ACT348H1F - TEST 2 - NOVEMBER 14, 2007

Write name and student number on each page. Write your solution for each question in the space provided.

1. (a) Formulate ${}_{10}V_{40}$ in each of the following ways:
5 (i) prospectively, (ii) retrospectively, (iii) in terms of life annuities,
(iv) in terms of life insurances, and (v) in terms of benefits premiums and the discount rate d.

(b) Mortality follows the Illustrative Table (attached at the end of the test) with annual effective interest rate 6%. A fully continuous whole life insurance of face amount 1000 with level benefit premiums is issued to (40).

3 (i) Assuming UDD, find ${}_{10}\overline{V}(\overline{A}_{40})$.

4 (ii) Continue to assume UDD and show algebraically that ${}_{10}\overline{V}(\overline{A}_{40}) = {}_{10}V_{40} \times C$,
where $C = \frac{i}{\delta} + \beta(\infty) \cdot \overline{P}(\overline{A}_{40})$
(recall that $\beta(\infty) = \frac{i-\delta}{\delta^2}$ and the $V^{(m)}$ formulation under UDD).
Verify this relationship using numerical values

2. A special fully discrete whole life insurance with face amount $100,000 is issued to (40). The annual benefit premium for the first 10 years is $100{,}000P^{1}_{40:\overline{10}|} = 359.40$ and for the next 10 years the annual benefit premium is $100{,}000P^{1}_{50:\overline{10}|} = 798.76$. After that (from age 60) the benefit premium is level at π for life. Mortality follows the Illustrative Table (attached at the end of the test) with annual effective interest rate 6%.

6 (a) Find ${}_{10}V$, ${}_{11}V$, ${}_{19}V$.

3 (b) Use ${}_{19}V$ and the recursive reserve relationship to show that ${}_{20}V = 0$.

3 (c) Find ${}_{40}V$.

3. A mortality model has constant mortality probability $q_y = q$ for all y. The annual effective rate of interest is i.

4 (a) A 10-year fully discrete term insurance with face amount 1 is issued to (x). Show that the benefit premium is vq . Show recursively that ${}_kV = 0$ for each k from 0 to 10.

4 (b) A 10-year fully discrete endowment insurance has death benefit equal to $b_{k+1} = {}_{k+1}V$ and has an endowment amount of 1. Find the level annual premium and find $Var[{}_kL|K(x) \geq k]$ for $k = 0, 1, ..., 9$.

ACT348H1 F - TEST 1 SOLUTIONS - NOVEMBER 14, 2007

1.(a) (i) $A_{50} - P_{40} \cdot \ddot{a}_{50}$ (ii) $P_{40} \cdot \ddot{s}_{40:\overline{10}|} - \frac{A^{\ 1}_{40:\overline{10}|}}{v^{10}\ {}_{10}p_{40}}$ (iii) $1 - \frac{\ddot{a}_{50}}{\ddot{a}_{40}}$

(iv) $\frac{A_{50}-A_{40}}{1-A_{40}}$ (v) $\frac{P_{50}-P_{40}}{P_{50}+d}$

(b) (i) ${}_{10}\overline{V}(\overline{A}_{40}) = \frac{\overline{A}_{50}-\overline{A}_{40}}{1-\overline{A}_{40}} = \frac{\frac{i}{\delta}(A_{50}-A_{40})}{1-\frac{i}{\delta}A_{40}} = \frac{\frac{.06}{ln\,1.06}(.24905-.16132)}{1-\frac{.06}{ln\,1.06}(.16132)} = .10833$.
$1000\,{}_{10}\overline{V}(\overline{A}_{40}) = 108.33$.

(ii) ${}_{10}\overline{V}(\overline{A}_{40}) = {}_{10}V^{(\infty)}(\overline{A}_{40})$. Under UDD this is ${}_{10}V(\overline{A}_{40}) + \beta(\infty) \cdot \overline{P}(\overline{A}_{40}) \cdot {}_{10}V_{40}$,
Under UDD we have ${}_{10}V(\overline{A}_{40}) = \frac{i}{\delta}\,{}_{10}V_{40}$, so
${}_{10}\overline{V}(\overline{A}_{40}) = \frac{i}{\delta}\,{}_{10}V_{40} + \beta(\infty) \cdot \overline{P}(\overline{A}_{40}) \cdot {}_{10}V_{40} = {}_{10}V_{40} \cdot [\frac{i}{\delta} + \beta(\infty) \cdot \overline{P}(\overline{A}_{40})\,]$.

