SECTION 19 - OPTION STRATEGIES (2)

Sections 3.3-3.5 and Chapter 4 of "Derivatives Markets"

In this section we will continue to use the XYZ stock and options listed in Example 78 in Section 18 along with the annual effective interest rate of 5%.

Bull and Bear Spreads

A **bull spread** based on call options is the combination of
(i) a purchased call with strike price K_1 and
(ii) a written call with strike price K_2, where $K_1 < K_2$.
A bull spread based on put options is the combination of
(i) a purchased put with strike price K_1 and
(ii) a written put with strike price K_2, where $K_1 < K_2$.

Example 79: A bull spread using call options based on XYZ option prices with options expiring at time 1 can be constructed as follows. Purchase a call with strike price 19 and write a call with strike price 23. The payoff at time 1 is

$$max\{S_1 - 19, 0\} - max\{S_1 - 23, 0\} = \begin{cases} 0 & \text{if } S_1 \leq 19 \\ S_1 - 19 & \text{if } 19 < S_1 \leq 23 \\ 4 & \text{if } S_1 > 23 \end{cases}.$$

The cost of this bull spread at time 0 is $4.06 - 2.45 = 1.61$, and the accumulated cost at time 1 is \$1.69. The profit at time 1 is

$$\text{payoff} - \text{accumulated cost of bull spread} = \begin{cases} -1.69 & \text{if } S_1 \leq 19 \\ S_1 - 20.69 & \text{if } 19 < S_1 \leq 23 \\ 2.31 & \text{if } S_1 > 23 \end{cases}.$$

A bull spread with the same profit can be constructed using put options. Purchase a put option with strike price 19 and write a put option with strike price 23. The payoff will be

$$max\{19 - S_1, 0\} - max\{23 - S_1, 0\} = \begin{cases} -4 & \text{if } S_1 \leq 19 \\ S_1 - 23 & \text{if } 19 < S_1 \leq 23 \\ 0 & \text{if } S_1 > 23 \end{cases}.$$

The cost of this bull spread at time 0 is $2.16 - 4.36 = -2.20$, and the accumulated cost at time 1 is -2.31. The profit at time 1 is

$$\text{payoff} - \text{accumulated cost of bull spread} = \begin{cases} -4 - (-2.31) = -1.69 & \text{if } S_1 \leq 19 \\ S_1 - 20.69 & \text{if } 19 < S_1 \leq 23 \\ 2.31 & \text{if } S_1 > 23 \end{cases}.$$

The two spreads do not have the same payoff, but they should have the same profit because of put-call parity. □

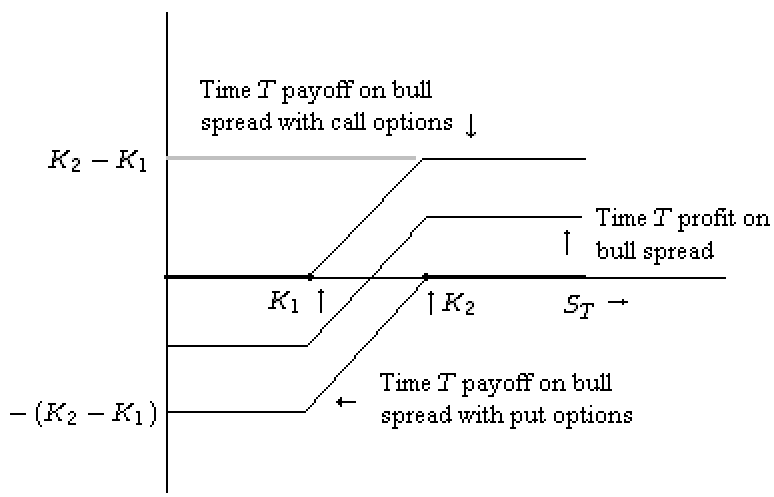
Suppose that we consider a bull spread made up of a purchased call with strike K_1 and a written call with strike K_2, where $K_1 < K_2$ and expiry is at time T. The payoff at time T is

$$max\{S_T - K_1, 0\} - max\{S_T - K_2, 0\} = \begin{cases} 0 & \text{if } S_T \le K_1 \\ S_T - K_1 & \text{if } K_1 < S_T \le K_2 \\ K_2 - K_1 & \text{if } S_T > K_2 \end{cases}.$$

The cost of the bull spread at time 0 is $C_{0,K_1} - C_{0,K_2} > 0$ (since the call option with the lower strike price will have the higher cost) and the profit at time T is payoff $-(C_{0,K_1} - C_{0,K_2})e^{rT}$.
Suppose that we consider a bull spread made up of a purchased put with strike K_1 and a written put with strike K_2, where $K_1 < K_2$ and expiry is at time T. The payoff at time T is

$$max\{K_1 - S_T, 0\} - max\{K_2 - S_T, 0\} = \begin{cases} -(K_2 - K_1) & \text{if } S_T \le K_1 \\ S_T - K_2 & \text{if } K_1 < S_T \le K_2 \\ 0 & \text{if } S_T > K_2 \end{cases}.$$

Under put-call parity, the profit at time T should be the same as the profit on the bull spread made up of call options. The payoff and profit diagrams of a bull spread are below.



A **bear spread** is the opposite of a bull spread. If $K_1 < K_2$, then a bear spread can be constructed by combining a written call with strike price K_1 and a purchased call with strike price K_2. The payoff would be the negative of a bull spread.

Box Spreads

A **box spread** is a combination of

(i) a synthetic long forward with forward price K_1, and

(ii) a synthetic short forward with forward price K_2.

The synthetic long forward with delivery price K_1 is constructed with a purchased call and written put, both with strike prices of K_1. Similarly, a written call and purchased put with strike prices of K_2 is a synthetic short forward with delivery price K_2. The payoff at time T is

$max\{S_T - K_1, 0\} - max\{K_1 - S_T, 0\} - max\{S_T - K_2, 0\} + max\{K_2 - S_T, 0\}$.

If $K_1 < K_2$, then this payoff is $K_2 - K_1$ for all values of S_T. This box spread has the same payoff as a long zero-coupon bond maturing for amount $K_2 - K_1$; there is no risk from the stock price The cost at time 0 should be the same as the present value of this certain payoff. This box spread is the same as lending. If $K_1 > K_2$, this box spread has the same payoff as a short zero-coupon bond, which is the same as borrowing with no risk from the stock price.

Suppose that $K_1 < K_2$. Another way to look at the box spread is

(i) a synthetic long forward with forward price $K_1 =$ purchased call and written put at strike price K_1, and

(i) a synthetic short forward with forward price $K_2 =$ written call and purchased put at strike price K_2, is the same as

(a) purchased call at K_1 and written call at $K_2 =$ bull spread using K_1/K_2 calls, combined with

(a) purchased put at K_2 and written put at $K_1 =$ bear spread using K_2/K_1 puts.

Example 80: We consider the following combination using XYZ options from the previous examples.

(i) purchase call and write put, both with strike price 17, and

(ii) write call and purchase put with strike price 21.

