(T^*(x) < Z) \cap (T^*(y) < Z)] = \int_0^\infty [1 - s_{T^*(x)}(t)][1 - s_{T^*(y)}(t)]\, \lambda e^{-\lambda t}\, dt$
$= \int_0^\infty (1 - e^{-.02t})(1 - e^{-.04t})(.01)e^{-.01t}\, dt = (.01)\int_0^\infty [e^{-.01t} - e^{-.03t} - e^{-.05t} + e^{-.07t})\, dt$
$= 1 - \frac{1}{3} - \frac{1}{5} + \frac{1}{7} = .6095$ . Answer: B

13. $P_{xy} = \frac{1}{\ddot{a}_{xy}} - d > \frac{1}{\ddot{a}_x} - d = P_x$ . (C) Is false. Answer: C