ACT 240, NOV. 10/08

Note Title

TEST 2 - FRI. NOV. 14, 10-11 AM

INC/DEC ANNUITIES, LOAN AMORTIZATION

RBC EXAMPLE

$(1+j)^6 = 1.04$

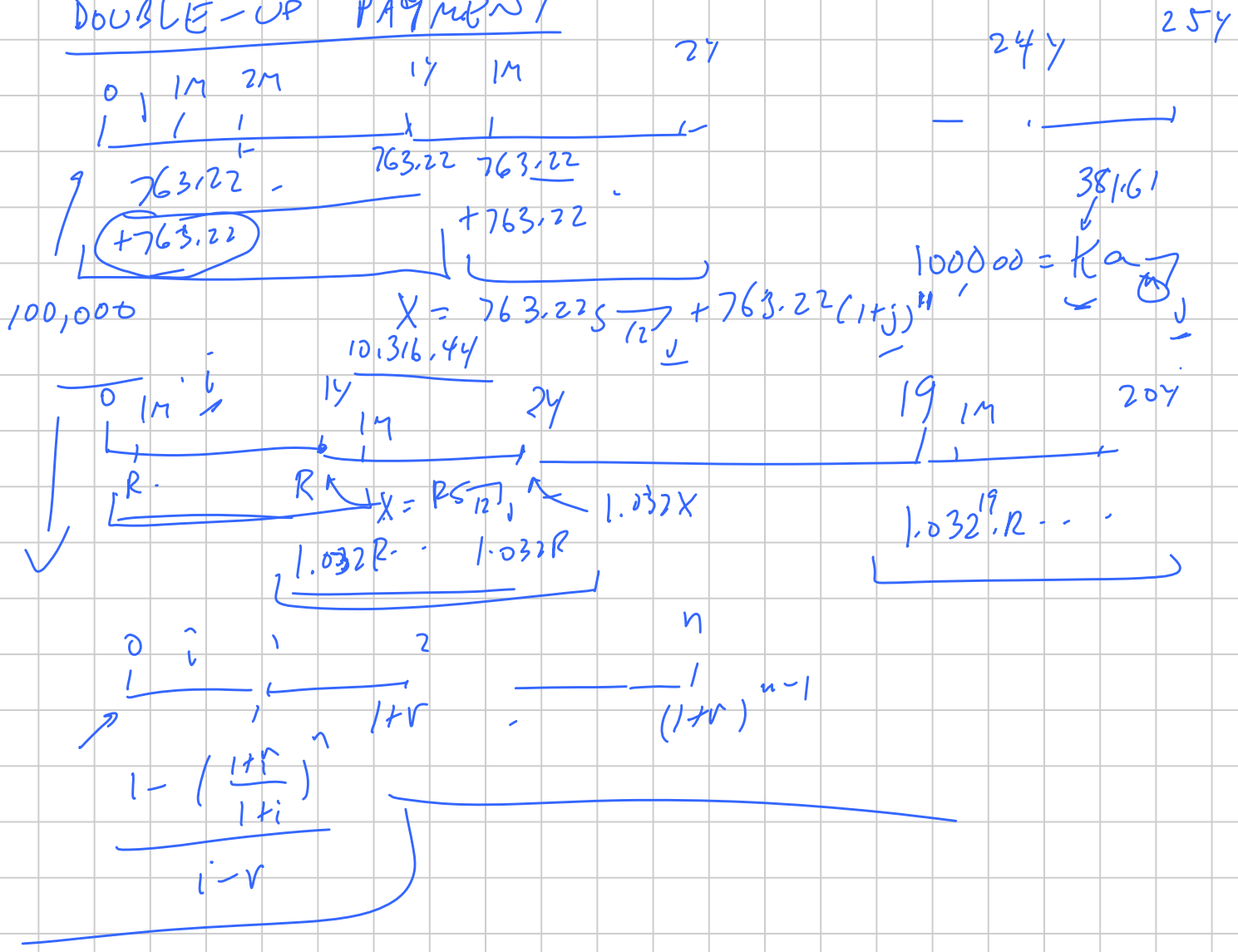
j = 1 MO. EFF INT RATE

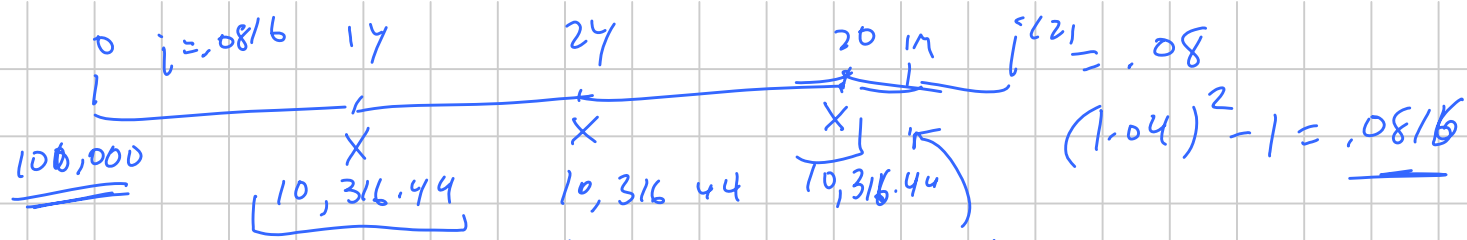
$= .006558$

100,000 LOAN, 25 YR MONTHLY PMTS, LOAN RATE 8%

NOMINAL CONV. SEMI-ANN

DOUBLE-UP PAYMENT





$100,000 = 10,316.44\, a_{\overline{n}|.0816}$

763.22
+763.22

$20 < n < 21$

$100,000(1.0816)^{20} - 10,316.44\, s_{\overline{20}|.0816} = OB$ 20 YEAR

240 MO

ADJUSTABLE RATE MORTGAGE

$j = .04/12$

$200,000 / a_{\overline{360}|j} = 954.83$

1 YEAR .06 → .005/MONTH

$OB_{1Y} = 954.83\, a_{\overline{348}|j}$

$OB_{1Y} / a_{\overline{348}|.005} =$

SINKING FUND METHOD OF LOAN REPAYMENT

BORROWER

MAKES INT PMTS TO LENDER

MAKES DEPOSITS TO DEPOSIT ACCT (SINKING FUND) TO ACCUMULATE L AT END OF n YRS