The payoff at time 1 is payoff on purchased call 17 + payoff on written put 17

+ payoff on written call 21 + payoff on purchased put 21

$$= \begin{cases} 0 - (17 - S_1) - 0 + (21 - S_1) = 4 & \text{if } S_1 \le 17 \\ (S_1 - 17) - 0 - 0 + (21 - S_1) = 4 & \text{if } 17 < S_1 \le 21 \\ (S_1 - 17) - 0 - (S_1 - 21) + 0 = 4 & \text{if } S_1 > 21 \end{cases} = 4 .$$

The payoff is 4, no matter what the stock price is at time 1. The cost at time 0 to create this box spread is $5.16 - 1.35 - 3.17 + 3.17 = 3.81$. Under put-call parity, this should be the present value of a payment of 4 to be made at time 1. The present value of the payment using the 5% annual rate of interest is $4v^{13} = 3.81$. □

Ratio Spread

A **ratio spread** is created by buying some number of calls at one strike price and selling another number of calls (not necessarily the same number of calls that were bought) at another strike price. A ratio spread can also be created using puts.

Example 81: We consider the following combination using XYZ options from previous examples.

5 call options with strike price 15 are purchased and 9 call options with strike price 20 are written. The overall premium cost is $5(6.46) - 9(3.59) = -.01$. The payoff at time 1 is

$$\begin{cases} 0 & \text{if } S_1 \leq 15 \\ 5(S_1 - 15) = 5S_1 - 75 & \text{if } 15 < S_1 \leq 20 \\ 5(S_1 - 15) - 9(S_1 - 20) = 105 - 4S_1 & \text{if } S_1 > 20 \end{cases}. \quad \square$$

Collars

It was seen earlier that the combination of a purchased put and a written call, both at the same strike price K and both expiring at the same time T is a synthetic short forward position with delivery price K at time T. If the written call option has a higher strike price than the purchased put option, the combination is referred to as a **collar**, and the difference between the call strike price and the put strike price is the **collar width**. If the stock is owned, the position is referred to as a collared stock.

Example 82: Create a collar using XYZ options with a purchased put at strike price 19 and a written call at strike price 23. Determine the payoff and profit at time 1. Suppose that the stock is owned. Find the payoff an profit of the collared stock position.

Solution: The payoff on the collar is

payoff on purchased put/19 + payoff on written call/23

$$= \begin{cases} 19 - S_1 + 0 = 19 - S_1 & \text{if } S_1 \leq 19 \\ 0 + 0 = 0 & \text{if } 19 < S_1 \leq 23 \\ 0 - (S_1 - 23) = 23 - S_1 & \text{if } S_1 > 23 \end{cases}.$$

The cost of the position at time 0 is $2.16 - 2.45 = -0.29$, and the accumulated cost at time 1 is -0.32 .

Example 82 continued

The profit on the collar is $\text{payoff} - (-.32) = \begin{cases} 19.32 - S_1 & \text{if } S_1 \leq 19 \\ .32 & \text{if } 19 < S_1 \leq 23 \\ 23.32 - S_1 & \text{if } S_1 > 23 \end{cases}$.

If the stock is owned, the payoff on the collared stock position is
payoff on collar + payoff on stock

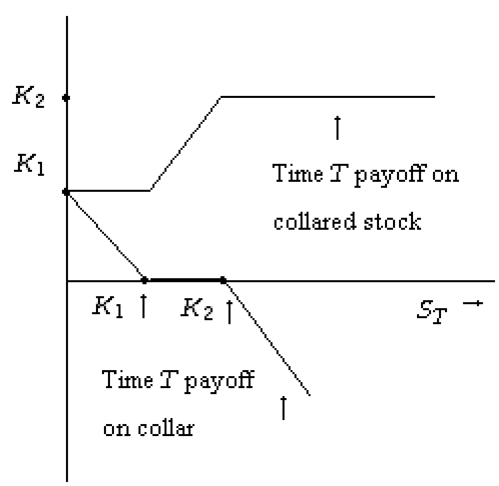
$$= \begin{cases} 19 - S_1 & \text{if } S_1 \leq 19 \\ 0 & \text{if } 19 < S_1 \leq 23 \\ 23 - S_1 & \text{if } S_1 > 23 \end{cases} + S_1 = \begin{cases} 19 & \text{if } S_1 \leq 19 \\ S_1 & \text{if } 19 < S_1 \leq 23 \\ 23 & \text{if } S_1 > 23 \end{cases}.$$

The initial cost of buying the stock at time 0 is 20, so the profit on the collared stock at time 1 is profit on collar + profit on the stock . The profit on the stock is $S_1 - 20(1.05)$, so the profit on the collared stock is $\begin{cases} -1.68 & \text{if } S_1 \leq 19 \\ S_1 - 20.68 & \text{if } 19 < S_1 \leq 23 \\ 2.32 & \text{if } S_1 > 23 \end{cases}$. □

For a general collar consisting of purchased put with strike K_1 and a written call with price K_2, and with $K_1 < K_2$, the payoff on the collar is $\begin{cases} K_1 - S_T & \text{if } S_T \leq K_1 \\ 0 & \text{if } K_1 < S_T \leq K_2 \\ K_2 - S_T & \text{if } S_T > K_2 \end{cases}$.

The profit will be the payoff − accumulated cost of the collar. Depending on the premiums for the two options, the accumulated cost of the collar might be positive or negative. In Example 82 above, the accumulated cost was positive (which means that the written call paid less in premium than the cost of the purchased put; this will not always be the case). If the stock is held along with the collar, the payoff of the collared stock will be $\begin{cases} K_1 & \text{if } S_T \leq K_1 \\ S_T & \text{if } K_1 < S_T \leq K_2 \\ K_2 & \text{if } S_T > K_2 \end{cases}$.

The graph of the payoff on the collar and the payoff on the collared stock are below.



The profit diagrams would have the same shape as the payoffs, but would be shifted vertically up or down depending upon the accumulated value of the cost of establishing the collar or collared stock. A combination of a written put option with strike K_1 and a purchased call option with strike K_2 will provide a collar for a short position in the asset.

The XYZ example just gives strike prices for a few options. With a more extensive set of option prices, we might see that a call option with a strike price of $24 has a premium of $2.15. If we create a collar with a purchased put with strike 19 and a written call with strike 24, the cost of the collar will be $.01 (very close to 0), since both options have almost the same premium. This is a **zero-cost collar**. The payoff and profit on a zero-cost collar are the same.

It is worth noting a more general relationship linking payoff and profit functions and diagrams. If two payoff functions differ by a constant, then this would be represented in the payoff graphs as one payoff graph being the other graph shifted either vertically up or vertically down. This corresponds to the two payoffs differing by the value of a zero-coupon bond. Since a zero-coupon bond always has zero profit, it follows that the profit function is the same for the two payoffs. This general relationship is true if no arbitrage opportunities exist. This can be seen in the following extension of the bull spread in Example 79 and the collared stock in Example 82.

The collared stock in Example 82 is a combination of a long (purchased) put with strike price 19, and a short (written) call with strike price 23, and a long position in the stock. The payoff function is $\begin{cases} 19 & \text{if } S_1 \leq 19 \\ S_1 & \text{if } 19 < S_1 \leq 23 \\ 23 & \text{if } S_1 > 23 \end{cases}$ and the profit function is $\begin{cases} -1.68 & \text{if } S_1 \leq 19 \\ S_1 - 20.68 & \text{if } 19 < S_1 \leq 23 \\ 2.32 & \text{if } S_1 > 23 \end{cases}$.

