ACT348H1F - TEST 1 - OCTOBER 10, 2007

Write name and student number on each page. Write your solution for each question in the space provided.

1. A 2-year fully discrete term insurance issued to (x) with a face amount of $100,000 has a premium of $13,000 in the first year and $26,000 in the second year. The annual effective rate of interest is 25% and the mortality probabilities at ages (x) and $(x+1)$ are $q_x = .2$ and $q_{x+1} = .25$.

6 (a) Describe the issue date loss random variable as a 3-point random variable with numerical outcomes and probabilities.

4 (b) Determine (i) the expected value of the issue date loss and.
(ii) determine $E[\,_1L \mid K(x) \geq 1]$ for the situation in part (a).

4 (c) Suppose that the premium is changed to Q in the first year and $2Q$ in the second year and that these are benefit premiums (equivalence principle premiums). Find Q.

4 (d) Suppose that the death benefit is $100,000 plus the return of premiums without interest and suppose that the premium is R in the first year and $2R$ in the second year and that these are benefit premiums. Find R.

4 (e) Suppose that the death benefit is $100,000 plus the return of premiums with interest at 25% suppose that the premium is P in the first year and $2P$ in the second year. Describe the issue date
loss random variable as a 3-point random variable in terms of P. Solve for P using the equivalence principle.

2. The force of mortality is constant at $\mu = .01$ for all ages, and the force of interest is .06. A fully continuous whole life insurance of 1 issued to (x) with level premiums for life.

2+4 (a) Suppose that the premium charged on the policy is based on the equivalence principle. Find the premium and the probability that the issue date loss is positive.

4 (b) Find the variance of the issue date loss based on the premium in (a).

4 (c) Find the minimum annual premium rate Q so that $P(L > 0) \leq .2$.

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1.(a) $L = \begin{cases} 100,000v - 13,000 = 67,000 & \text{Prob. } q_x = .2 \\ 100,000v^2 - 13,000(1+2v) = 30,200 & \text{Prob. } {}_{1|}q_x = (.8)(.25) = .2 \\ -13,000(1+2v) = -33,800 & \text{Prob. } {}_2p_x = (.8)(.75) = .6 \end{cases}$

(b) (i) $E[L] = (67,000)(.2) + (30,200)(.2) + (-33,800)(.6) = -840$.

(ii) $E[{}_1L \mid K(x) \geq 1] = 100,000vq_{x+1} - 26,000 = -6,000$.

(c) $100,000A^{\;1}_{x:\overline{2}|} = 100,000(vq_x + v^2\ {}_{1|}q_x) = Q(1 + 2vp_x)$

$\rightarrow Q = \frac{100,000(\frac{.2}{1.25} + \frac{.2}{1.25^2})}{1 + 2\cdot\frac{.8}{1.25}} = 12,632$.

(d) $R(1 + 2vp_x) = 100,000(vq_x + v^2\ {}_{1|}q_x) + R(vq_x + 3v^2\ {}_{1|}q_x)$

$\rightarrow R = \frac{100,000(\frac{.2}{1.25} + \frac{.2}{1.25^2})}{1 + 2\cdot\frac{.8}{1.25} - \frac{.2}{1.25} - 3\cdot\frac{.2}{1.25^2}} = 16,590$

(e) $L = \begin{cases} [100,000 + P(1+i)]v - P = 100,000v & \text{Prob. } .2 \\ 100,000 + P(1+i)^2 + 2P(1+i)]v^2 - P(1+2v) = 100,000v^2 & \text{Prob. } .2 \\ -P(1+2v) & \text{Prob. } .6 \end{cases}$

We can find P by setting $E[L] = 0$.

$\frac{100,000}{1.25}(.2) + \frac{100,000}{1.25^2}(.2) - P(1 + \frac{2}{1.25})(.6) = 0 \rightarrow P = 18,462$.

2. (a) The equivalence principle premium is $\overline{P}(\overline{A}_x) = \frac{\overline{A}_x}{\overline{a}_x} = \frac{\frac{\mu}{\delta+\mu}}{\frac{1}{\delta+\mu}} = \mu = .01$.

We find the point t_0 at which the issue date loss is 0 if death occurs at that point.
$e^{-.06t_0} = .01\overline{a}_{\overline{t_0|}} = .01 \cdot \frac{1-e^{-.06t_0}}{.06}$. This equation can be written in the form
$\frac{e^{.06t_0}-1}{.06} = 100$, which then can be written as $e^{.06t_0} = 7$, so that $t_0 = \frac{ln\,7}{.06} = 32.43$.
The loss is positive if death occurs before t_0 , and the probability of that is
$P(L > 0) = P(T < t_0) = {}_{t_0}q_x = 1 - {}_{t_0}p_x = 1 - e^{-.01t_0} = 1 - e^{-.01(32.43)} = .277$.
Alternatively, since $e^{.06t_0} = 7$, it follows that $e^{-.01t_0} = 7^{-1/6}$, and
$P(L > 0) = 1 - 7^{-1/6} = .277$.

(b) $Var[L] = [1 + \frac{.01}{\delta}]^2 \, Var(Z) = [1 + \frac{.01}{.06}]^2 \cdot [{}^2\overline{A}_x - \overline{A}_x^2]$
$= [1 + \frac{.01}{.06}]^2 \cdot [\frac{\mu}{2\delta+\mu} - (\frac{\mu}{\delta+\mu})^2] = [1 + \frac{.01}{.06}]^2 \cdot [\frac{.01}{.12+.01} - (\frac{.01}{.06+.01})^2]$
$= .0769$.
Alternatively, since the force of mrotality is constant and the equivalence princple premium is used, the variance is equal to ${}^2\overline{A}_x = \frac{.01}{.12+.01} = \frac{1}{13}$.

(c) We find the time point t so that ${}_tq_x = .2$. This point is the solution of the equation
${}_tp_x = e^{-.01t} = .8$, so $t = \frac{ln\,.8}{-.01} = 22.3144$.
We find the premium Q from the equation $e^{-.06t} = Q\overline{a}_{\overline{t|}}$, so that $e^{-.06t} = Q \cdot \frac{1-e^{-.06t}}{.06}$.
Since $e^{-.01t} = .8$ it follows that $e^{-.06t} = (.8)^6$, so that $(.8)^6 = Q \cdot \frac{1-(.8)^6}{.06}$, from which we get $Q = .021317$.