SINKING FUND RATE IS j → ANNUAL DEP = $L / s_{\overline{n}|j}$

TOTAL AMT. /YR REQUIRED BY BORROWER

$L\left[i + \frac{1}{s_{\overline{n}|j}}\right]$

$(1+j)^k < (1+i)^k$

SUPPOSE $j < i \rightarrow s_{\overline{n}|j} < s_{\overline{n}|i} \rightarrow \frac{1}{s_{\overline{n}|j}} > \frac{1}{s_{\overline{n}|i}}$

$L\left[i + \frac{1}{s_{\overline{n}|j}}\right] > L\left[i + \frac{1}{s_{\overline{n}|i}}\right]$

$i = .10 \quad j = .08$

$i' > i$

LEVEL PMT FOR AMORT AT i

$L \equiv PMT$ FOR $a_{\overline{n}|i}$

$\frac{1}{a_{\overline{n}|i}} = i + \frac{1}{s_{\overline{n}|i}}$

$i + iv^n - iv^n$

$1 - v^n$

	0	1	2	4	5	10
LENDER, $i = .1$	L	Li	Li		Li	Li, L
	100,000	10,000	10,000			10,000
		I_1	I_2			I_n
		$PR_1 = 0$	$PR_2 = 0$			$PR_n =$ 100,000
BORROWER $j = .08$		10,000	10,000		10,000	10,000
		6903	6903		6903	6903 → $6903 s_{\overline{10}|.08}$ = 100,000