The shape of the graph of the payoff function of the collared stock can be seen in the diagram at the bottom of the previous page. This is the same shape as the graph of a bull spread based on a purchased call with strike price 19 and a written call with strike price 23. The payoff function of this bull spread is $\begin{cases} 0 & \text{if } S_1 \leq 19 \\ S_1 - 19 & \text{if } 19 < S_1 \leq 23 \\ 4 & \text{if } S_1 > 23 \end{cases}$.

We see that Bull Spread Payoff $=$ Collared Stock Payoff $- 19$.

The premium for the call with strike price 19 is 4.06 (from the previous section) and the premium for the call with strike price 23 is 2.45, so the cost (at time 0) to establish this position is $4.06 - 2.45 = 1.61$, and the accumulated cost at time 1 is $1.61(1.05) = 1.69$.

The profit function for this bull spread is $\begin{cases} -1.69 & \text{if } S_1 \leq 19 \\ S_1 - 20.29 & \text{if } 19 < S_1 \leq 23 \\ 2.31 & \text{if } S_1 > 23 \end{cases}$. This is the same as the profit function on the collared stock (with a .01 difference due to roundoff error).

Another way of seeing that the profit on these two will be the same comes from the put/call parity relationship. According to put/call parity

long stock $+$ long put$(K,T) =$ long call$(K,T) +$ long zero bond paying K at time T .

The collared stock position is long stock $+$ long put 19 $+$ short call 23 .

Using the put/call parity relationship, this is the same as

long call 19 $+$ long zero paying 19 at time $T +$ short call 23 .

The bull spread is long call 19 $+$ short call 23 , so we see that the collared stock position is the same as the bull spread position $+$ long zero paying 19 at time T ($T = 1$ in this case). This is why the payoff on the collared stock is 19 $+$ the payoff on the bull spread. Also, since the profit on a zero (short or long) is 0, it follows that the profit on the collared stock is the same as the profit on the bull spread.

Straddle

A **straddle** is combination of a purchased call and purchased put with the same expiry and strike price. A written straddle would be the combination of a written call and a written put with the same strike price.

Example 83: Formulate the payoff and profit using XYZ options from earlier examples for straddles with strike prices of 20 and 25.

Solution: For strike price 20, the payoff is

payoff on purchased call/20 $+$ payoff on purchased put/20

$= max\{S_1 - 20, 0\} + max\{20 - S_1, 0\} = \begin{cases} 20 - S_1 & \text{if } S_1 \leq 20 \\ S_1 - 20 & \text{if } S_1 > 20 \end{cases}$.

The cost of this straddle at time 0 is $3.59 + 2.64 = 6.23$ and the accumulated cost at time 1 is 6.54. The profit at time 1 is $\begin{cases} 13.46 - S_1 & \text{if } S_1 \leq 20 \\ S_1 - 26.54 & \text{if } S_1 > 20 \end{cases}$.

This strategy produces a profit if the stock price is either below 13.37 or above 26.23 at time 1.

For strike price 25, the payoff on the straddle is

$$max\{S_1-25,0\}+max\{25-S_1,0\}=\begin{cases}25-S_1 & \text{if } S_1\le 25\\ S_1-25 & \text{if } S_1>25\end{cases}.$$

The cost of this straddle at time 0 is $1.89+5.70=7.59$ and the accumulated cost is 7.97.

The profit at time 1 is $\begin{cases}17.03-S_1 & \text{if } S_1\le 25\\ S_1-32.97 & \text{if } S_1>25\end{cases}$.

This strategy produces a profit if the stock price is either below 17.03 or above 32.97 at time 1.□

Strangle

A purchased **strangle** is a combination of purchased call and purchased put options expiring at the same time but with different strike prices. The usual strangle would be a combination of out-of-the-money options, so the put strike would be less than the call strike.

Example 84: Formulate the payoff and profit using XYZ options from earlier examples for a strangle consisting of a purchased call with strike 25 and a purchased put with strike 15.

Solution: The payoff is

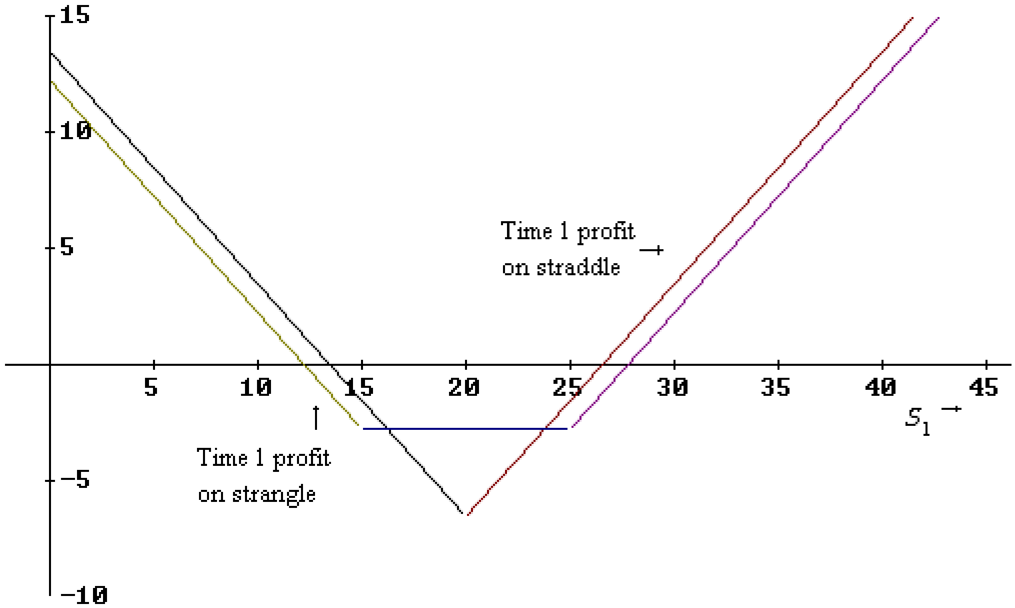
$$max\{S_1-25,0\}+max\{15-S_1,0\}=\begin{cases}15-S_1 & \text{if } S_1\le 15\\ 0 & \text{if } 15<S_1\le 25\\ S_1-25 & \text{if } S_1>25\end{cases}.$$

The cost of this straddle at time 0 is $1.89+.75=2.64$, and the accumulated cost is 2.77.

The profit at time 1 is $\begin{cases}12.23-S_1 & \text{if } S_1\le 15\\ -2.77 & \text{if } 15<S_1\le 25\\ S_1-27.77 & \text{if } S_1>25\end{cases}$.

This strategy produces a profit if the stock price is either below 12.23 or above 27.77 at time 1.□

The graph below shows the profit of the straddle with strike 20 from Example 83 and the strangle from Example 84.



A **written strangle** is the opposite of a purchased strangle. A written strangle consists of a written put and a written call, with the put strike price less than the call strike price (both options are usually out-of-the-money). The payoff and profit would be the negative of a straddle.

Butterfly Spreads

A **butterfly spread** is a combination of a written straddle with a purchased strangle. The purchased strangle provides some insurance against the written straddle.

Example 85: Use the XYZ options to find the payoff and profit on a butterfly spread consisting of a written straddle with strike price 20 combined with a purchased put with strike price 15 and a purchased call with strike price 25 (purchased strangle).