Numerically, we have from (i) ${}_{10}\overline{V}(\overline{A}_{40}) = .10833$. From the table we get
${}_{10}V_{40} = 1 - \frac{\ddot{a}_{50}}{\ddot{a}_{40}} = .10460$. Also, $\frac{i}{\delta} = \frac{.06}{ln\,1.06} = 1.029709$ (or from the table it is 1.02971).
$\beta(\infty) = \frac{\frac{i}{\delta}-1}{\delta} = \frac{\frac{.06}{ln\,1.06}-1}{ln\,1.06} = .509845$ (or from the table is .50985).
$\overline{P}(\overline{A}_{40}) = \frac{\delta\overline{A}_{40}}{1-\overline{A}_{40}} = \frac{\delta\cdot\frac{i}{\delta}A_{40}}{1-\frac{i}{\delta}A_{40}} = .011607$.
Then
${}_{10}V_{40} \cdot [\frac{i}{\delta} + \beta(\infty) \cdot \overline{P}(\overline{A}_{40})\,] = (.10460)[1.02971 + (.50985)(.01161)] = .10833 = {}_{10}\overline{V}(\overline{A}_{40})$.

2.(a) Since the benefit and premium for the first 10 years are that of a 10-year term insurance issued to (40), the reserve at the end of 10 years is 0, so ${}_{10}V = 0$. The next 10 years have the same premium and benefit as a 10 year term insurance issued to (50). Since ${}_{10}V = 0$, it follows that the reserves for the second 10 years are the same as reserves for a 10 year term insurance at age 50. Using the recursive relationship, we have

$({}_{10}V + 100{,}000P_{\overset{1}{50}:\overline{10|}})(1+i) - 100,000q_{50} = p_{50} \cdot {}_{11}V$, so

$(0 + 798.76)(1.06) - 100,000(.00592) = (.99408)\,{}_{11}V \rightarrow {}_{11}V = 256.20$.

The same reasoning leading to ${}_{10}V = 0$ also leads to ${}_{20}V = 0$.

So ${}_{19}V$ is the same as the 9-th year reserve on a 10-year term insurance of (50);

${}_{19}V = 100,000\,{}_{9}V_{\overset{1}{50}:\overline{10|}} = 100,000[\,A_{\overset{1}{59}:\overline{1|}} - P_{\overset{1}{50}:\overline{10|}} \cdot \ddot{a}_{59:\overline{1|}}\,]$

$= 100,000[\,vq_{59} - P_{\overset{1}{50}:\overline{10|}}\,] = 100,000[\,\frac{.01262}{1.06} - .007986\,] = 391.81$.

This part can also be solved by finding reserve retrospectively, or by finding π from

$100,000A_{40} = P_{\overset{1}{40}:\overline{10|}} \cdot \ddot{a}_{40:\overline{10|}} + P_{\overset{1}{50}:\overline{10|}} \cdot {}_{10}E_{40} \cdot \ddot{a}_{50:\overline{10|}} + \pi \cdot {}_{20}E_{40} \cdot \cdot \ddot{a}_{60}$

and then finding reserves prospectively.

(b) Since ${}_{20}V = 0$, we have $100,000A_{60} - \pi \cdot \ddot{a}_{60} = 0$, so that $\pi = 100,000P_{60}$ and ${}_{40}V$ is the same as the 20-th year reserve on an ordinary whole life policy issued at age 60. So ${}_{40}V = 100,000\,{}_{20}V_{60} = 100,000[\,1 - \frac{\ddot{a}_{80}}{\ddot{a}_{60}}] = 47,018.50$.

3.(a) $\ddot{a}_{x:\overline{10|}} = 1 + vp_x + v^2\,{}_2p_x + \cdots + v^9\,{}_9p_x = 1 + vp + v^2p^2 + \cdots + v^9p^9 = \frac{1-v^{10}p^{10}}{1-vp}$.

$A_{\overset{1}{x}:\overline{10|}} = vq_x + v^2\,{}_{1|}q_x + \cdots + v^{10}\,{}_{9|}q_x = vq + v^2pq + \cdots + v^{10}p^9q$

$= vq[1 + vp + \ + v^2p^2 + \cdots + v^9p^9] = vq[\frac{1-v^{10}p^{10}}{1-vp}]$. $\quad P_{\overset{1}{x}:\overline{10|}} = \frac{A_{\overset{1}{x}:\overline{10|}}}{\ddot{a}_{x:\overline{10|}}} = vq$.

$vq(1+i) - q = p\,{}_1V \rightarrow 0 = p\,{}_1V \rightarrow {}_1V = 0$.

Same equation continues for $k = 2, 3, \ldots$, so ${}_kV = 0$ for all k.

(b) Since $b_{k+1} = {}_{k+1}V$, the level premium π satisfies the equation $\pi\ddot{s}_{\overline{10|}} = 1$, so $\pi = \frac{1}{\ddot{s}_{\overline{10|}}}$.

$Var[{}_kL|K(x) \geq k] = \sum_{j=0}^{10-k-1} v^2v^{2j}[b_{k+j+1} - {}_{k+j+1}V]^2\,{}_jp_{x+k} \cdot p_{x+k+j} \cdot q_{x+k+j}$.

Since $b_{k+j+1} - {}_{k+j+1}V = 0$, it follows that $Var[{}_kL|K(x) \geq k] = 0$ for all k.