$OB_5 = 100{,}000 - 6903\, s_{\overline{5}|.08}$

$I_5 = 10{,}000 - SF_4 \times j$

$6903\, s_{\overline{4}|.08} \times (.08)$

DOLLAR-WEIGHTED RATE OF RETURN (APPROX INTERNAL RATE OF RETURN)

$+100$ $+100$ -50

$$1000(1+i) + 100(1+i)^{5/6} + 100(1+i)^{1/4} - 50(1+i)^{1/6} = 1300$$

USES SIMPLE INTEREST

$i = .1366$

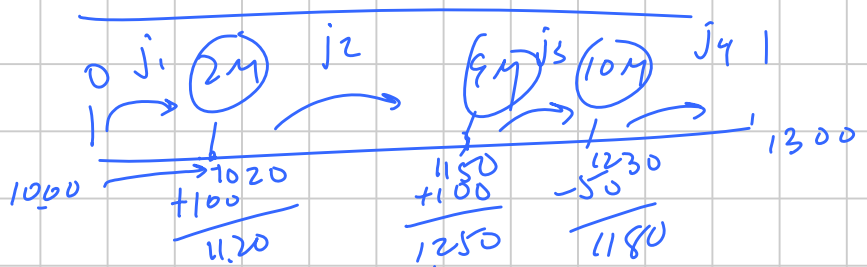
$$1000(1+i) + 100\left(1+\tfrac{5}{6}i\right) + 100\left(1+\tfrac{1}{4}i\right) - 50\left(1+\tfrac{1}{6}i\right) = 1300$$

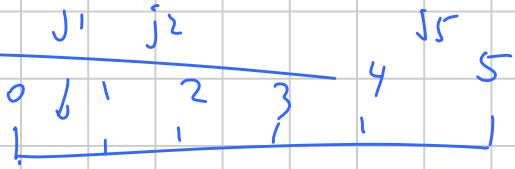
$$i_D = \frac{150}{1000 + 100\left(\frac{5}{6}\right) + 100\left(\frac{1}{4}\right) - 50\left(\frac{1}{6}\right)}$$

$i_D = .1364$

$$\frac{150}{1000 + 150\left(\frac{1}{2}\right)}$$

TIME-WEIGHTED RETURN





$$= (1+i_5)^5$$

$$(1+j_1)(1+j_2)\cdots(1+j_5)$$

$$(1+j_1)(1+j_2)(1+j_3)(1+j_4) = 1+i_T$$

$$\left(\frac{1020}{1000}\right)\left(\frac{1150}{1120}\right)\left(\frac{1230}{1250}\right)\left(\frac{1300}{1180}\right) = 1.1354$$

i_T

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Close

Full-Amortization Table - Monthly

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Show Yearly Amortization Table