Solution: The combined payoff is

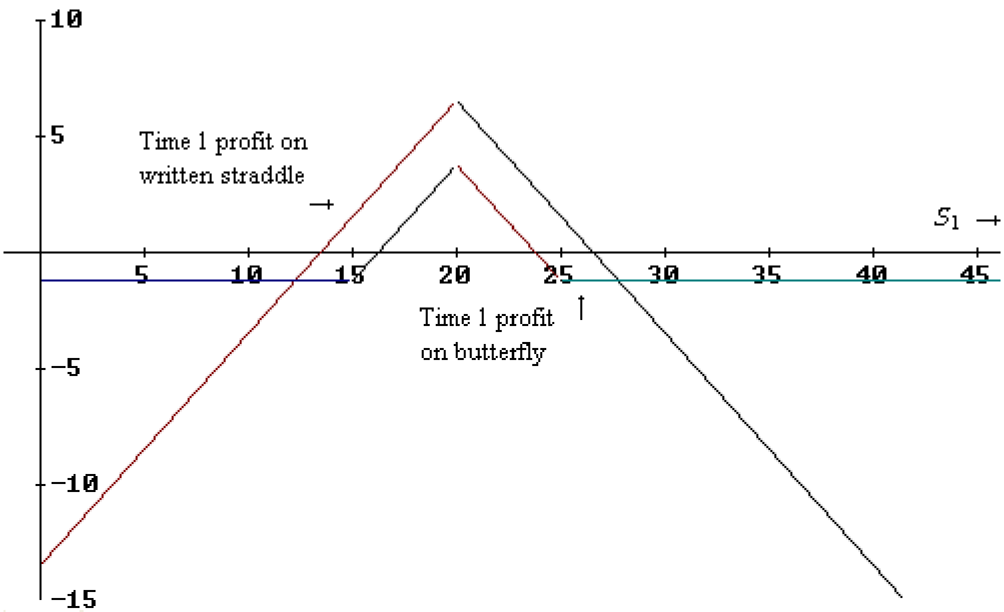
payoff on straddle + payoff on strangle

$$= \begin{cases} S_1 - 20 & \text{if } S_1 \leq 20 \\ 20 - S_1 & \text{if } S_1 > 20 \end{cases} + \begin{cases} 15 - S_1 & \text{if } S_1 \leq 15 \\ 0 & \text{if } 15 < S_1 \leq 25 \\ S_1 - 25 & \text{if } S_1 > 25 \end{cases} = \begin{cases} -5 & \text{if } S_1 \leq 15 \\ S_1 - 20 & \text{if } 15 < S_1 \leq 20 \\ 20 - S_1 & \text{if } 20 < S_1 \leq 25 \\ -5 & \text{if } S_1 > 25 \end{cases}.$$

The cost at time 0 to create this butterfly is $-6.23 + 2.64 = -3.59$, and the accumulated cost is -3.77. (a net amount of premium of 3.59 is received at time 0). The profit at time 1 is

$$= \begin{cases} -5 & \text{if } S_1 \leq 15 \\ S_1 - 20 & \text{if } 15 < S_1 \leq 20 \\ 20 - S_1 & \text{if } 20 < S_1 \leq 25 \\ -5 & \text{if } S_1 > 25 \end{cases} + 3.77 = \begin{cases} -1.23 & \text{if } S_1 \leq 15 \\ S_1 - 16.23 & \text{if } 15 < S_1 \leq 20 \\ 23.77 - S_1 & \text{if } 20 < S_1 \leq 25 \\ -1.23 & \text{if } S_1 > 25 \end{cases}.$$

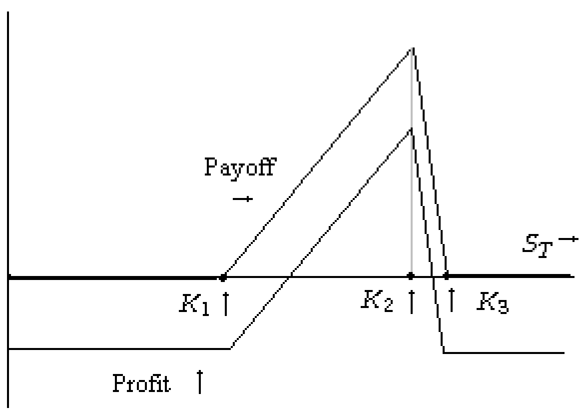
The graph of the profit on the written straddle and the profit on the butterfly spread is below.



□

Asymmetric Butterfly Spread

Given $K_1 < K_2 < K_3$ it is possible to create a butterfly spread that has the following payoff and profit graphs.



This can be done by purchasing λ calls with strike price K_1 and $1-\lambda$ calls with strike price K_3 and writing 1 call with strike price K_2, where $\lambda = \frac{K_3-K_2}{K_3-K_1}$. The payoff will be

$$\begin{cases} 0 & \text{if } S_T \le K_1 \\ \lambda(S_T - K_1) & \text{if } K_1 < S_T \le K_2 \\ \lambda(S_T - K_1) - (S_T - K_2) = K_2 - \lambda K_1 - (1-\lambda)S_T & \text{if } K_2 < S_T \le K_3 \\ K_2 - \lambda K_1 - (1-\lambda)S_T + (1-\lambda)(S_T - K_3) = 0 & \text{if } S_T > K_3 \end{cases} .$$

This is referred to as an asymmetric butterfly spread.

Hedging And Insurance For The Seller Of An Asset

Someone who holds an asset (or a producer of an asset) that will be selling the asset at a later time has various ways to limit the risk on the price to be received at the later time. Limiting the risk is referred to as hedging the risk.

A short forward contract with forward price $F_{0,T}$ locks in the amount that will be received when the asset is sold at time T. Assuming that the forward price is based on no arbitrage opportunities existing, the forward contract eliminates the chance for a profit or loss. The no arbitrage forward price will be $F_{0,T} = S_0 e^{rT}$, so the "return" will be the same as the risk free rate. There is no cost for this forward contract that guarantees a price for the asset.

Buying a put option with strike price K will limit, on the downside, the price that will be received when the asset is sold at time T. Since it is assumed that the asset is being held until time T, the payoff at time T is $max\{K - S_T, 0\} + S_T \geq K$. We have put in place a minimum payoff, but there is the cost of the put option needed to create this insurance hedge. The seller can trade off the potential high price for the asset by selling a call at a higher strike and creating a collared asset. The premium received from selling the call offsets the cost of the put.

Selling a call option with strike price K provides income now and reduces the minimum payoff that will be received when the asset is sold at time T. But the payoff at time T is limited to $S_T - max\{S_T - K, 0\}$ which is $\leq K$, which occurs if the asset value at time T is $> K$ (so the call will be exercised). We are guaranteed at least the accumulated premium on the written call option, but we have an upper limit on the payoff that we will receive when the asset is sold.

A **paylater** strategy for the seller of an asset consists of buying m puts at strike K_1 and selling n puts at strike $K_2 > K_1$, so that the premium at time 0 is $mC_{0,K_1} - nC_{0,K_2} = 0$.
The payoff at time T is $m\,max\{K_1 - S_T, 0\} - n\,max\{K_2 - S_T, 0\}$

$$= \begin{cases} m(K_1 - S_1) - n(K_2 - S_1) & \text{if } S_T \leq K_1 \\ -n(K_2 - S_1) & \text{if } K_1 < S_T \leq K_2 \\ 0 & \text{if } S_1 > K_2 \end{cases}.$$

Since $K_1 < K_2$, it follows that $C_{0,K_1} < C_{0,K_2}$, and we must have $m > n$ in order to have premium 0 at time 0. This arrangement provides no hedging for asset price above K_2. For asset prices between K_1 and K_2 there is a negative payoff and for asset price below K_1 there will be a positive payoff.

