TEST 2 SOLUTIONS

1. (a) $\ell(\beta_0, \beta_1) = \sum_{i=1}^{n} y_i \log \mu_i - \mu_i$

$$= \sum y_i(\beta_0 + \beta_1 x_i) - \exp(\beta_0 + \beta_1 x_i)$$

(5)

$$\ell_{\beta_0} = \sum y_i - \sum \exp(\beta_0 + \beta_1 x_i)$$

$$\ell_{\beta_1} = \sum x_i y_i - \sum x_i \exp(\beta_0 + \beta_1 x_i)$$

$$y_+ = \sum e^{\hat\beta_0 + \hat\beta_1 x_i} = \sum \mu_i(\hat\beta)$$

$$(xy)_+ = \sum x_i e^{\hat\beta_0 + \hat\beta_1 x_i} = \sum x_i \mu_i(\hat\beta)$$

(b)
- log-linear regression of # of faults on length, using Poisson dist
- estimate of β_0 is 0.917 i.e. background level of faults $(x=0) \simeq e^{.917} \simeq 2.7$
- increase in # of faults / unit length (m) is $e^{0.002} \simeq 1$

(5)

- these are sign different from 0
- 61.758 is the value of the maxi'd log-lik. 32 obs − 2 pars ∴ 30 df (not much better than fit without β_1)
- (− AIC measures fit + # pars)
- iterative method to compute mle's converged in 4 steps

2 (a) 5

1. fit model spec'd; $\theta_1^0 = 44$, $\theta_2^0 = 6$, $\theta_3^0 = -0.03$; trace
2. "converged" but est. wrong
3. RSS = $\sum\{y_i - \eta(x_i, \hat\theta)\}^2$ min'd
4. define RSS(θ) directly
5. use optim (Nelder-Mead) to find $\hat\theta$
6. didn't converge
7. use a better (!) method
8. estimates $\hat\theta$ now sensible
9. RSS = 7.302 not min'd !
10. converged

2 (b) bec. model doesn't fit well; practically linear

3 (a) $\ell(\theta) = n\log\theta - \theta\sum_{i=1}^{n} y_i - n\log(1-e^{-\theta T})$

$\ell'(\theta) = \frac{n}{\theta} - y_+ - \frac{nTe^{-\theta T}}{1-e^{-\theta T}}$

(5) $\frac{1}{n}y_+ = \frac{1}{\hat\theta} - \frac{Te^{-\hat\theta T}}{1-e^{-\hat\theta T}} = \frac{1-e^{-\hat\theta T} - \hat\theta Te^{-\hat\theta T}}{\hat\theta(1-e^{-\hat\theta T})} = \frac{1-e^{-\hat\theta T}(1+\hat\theta T)}{1-e^{-\hat\theta T}\,\hat\theta}$

$\ell''(\theta) = -\frac{n}{\theta^2} - \frac{-nT^2e^{-\theta T}(1-e^{-\theta T}) - nTe^{-\theta T}e^{-\theta T}T}{(1-e^{-\theta T})^2}$

$= -\frac{n}{\theta^2} + \frac{nT^2e^{-\theta T} - nT^2e^{-2\theta T} + nT^2e^{-2\theta T}}{(1-e^{-\theta T})^2}$

$= -\frac{n}{\theta^2} + \frac{nT^2e^{-\theta T}}{(1-e^{-\theta T})^2}$

(b)

```
loglik ~~(th, y)~~ <- function (th, y){
                n ~~* log(th)~~ <- length(y)
                n * log (th) - θ * sum(y) - n* log (1-exp(-θ*T))}
```

(5)

```
loglikder <- deriv (loglik ~~(th,y)~~ ~~x~~, th) y=y
loglikder2 <- deriv (loglikder, th)
truncexp <- function ( y, th.start) {
    th <- th.start
    for ( i in 1:n ) {
        thnew <- th - loglikder(th,y)/ loglikder2(th,y)
        th <- thnew }
    list (th, loglikder2(th,y))}
```

4. (10)

- generate $y \sim$ truncated exp ($\theta = \theta_0$, say)
- for (j in 1:N) {

```
      y ← ranexp(θ₀) ; thstart ← 1/mean(y)
      that[j] ← truncexp(ty, th.start)[[1]]
      se[j] ← "                        " [[2]]
      }
```

hist(that) ; var(that) ; mean(se) ; etc.

5. (10)

$$f(y_i) = \frac{\theta_i e^{-\theta_i y_i}}{1 - e^{-\theta_i T}} \qquad \log \theta_i = x_i^T \beta$$

$$\ell(\beta) = \sum_{i=1}^{n} \left(x_i^T \beta - \exp(x_i^T \beta) y_i - (1 - \exp(x_i^T \beta) T) \right)$$

NR $\quad \hat\beta^{(t+1)} = \hat\beta^{(t)} - \left[\ell''(\hat\beta^{(t)}) \right]^{-1} \ell'(\hat\beta^{(t)})$

1	442
1	65
2	4422243
2	867756
3	110
3	8

1	4
1	
2	21
2	69
3	002
3	

410   2102