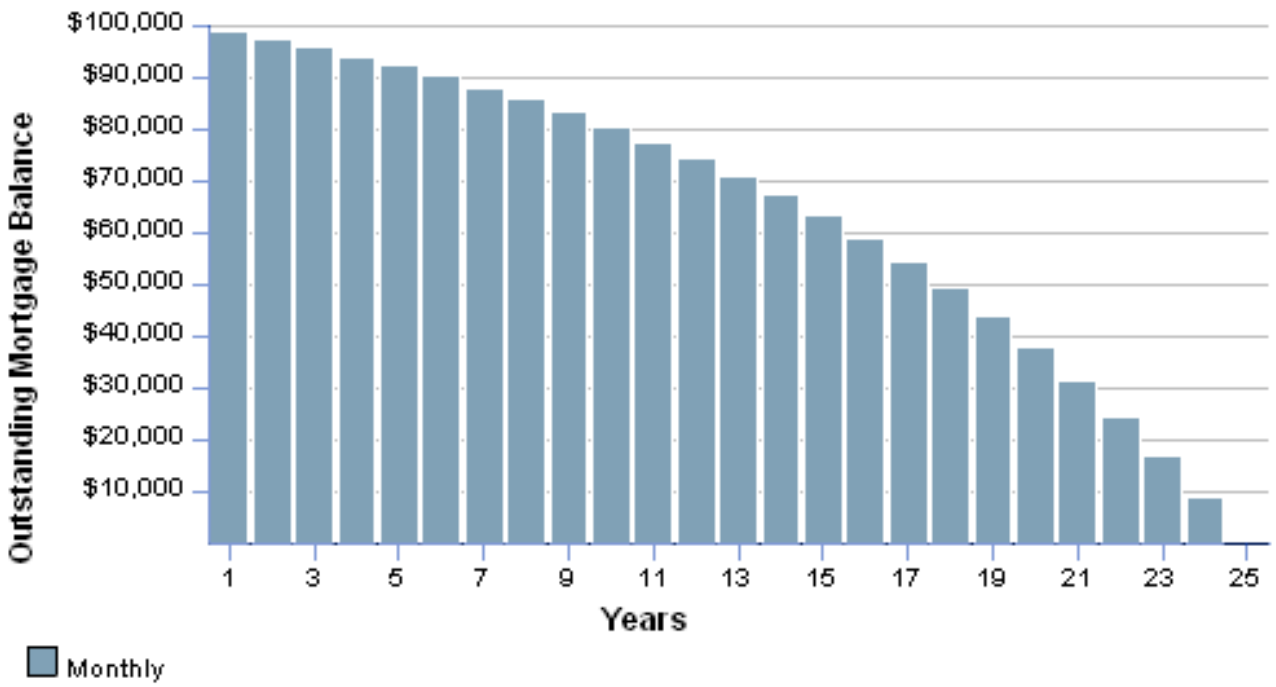
#	Principal Outstanding	Principal & Interest Payment	Interest	Principal	New Principal Outstanding	Payment Options Selected
1	$100,000.00	$1,526.44	$655.82	$870.62	$99,129.38	
2	$99,129.38	$763.22	$650.11	$113.11	$99,016.27	
3	$99,016.27	$763.22	$649.37	$113.85	$98,902.42	
4	$98,902.42	$763.22	$648.62	$114.60	$98,787.82	
5	$98,787.82	$763.22	$647.87	$115.35	$98,672.47	
6	$98,672.47	$763.22	$647.11	$116.11	$98,556.36	
7	$98,556.36	$763.22	$646.35	$116.87	$98,439.50	
8	$98,439.50	$763.22	$645.59	$117.63	$98,321.86	
9	$98,321.86	$763.22	$644.81	$118.41	$98,203.45	
10	$98,203.45	$763.22	$644.04	$119.18	$98,084.27	
11	$98,084.27	$763.22	$643.26	$119.96	$97,964.31	
12	$97,964.31	$763.22	$642.47	$120.75	$97,843.56	
13	$97,843.56	$1,526.44	$641.68	$884.76	$96,958.79	
14	$96,958.79	$763.22	$635.87	$127.35	$96,831.45	
15	$96,831.45	$763.22	$635.04	$128.18	$96,703.27	
16	$96,703.27	$763.22	$634.20	$129.02	$96,574.25	
17	$96,574.25	$763.22	$633.35	$129.87	$96,444.38	

Mortgage Payment Calculator

Print Page

Payment Frequency	Payment Amount	Amortization	Term Interest Cost	Amortization Interest Cost	Amortization Interest Savings vs. Monthly Payment
Monthly	$763.22	25.0 yrs	$128,959.91	$128,959.91	$0.00
Semi-monthly	$381.61	24.9 yrs	$127,800.58	$127,800.58	$1,165.66
Bi-weekly	$352.26	24.7 yrs	$126,090.92	$126,090.92	$2,875.32
Weekly	$176.13	24.6 yrs	$125,573.50	$125,573.50	$3,392.74
Accelerated Bi-weekly	$381.61	19.9 yrs	$97,520.79	$97,520.79	$31,445.45
Accelerated Weekly	$190.81	19.9 yrs	$97,157.05	$97,157.05	$31,809.19