Hedging And Insurance For The Purchaser Of An Asset

The strategies that can be employed by a purchaser to hedge the price to be paid when the asset is bought, are the reverse of the strategies that the seller can use. The purchaser can enter a long forward contract, guaranteeing a specific price. The purchaser can guarantee a maximum price of K by buying a call option with strike price K.

PROBLEM SET 19

Option Strategies (2)

Use the following information for problems 1 to 8. Price of XYZ stock at time 0 is 20. Annual effective interest is at rate 5%. Call and put option (European) values for various strike prices are:

Strike Price	Call Price	Put Price
15	6.46	0.75
17	5.16	1.35
19	4.06	2.16
20	3.59	2.64
21	3.17	3.17
23	2.45	4.36
25	1.89	5.70

It is assumed that XYZ stock pays no dividends.

1.(a) Describe the payoff and profit at time 1 of a bull spread consisting of a purchased call with strike price 21 and a written call with strike price 25.
(b) Using put options, construct a bull spread with the same profit as the one in part (a).
(c) Describe the payoff and profit of a bear spread consisting of a written call with strike 21 and a purchased call with strike 25.
(d) Use put-call parity to show that the profit on a bull spread made up of call options with strike prices $K_1 < K_2$ has the same profit as a bull spread made up of put options with the same strike prices.

2. Construct a box spread that has a certain payoff of 4 at time 1 at least two ways. Show that a box spread with a certain payoff of 4 at time 1 has cost $4v$ (use put-call parity).

3. An investor creates a ratio spread with call options with strike prices of 15 and 20.
The minimum payoff is -40, and the payoff is positive for $S_1 > 40$. Determine the number of calls purchased/ written for each strike price. Describe the payoff and profit at time 1.

4. Verify that a collared stock based on a purchased put with strike price K_1 and a written call with strike price $K_2 > K_2$ is the same as a floor with strike price K_1 combined with a written call at K_2.

5. Formulate the payoff and profit on a collar and a collared stock based on strike prices 20 and 25.

6. Based on the table of XYZ option values, what are the approximate strike prices for the purchased put and written call for a zero-cost collar with collar width 2?

7.(a) Formulate the payoff and profit for a straddle with strike 15, and then with strike 25.
(b) Formulate the payoff and profit for the combination of a long straddle at strike 20 and a short straddle at strike 15.
(c) Formulate the payoff and profit for the combination of a long straddle at strike 20 and a short straddle at strike 25.
(d) Compare the profit on the combination in part (c) with the profit on the collared stock in Problem 5. Verify that the profit in (c) is double the profit of the collared stock in Problem 5.

8.(a) Formulate the payoff and profit on a strangle consisting of a purchased put with strike price 19 and a call strike price of 23.
(b) Use the strangle in (a) combined with a written straddle on the stock with strike price 20 to create a butterfly spread. Formulate the payoff and profit for the butterfly spread.

Problems 9 and 10 relate to the following information. The current price of silver is $12 per ounce. The silver mining and refining company SC sells refined silver. SC has a cost of $11 per ounce to mine and refine silver. EP is a company that coats metals in silver with a technique called electroplating. EP electroplates metal at a cost of $1 per ounce plus the cost of silver. EP charges customers $14 per ounce of silver used in the electroplating process.
The 6-month interest rate is 2.5% and you are given the following option prices for options on one ounce of silver expiring in 6 months.

Strike	Call Premium	Put Premium
11	1.464	.196
11.5	1.117	.337
12	.825	.532
12.3	.676	.676
12.5	.589	.784
13	.407	1.090

You are also given that the 6-month forward price of silver is $12.30.

9. SC will be selling refined silver in 6 months. Formulate the profit per ounce of silver for SC 6 months from now in each of the following cases.
(a) SC sells silver in 6 months at the market price.
(b) SC enters a forward contract to sell silver in 6 months.
(c) SC purchases a put option with strike price 11 , 12.3, or 13.
(d) SC writes a call option with strike price 11 , 12.3 , or 13
(e) SC buys a collar with put strike 11 and call strike 13 .
(f) SC buys a paylater by selling a 13-strike put and buying 2 12-strike puts.

10. EP will be buying refined silver in 6 months. Formulate the profit per ounce of silver used for EP 6 months from now in each of the following cases.
(a) EP buys silver in 6 months at the market price.
(b) EP enters a forward contract to buy silver in 6 months.
(c) EP purchases a call option with strike price 11 , 12.3, or 13.
(d) EP writes a put option with strike price 11 , 12.3 , or 13
(e) EP sells a collar with put strike 11 and call strike 13 .
(f) EP buys a paylater by selling a 12-strike call and buying 2 13-strike calls.

PROBLEM SET 19 SOLUTIONS

1.(a) The payoff is $max\{S_1-21,0\}-max\{S_1-25,0\}=\begin{cases}0 & \text{if } S_1\leq 21\\ S_1-21 & \text{if } 21<S_1\leq 25\\ 4 & \text{if } S_1>25\end{cases}$.

The cost of the spread at time 0 is $3.17-1.89=1.28$, and the accumulated cost at time 1 is 1.34. The profit is $\text{payoff}-1.34=\begin{cases}-1.34 & \text{if } S_1\leq 21\\ S_1-22.34 & \text{if } 21<S_1\leq 25\\ 2.66 & \text{if } S_1>25\end{cases}$.

(b) Purchase put option with strike 21 and write a put option with strike 25.

Payoff is $max\{21-S_1,0\}-max\{25-S_1,0\}=\begin{cases}-4 & \text{if } S_1\leq 21\\ S_1-25 & \text{if } 21<S_1\leq 25\\ 0 & \text{if } S_1>25\end{cases}$.

The cost of the spread at time 0 is $3.17-5.70=-2.53$, and the accumulated cost at time 1 is -2.66. The profit is $\text{payoff}-(-2.66)=\begin{cases}-1.34 & \text{if } S_1\leq 21\\ S_1-22.34 & \text{if } 21<S_1\leq 25\\ 2.66 & \text{if } S_1>25\end{cases}$.

(c) Payoff is $-max\{S_1-21,0\}+max\{S_1-25,0\}=\begin{cases}0 & \text{if } S_1\leq 21\\ -(S_1-21) & \text{if } 21<S_1\leq 25\\ -4 & \text{if } S_1>25\end{cases}$.

The cost of the spread at time 0 is $-3.17+1.89=-1.28$, and the accumulated cost at time 1 is -1.34. The profit is $\text{payoff}+1.34=\begin{cases}1.34 & \text{if } S_1\leq 21\\ 22.34-S_1 & \text{if } 21<S_1\leq 25\\ -2.66 & \text{if } S_1>25\end{cases}$.

(d) The profit on the bull spread made up of call options is

$$\begin{cases}0 & \text{if } S_T\leq K_1\\ S_T-K_1 & \text{if } K_1<S_T\leq K_2\\ K_2-K_1 & \text{if } S_T>K_2\end{cases}-(C_{0,K_1}-C_{0,K_2})e^{rT}.$$

The profit on the bull spread made up of put option is

$$\begin{cases}-(K_2-K_1) & \text{if } S_T\leq K_1\\ S_T-K_2 & \text{if } K_1<S_T\leq K_2\\ 0 & \text{if } S_T>K_2\end{cases}-(P_{0,K_1}-P_{0,K_2})e^{rT}.$$

From put-call parity, we have $C_{0,K_1}+K_1e^{-rT}=P_{0,K_1}+S_0$, and
$C_{0,K_2}+K_2e^{-rT}=P_{0,K_2}+S_0$. Therefore,
$C_{0,K_1}-C_{0,K_2}+(K_1-K_2)e^{-rT}=P_{0,K_1}-P_{0,K_2}$.
Substituting this left side of this equation for $P_{0,K_1}-P_{0,K_2}$ in the profit based on put options, results in the profit based on call options.

2. A box spread is a synthetic long forward with forward price K_1 combined with a synthetic short forward with forward price K_2. The payoff is a certain payment of $K_2 - K_1$. We want $K_2 - K_1 = 4$. There are several ways to do construct the box spread.
(i) $K_1 = 17$, $K_2 = 21$. The synthetic long forward with forward price 17 is a purchased call and written put both with strike price 17. The synthetic short forward with forward price 21 is a written call and purchased put, both with strike price 21. The cost at time 0 is
$C_{0,17} - P_{0,17} - C_{0,21} + P_{0,21} = 5.16 - 1.35 - 3.17 + 3.17 = 3.81$.
(ii) $K_1 = 23$, $K_2 = 19$. The synthetic long forward with forward price 19 is a purchased call and written put both with strike price 19. The synthetic short forward with forward price 23 is a written call and purchased put, both with strike price 23. The cost at time 0 is
$C_{0,19} - P_{0,19} - C_{0,23} + P_{0,23} = 4.06 - 2.16 - 2.45 + 4.36 = 3.81$.

A synthetic long forward with forward price K_1 has cost at time 0 of $C_{0,K_1} - P_{0,K_1}$,
and the synthetic short forward with forward price K_2 has cost at time 0 of $-C_{0,K_2} + P_{0,K_2}$.
The overall cost of the box spread is $C_{0,K_1} - P_{0,K_1} - C_{0,K_2} + P_{0,K_2}$.
From put-call parity, we have $C_{0,K_1} + K_1 e^{-rT} = P_{0,K_1} + S_0$, and
$C_{0,K_2} + K_2 e^{-rT} = P_{0,K_2} + S_0$. Then,
$C_{0,K_1} - P_{0,K_1} = S_0 - K_1 e^{-rT}$ and $-C_{0,K_2} + P_{0,K_2} = -S_0 + K_2 e^{-rT}$,
so that $C_{0,K_1} - P_{0,K_1} - C_{0,K_2} + P_{0,K_2} = S_0 - K_1 e^{-rT} - S_0 + K_2 e^{-rT}$
$= (K_2 - K_1)e^{-rT}$.

3. With m calls at 15 and n calls at 20, the payoff at time 1 is

$$\begin{cases} 0 & \text{if } S_1 \le 15 \\ m(S_1 - 15) = mS_1 - 15m & \text{if } 15 < S_1 \le 20 \\ m(S_1 - 15) + n(S_1 - 20) = (m+n)S_1 - 15m - 20n & \text{if } S_1 > 20 \end{cases}$$

Since there is a minimum payoff of -40, the payoff must decrease from 0 when $S_1 = 15$ to $-40 = m(20 - 15) = 5m$. It follows that $m = -8$, which means 8 options are written with strike 15. The payoff must increase for $S_1 > 15$, and in order for the payoff to become positive when $S_1 > 40$, it must be the case that the payoff is 0 if $S_1 = 40$. Therefore,
$(m+n)(40) - 15m - 20n = (-8+n)(40) - 15(-8) - 20n = 0$,
from which we get $n = 10$. Therefore, the ratio spread consists of 8 written calls at 15 and 10 purchased calls at 20. The payoff is

$$\begin{cases} 0 & \text{if } S_1 \le 15 \\ -8(S_1 - 15) = 120 - 8S_1 & \text{if } 15 < S_1 \le 20 \\ -8(S_1 - 15) + 10(S_1 - 20) = 2S_1 - 80 & \text{if } S_1 > 20 \end{cases} .$$

3 continued

The cost of the ratio spread at time 0 is $-8(6.46)+10(3.59)=-15.78$, and the accumulated cost at time 1 is -16.57 . The profit at time 1 is

$$\begin{cases} 16.57 & \text{if } S_1 \leq 15 \\ 136.57-8S_1 & \text{if } 15 < S_1 \leq 20 \\ 2S_1-79.21 & \text{if } S_1 > 20 \end{cases}.$$

4. A floor is a combination of owning the stock along with a purchased put at strike K_1. Combining this with a written call at $K_2 > K_1$ results in the purchased put and written call, which is a collar, and then with the owned stock the position becomes a collared stock.

5. The collar consists of a purchased put 20 and a written call 25. The payoff is

$$max\{20-S_1,0\}-max\{S_1-25,0\}=\begin{cases} 20-S_1 & \text{if } S_1 \leq 20 \\ 0 & \text{if } 20 < S_1 \leq 25 \\ 25-S_1 & \text{if } S_1 > 25 \end{cases}.$$

The cost at time 0 for the collar is $2.64-1.89=0.75$, and the accumulated cost at time 1 is 0.79. The profit at time 1 is $\begin{cases} 19.21-S_1 & \text{if } S_1 \leq 20 \\ -.79 & \text{if } 20 < S_1 \leq 25 \\ 24.21-S_1 & \text{if } S_1 > 25 \end{cases}$.

The payoff on a long stock is S_1 at time S_1 and the profit is S_1-21.

The payoff on the collared stock is $\begin{cases} 20 & \text{if } S_1 \leq 20 \\ S_1 & \text{if } 20 < S_1 \leq 25 \\ 25 & \text{if } S_1 > 25 \end{cases}$,

and the profit is $\begin{cases} -1.79 & \text{if } S_1 \leq 20 \\ S_1-21.79 & \text{if } 20 < S_1 \leq 25 \\ 3.21-S_1 & \text{if } S_1 > 25 \end{cases}$.

6. The cost of the collar with purchased put at K and written call at $K+2$, is $P_{0,K}-C_{0,K+2}$. By trial and error, we see that with $K=19$,

$P_{0,19}-C_{0,21}=2.16-3.17=-1.01$, and with $K=21$,

$P_{0,21}-C_{0,23}=2.16-3.17=3.17-2.45=0.72$.

It appears that to get a zero-cost collar, the purchased put should have strike K between 21 and 22.

7.(a)(i) Strike 15, payoff $= max\{S_1-15,0\}+max\{15-S_1,0\} = \begin{cases} 15-S_1 & \text{if } S_1 \le 15 \\ S_1-15 & \text{if } S_1 > 15 \end{cases}$.

Cost of straddle at time 0 is $6.46+0.75=7.21$, accumulate value is 7.57.

Profit is $\begin{cases} 7.43-S_1 & \text{if } S_1 \le 15 \\ S_1-22.57 & \text{if } S_1 > 15 \end{cases}$.

(ii) Strike 25, payoff $= max\{S_1-25,0\}+max\{25-S_1,0\} = \begin{cases} 25-S_1 & \text{if } S_1 \le 25 \\ S_1-25 & \text{if } S_1 > 25 \end{cases}$.

Cost of straddle at time 0 is $1.89+5.70=7.59$, accumulate value is 7.97.