Mortgage Amortization Paydown



Results Summary

Mortgage Amount:
$100,000.00

Interest Rate Type:
Fixed

Payment Amount:
$763.22

Amortization Period:
25 years 0 months

Payment Frequency:
Monthly

Interest Term:
25 years 0 months

Interest Rate:
8.000%

Update Your Calculation

- Double-Up Payment
- Anniversary Payment
- Skip-A-Payment
- Change & Compare Scenarios

Show Amortization Table

34. An investor took out a loan of 150,000 at 8% compounded quarterly, to be repaid over 10 years with quarterly payments of 5483.36 at the end of each quarter. After 12 payments, the interest rate dropped to 6% compounded quarterly. The new quarterly payment dropped to 5134.62 .

After 20 payments in total, the interest rate on the loan increased to 7% compounded quarterly. The investor decided to make an additional payment of X at the time of his 20th payment. After the additional payment was made, the new quarterly payment was calculated to be 4265.73, payable for five more years.

Determine X .

(A) 11,047

(B) 13,369

(C) 16,691

(D) 20,152

(E) 23,614

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View Rates

Fixed Rate Mortgages*

Convertible

Term	Posted Rate	Special Offers‡
6 Month	6.35%	5.29%[1]

Closed

Term	Posted Rate	Special Offers‡
1 Year	6.35%	Call for details
2 Year	6.70%	Call for details
3 Year	6.70%	Call for details
4 Year	7.04%	5.98%[2]
5 Year	7.20%	6.14%[3]
7 Year	7.65%	6.20%[4]
10 Year	7.80%	Call for details
25 Year	8.70%	Call for details

Open

Term	Posted Rate
6 month	8.15%
1 year	8.50%

Variable Rate Mortgages†

Closed

Term	Posted Rate	Special Offers‡
5 year	4.75%	4.75%[5]
5 year RateCapper®	4.75%	--

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RBC Homeline Plan
Flash Video HTML

Client Story
Randy & Neil using RBC Homeline Plan
Video Transcript

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RateCapper Maximum Rate	7.75%	--

Open

Term	Posted Rate
5 year	4.75%

NOTES

* Interest rate compounded half-yearly, not in advance. Interest rates are subject to change without notice at any time.

† Interest rate is compounded monthly, not in advance. This rate may change at any time without notice. Royal Bank of Canada prime rate is an annual variable rate of interest announced by Royal Bank of Canada from time to time as its prime rate.

‡ The annual percentage rate (APR) is based on a $100,000 mortgage for the applicable term assuming certain cost of borrowing charges (for example, appraisal fees). If there are no cost of borrowing charges, the APR and the interest rate will be the same.

1. **APR 5.62%**. Offer expires October 31, 2008.

2. **APR 6.02%**. Offer expires October 31, 2008.

3. **APR 6.17%**. Offer expires October 31, 2008.

4. **APR 6.22%**. Offer expires October 31, 2008.

5. **APR 4.75%**. Offer expires October 31, 2008.

For mortgages approved on or before October 31, 2008 funds must be advanced within 90 days of date of application in order to qualify for the Special Offer rate. Offer may be changed, withdrawn or extended at any time, without notice.

Applicable to residential mortgages only and subject to Royal Bank of Canada lending criteria for residential properties. Some conditions apply. Rates shown for the applicable term are Royal Bank of Canada's special discounted rates and are not the posted rates of Royal Bank of Canada. Offer may be changed, withdrawn or extended at any time, without notice.

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SOLUTION

The APR will be i', with corresponding monthly rate $j' = \frac{i'}{12}$, satisfying the relationship $250{,}000 - 5{,}000 = 2{,}011.56\, a_{\overline{360}|j'}$. Using a calculator unknown interest function, we get $j' = .007689577$, so that the APR is $i' = 12\, j' = .0922749$ (nominal annual rate of interest compounded monthly).