Profit is $\begin{cases} 17.03-S_1 & \text{if } S_1 \le 25 \\ S_1-32.97 & \text{if } S_1 > 25 \end{cases}$.

(b) Long straddle 20 payoff is $\begin{cases} 20-S_1 & \text{if } S_1 \le 20 \\ S_1-20 & \text{if } S_1 > 20 \end{cases}$,

and short straddle 15 payoff is $-\begin{cases} 15-S_1 & \text{if } S_1 \le 15 \\ S_1-15 & \text{if } S_1 > 15 \end{cases} = \begin{cases} S_1-15 & \text{if } S_1 \le 15 \\ 15-S_1 & \text{if } S_1 > 15 \end{cases}$.

Combined payoff is $\begin{cases} 5 & \text{if } S_1 \le 15 \\ 35-2S_1 & \text{if } 15 < S_1 \le 20 \\ -5 & \text{if } S_1 > 20 \end{cases}$.

Long straddle 20 profit (from Example in notes) is $\begin{cases} 13.46-S_1 & \text{if } S_1 \le 20 \\ S_1-26.54 & \text{if } S_1 > 20 \end{cases}$,

and short straddle 15 profit is $-\begin{cases} 7.43-S_1 & \text{if } S_1 \le 15 \\ S_1-22.57 & \text{if } S_1 > 15 \end{cases}$.

Combined profit is $\begin{cases} 6.03 & \text{if } S_1 \le 15 \\ 36.03-2S_1 & \text{if } 15 < S_1 \le 20 \\ -3.97 & \text{if } S_1 > 20 \end{cases}$.

(c) From the Example in the notes, the payoff and profit of a long straddle 25 is

payoff $= \begin{cases} 25-S_1 & \text{if } S_1 \le 25 \\ S_1-25 & \text{if } S_1 > 25 \end{cases}$, and profit $= \begin{cases} 17.03-S_1 & \text{if } S_1 \le 25 \\ S_1-32.97 & \text{if } S_1 > 25 \end{cases}$.

Combined payoff on long straddle 20 and short straddle 25 is

$$\begin{cases} 20-S_1 & \text{if } S_1 \le 20 \\ S_1-20 & \text{if } S_1 > 20 \end{cases} - \begin{cases} 25-S_1 & \text{if } S_1 \le 25 \\ S_1-25 & \text{if } S_1 > 25 \end{cases} = \begin{cases} -5 & \text{if } S_1 \le 20 \\ 2S_1-45 & \text{if } 20 < S_1 \le 25 \\ 5 & \text{if } S_1 > 25 \end{cases}.$$

Combined profit on long straddle 20 and short straddle 25 is

$$\begin{cases} 13.46-S_1 & \text{if } S_1 \le 20 \\ S_1-26.54 & \text{if } S_1 > 20 \end{cases} - \begin{cases} 17.03-S_1 & \text{if } S_1 \le 25 \\ S_1-32.97 & \text{if } S_1 > 25 \end{cases}$$

$$= \begin{cases} -3.57 & \text{if } S_1 \le 20 \\ 2S_1-43.57 & \text{if } 20 < S_1 \le 25 \\ 6.43 & \text{if } S_1 > 25 \end{cases}.$$

7.(d) $\begin{cases} -3.57 & \text{if } S_1 \le 20 \\ 2S_1 - 43.57 & \text{if } 20 < S_1 \le 25 \\ 6.43 & \text{if } S_1 > 25 \end{cases} = 2 \times \begin{cases} -1.79 & \text{if } S_1 \le 20 \\ S_1 - 21.79 & \text{if } 20 < S_1 \le 25 \\ 3.21 - S_1 & \text{if } S_1 > 25 \end{cases}$.

8.(a) Payoff $= max\{19 - S_1, 0\} + max\{S_1 - 23, 0\} = \begin{cases} 19 - S_1 & \text{if } S_1 \le 19 \\ 0 & \text{if } 19 < S_1 \le 23 \\ S_1 - 23 & \text{if } S_1 > 23 \end{cases}$.

Cost of strangle at time 0 is $2.16 + 2.45 = 4.61$, and accumulated cost at time 1 is 4.84.

Profit at time 1 on strangle is $\begin{cases} 14.16 - S_1 & \text{if } S_1 \le 19 \\ -4.84 & \text{if } 19 < S_1 \le 23 \\ S_1 - 27.84 & \text{if } S_1 > 23 \end{cases}$.

(b) Written straddle has payoff $\begin{cases} S_1 - 20 & \text{if } S_1 \le 20 \\ 20 - S_1 & \text{if } S_1 > 20 \end{cases}$

and profit $\begin{cases} S_1 - 13.46 & \text{if } S_1 \le 20 \\ 26.54 - S_1 & \text{if } S_1 > 20 \end{cases}$ (from example in the notes).

The payoff on the butterfly spread is

$$\begin{cases} S_1 - 20 & \text{if } S_1 \le 20 \\ 20 - S_1 & \text{if } S_1 > 20 \end{cases} + \begin{cases} 19 - S_1 & \text{if } S_1 \le 19 \\ 0 & \text{if } 19 < S_1 \le 23 \\ S_1 - 23 & \text{if } S_1 > 23 \end{cases} = \begin{cases} -1 & \text{if } S_1 \le 19 \\ S_1 - 20 & \text{if } 19 < S_1 \le 20 \\ 20 - S_1 & \text{if } 20 < S_1 \le 23 \\ -3 & \text{if } S_1 > 23 \end{cases}.$$

The profit on the butterfly spread is

$$\begin{cases} S_1 - 13.46 & \text{if } S_1 \le 20 \\ 26.54 - S_1 & \text{if } S_1 > 20 \end{cases} + \begin{cases} 14.16 - S_1 & \text{if } S_1 \le 19 \\ -4.84 & \text{if } 19 < S_1 \le 23 \\ S_1 - 27.84 & \text{if } S_1 > 23 \end{cases}$$

$$= \begin{cases} .70 & \text{if } S_1 \le 19 \\ S_1 - 18.30 & \text{if } 19 < S_1 \le 20 \\ 21.70 - S_1 & \text{if } 20 < S_1 \le 23 \\ -1.30 & \text{if } S_1 > 23 \end{cases}.$$

9.(a) Cost per ounce is 11. Profit per ounce at time .5 is $S_{.5} - 11(.1025) = S_{.5} - 11.275$.

(b) Profit at time .5 is $12.3 - 11.275 = 1.025$.

(c)(i) 11-strike put has premium .196. Profit at time .5 is

$$S_{.5} - 11.275 + \begin{cases} 11 - S_{.5} & \text{if } S_{.5} \le 11 \\ 0 & \text{if } S_{.5} > 11 \end{cases} - (.196)(1.025) = \begin{cases} -.476 & \text{if } S_{.5} \le 11 \\ S_{.5} - 11.476 & \text{if } S_{.5} > 11 \end{cases}.$$

(ii) 12.3-strike put has premium .676. Profit at time .5 is

9 continued

$$S_{.5}-11.275+\begin{cases}12.3-S_{.5} & \text{if } S_{.5}\le 12.3\\ 0 & \text{if } S_{.5}>12.3\end{cases}-(.676)(1.025)=\begin{cases}.332 & \text{if } S_{.5}\le 12.3\\ S_{.5}-11.978 & \text{if } S_{.5}>12.3\end{cases}$$

(ii) 13-strike put has premium 1.09. Profit at time .5 is

$$S_{.5}-11.275+\begin{cases}13-S_{.5} & \text{if } S_{.5}\le 13\\ 0 & \text{if } S_{.5}>13\end{cases}-(1.09)(1.025)=\begin{cases}.608 & \text{if } S_{.5}\le 13\\ S_{.5}-12.392 & \text{if } S_{.5}>13\end{cases}$$

(d)(i) 11-strike call has premium 1.464. Profit at time .5 is

$$S_{.5}-11.275+(1.464)(1.025)-\begin{cases}0 & \text{if } S_{.5}\le 11\\ S_{.5}-11 & \text{if } S_{.5}>11\end{cases}=\begin{cases}S_{.5}-9.774 & \text{if } S_{.5}\le 11\\ 1.226 & \text{if } S_{.5}>11\end{cases}.$$

(ii) 12.3-strike call has premium .676. Profit at time .5 is

$$S_{.5}-11.275+(.676)(1.025)-\begin{cases}0 & \text{if } S_{.5}\le 12.3\\ S_{.5}-12.3 & \text{if } S_{.5}>12.3\end{cases}=\begin{cases}S_{.5}-10.582 & \text{if } S_{.5}\le 12.3\\ 1.718 & \text{if } S_{.5}>12.3\end{cases}$$

(ii) 13-strike call has premium .407. Profit at time .5 is

$$S_{.5}-11.275+(.407)(1.025)\begin{cases}0 & \text{if } S_{.5}\le 13\\ S_{.5}-13 & \text{if } S_{.5}>13\end{cases}=\begin{cases}S_{.5}-10.858 & \text{if } S_{.5}\le 13\\ 2.142 & \text{if } S_{.5}>13\end{cases}$$

(e) Collar consists of buying 11-strike put and selling 13-strike call. Premium is $.196-.407=-.211$. Profit at time .5 is

$$S_{.5}-11.275+(.211)(1.025)+\begin{cases}11-S_{.5} & \text{if } S_1\le 11\\ 0 & \text{if } 11<S_1\le 13\\ -(S_{.5}-13) & \text{if } S_1>13\end{cases}$$

$$=\begin{cases}-.059 & \text{if } S_1\le 11\\ S_{.5}-11.059 & \text{if } 11<S_1\le 13\\ 1.941 & \text{if } S_1>13\end{cases}.$$

(f) Paylater premium is $2(.532)-1.09=-.026$. Profit at time .5 is

$$S_{.5}-11.275+(.026)(1.025)+\begin{cases}2(12-S_{.5})-(13-S_{.5}) & \text{if } S_1\le 12\\ -(13-S_{.5}) & \text{if } 12<S_1\le 13\\ 0 & \text{if } S_1>13\end{cases}$$

$$=\begin{cases}-.248 & \text{if } S_1\le 12\\ 2S_{.5}-24.248 & \text{if } 12<S_1\le 13\\ S_{.5}-11.248 & \text{if } S_1>13\end{cases}.$$

10.(a) Cost per ounce is at time .5 is $S_{.5}$. Profit per ounce at time .5 is $14 - S_{.5} - 1 = 13 - S_{.5}$.

(b) Profit at time .5 is $14 - 1 - 12.3 = 0.70$ per ounce .

(c)(i) 11-strike call has premium 1.464. Profit at time .5 is

$$14 - 1 - S_{.5} - (1.464)(1.025) + \begin{cases} 0 & \text{if } S_{.5} \le 11 \\ S_{.5} - 11 & \text{if } S_{.5} > 11 \end{cases} = \begin{cases} 11.499 - S_{.5} & \text{if } S_{.5} \le 11 \\ .499 & \text{if } S_{.5} > 11 \end{cases}$$

(ii) 12.3-strike call has premium .676. Profit at time .5 is

$$14 - 1 - S_{.5} - (.676)(1.025) + \begin{cases} 0 & \text{if } S_{.5} \le 12.3 \\ S_{.5} - 12.3 & \text{if } S_{.5} > 12.3 \end{cases} = \begin{cases} 12.307 - S_{.5} & \text{if } S_{.5} \le 12.3 \\ .007 & \text{if } S_{.5} > 12.3 \end{cases}$$

(iii) 13-strike call has premium .407. Profit at time .5 is

$$14 - 1 - S_{.5} - (.407)(1.025) + \begin{cases} 0 & \text{if } S_{.5} \le 13 \\ S_{.5} - 13 & \text{if } S_{.5} > 13 \end{cases} = \begin{cases} 12.583 - S_{.5} & \text{if } S_{.5} \le 13 \\ -.417 & \text{if } S_{.5} > 13 \end{cases}$$

(d)(i) 11-strike put has premium .196. Profit at time .5 is

$$14 - 1 - S_{.5} + (.196)(1.025) - \begin{cases} 11 - S_{.5} & \text{if } S_{.5} \le 11 \\ 0 & \text{if } S_{.5} > 11 \end{cases} = \begin{cases} 2.201 & \text{if } S_{.5} \le 11 \\ 13.201 - S_{.5} & \text{if } S_{.5} > 11 \end{cases}.$$

(ii) 12.3-strike put has premium .676. Profit at time .5 is

$$14 - 1 - S_{.5} + (.676)(1.025) - \begin{cases} 12.3 - S_{.5} & \text{if } S_{.5} \le 12.3 \\ 0 & \text{if } S_{.5} > 12.3 \end{cases} = \begin{cases} 1.393 & \text{if } S_{.5} \le 12.3 \\ 13.693 - S_{.5} & \text{if } S_{.5} > 12.3 \end{cases}.$$

(iii) 13-strike put has premium 1.090. Profit at time .5 is

$$14 - 1 - S_{.5} + (1.09)(1.025) - \begin{cases} 13 - S_{.5} & \text{if } S_{.5} \le 13 \\ 0 & \text{if } S_{.5} > 13 \end{cases} = \begin{cases} 1.117 & \text{if } S_{.5} \le 13 \\ 14.117 - S_{.5} & \text{if } S_{.5} > 12.3 \end{cases}.$$

(e) Collar consists of selling 11-strike put and buying 13-strike call. Premium is $.407 - .196 = .211$. Profit at time .5 is

$$14 - 1 - S_{.5} - (.211)(1.025) + \begin{cases} -(11 - S_{.5}) & \text{if } S_1 \le 11 \\ 0 & \text{if } 11 < S_1 \le 13 \\ S_{.5} - 13 & \text{if } S_1 > 13 \end{cases}$$

$$= \begin{cases} 1.784 & \text{if } S_1 \le 11 \\ 12.784 - S_{.5} & \text{if } 11 < S_1 \le 13 \\ -.216 & \text{if } S_1 > 13 \end{cases} .$$

(f) Paylater premium is $2(.407) - .825 = -.011$. Profit at time .5 is

$$14 - 1 - S_{.5} + (.011)(1.025) + \begin{cases} 0 & \text{if } S_1 \le 12 \\ -(S_{.5} - 12) & \text{if } 12 < S_1 \le 13 \\ 2(S_{.5} - 13) - (S_{.5} - 12) & \text{if } S_1 > 13 \end{cases}$$

$$= \begin{cases} 13.011 - S_{.5} & \text{if } S_1 \le 12 \\ 25.011 - 2S_{.5} & \text{if } 12 < S_1 \le 13 \\ -.989 & \text{if } S_1 > 13 \end{cases} .$$