□

3.3 THE SINKING-FUND METHOD OF LOAN REPAYMENT

The final comments in Section 3.1 considered the case of a loan which called for periodic payments of interest only during the term of the loan, along with repayment of the full principal amount at the end of the term. For such a loan of amount L at periodic rate of interest i for n periods, the borrower would have to make a series of n interest payments to the lender, each of amount $L \cdot i$, along with a payment of L at time n. The borrower might offset the obligation to pay the single lump sum of amount L at time n by accumulating that amount during the term of the loan by means of n periodic deposits into an interest-bearing savings account called a *sinking fund*. This method of loan repayment is called the **sinking-fund method**.

There is no guarantee that the rate earned in the sinking fund, say j, is the same as the periodic interest rate on the loan, i. In a practical situation it would usually be the case that the interest rate charged by the lender is larger than the rate that can be earned in a deposit account, so that $i > j$.

A standard way of accumulating the principal amount in the sinking fund is by using level deposits. If this is the case, then for the loan situation just described the borrower's payment would be $L \cdot i$, which is the interest payment to the lender, plus $\frac{L}{s_{\overline{n}|j}}$, which is the level sinking fund deposit, producing a total periodic outlay of $L\left[i + \frac{1}{s_{\overline{n}|j}}\right]$.

EXAMPLE 3.4 (*Sinking fund*)

A loan of 100,000 is to be repaid by ten annual payments beginning one year after the loan is made. The lender wants annual payments of only

interest at a rate of 10% and repayment of the principal in a single lump sum at the end of 10 years. The borrower can accumulate the principal in a sinking fund earning an annual interest rate of 8%, and decides to do this by means of 10 level deposits starting one year after the loan is made.

(a) Find the borrowers' total annual outlay and compare this to the level annual payment required by the amortization method at 10%. Find the annual rate of interest i' for which the amortization method at rate i results in the same total annual outlay as the borrower pays in the sinking fund method in this example.

(b) Suppose that the lender's rate is 8% and the sinking fund rate is 10%. Repeat part (a), comparing this to the amortization method at 8%.

SOLUTION

(a) The total annual outlay under the sinking fund method is $100{,}000\left[.1+\frac{1}{s_{\overline{10}|.08}}\right] = 16{,}902.95,$ and the annual payment under amortization at 10% is $\frac{100{,}000}{a_{\overline{10}|.10}} = 16{,}274.54.$ To find i' we have $100{,}000 = 16{,}902.95 \cdot a_{\overline{10}|i'}$ which results in $i' = .1089.$

(b) The total annual outlay under the sinking fund method is $100{,}000\left[.08+\frac{1}{s_{\overline{10}|.10}}\right] = 14{,}274.54,$ and the annual payment under amortization at 8% is $\frac{100{,}000}{a_{\overline{10}|.08}} = 14{,}902.95.$ To find i' we have $100{,}000 = 14{,}274.54 \cdot a_{\overline{10}|i'}$ which results in $i' = .0706.$ □

As deposits are made to the sinking fund, the fund balance grows toward the target value of L. For instance, in part (a) of Example 3.4, just after the fifth deposit into the sinking fund, the fund balance is $6902.95 \cdot s_{\overline{5}|.08} = 40{,}496.85.$ This is the accumulated value after five years in the fund that will eventually pay back the principal amount. The value of the net debt outstanding after 5 years can be regarded as the initial loan amount minus the amount for repayment of principal that has already been accumulated to that point. This is

48. A 12-year loan of 8000 is to be repaid with payments to the lender of 800 at the end of each year and deposits of X at the end of each year into a sinking fund.

Interest on the loan is charged at an 8% annual effective rate. The sinking fund annual effective interest rate is 4% .

Calculate X .

(A) 298

(B) 330

(C) 361

(D) 385

(E) 411