Today

- HW 2 due March 6
- random effects and mixed effects models ELM Ch. 8
- R Studio Cheatsheets
- In the News: homeopathic vaccines

A general framework

$$y \mid \gamma = X\beta + Z\gamma + \epsilon, \quad \epsilon \sim N(0, \sigma^2 \Lambda)$$

- γ: $q-$vector of random effects β: p-vector of fixed effects
- assumption $\gamma \sim N(0, \sigma^2 D)$
- marginal distribution

$$y \sim N(X\beta, \sigma^2(\Lambda + ZDZ^{\mathrm{T}})) = N(X\beta, \sigma^2 V), \text{say}$$

- applications
  - multi-level models
  - repeated measures
  - longitudinal data
  - components of variance

Illustration

SM Example 9.16

Example 9.16 (Longitudinal data) A short longitudinal study has one individual allocated to the treatment and two to the control, with observations

$$y_{1j} = \beta_0 + b_1 + \varepsilon_{1j}, \quad y_{21} = \beta_0 + b_2 + \varepsilon_{21}, \quad y_{3j} = \beta_0 + \beta_1 + b_3 + \varepsilon_{3j}, \quad j = 1, 2.$$

Thus there are two measurements on the first and third individuals, and just one on the second. The b_j represent variation among individuals and the ε_{ij} variation between measures on the same individuals. If the b's and ε's are all mutually independent with variances σ_b^2 and σ^2, then

$$\begin{pmatrix} y_{11} \\ y_{12} \\ y_{21} \\ y_{31} \\ y_{32} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{21} \\ \varepsilon_{31} \\ \varepsilon_{32} \end{pmatrix},$$

and this fits into formulation (9.12) with $\Omega_b = \sigma_b^2 I_3$ and $\Omega = \sigma^2 I_5$. Here ψ comprises the scalar σ_b^2/σ^2, and hence the variance matrix

$$\Omega + Z\Omega_b Z^{\mathrm{T}} = \begin{pmatrix} \sigma_b^2+\sigma^2 & \sigma_b^2 & 0 & 0 & 0 \\ \sigma_b^2 & \sigma_b^2+\sigma^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_b^2+\sigma^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_b^2+\sigma^2 & \sigma_b^2 \\ 0 & 0 & 0 & \sigma_b^2 & \sigma_b^2+\sigma^2 \end{pmatrix}$$

Estimation

- $y \sim N(X\beta, \sigma^2(\Lambda + ZDZ^{\mathrm{T}})) = N(X\beta, \sigma^2 V)$
- $$\ell(\beta; y) = -\frac{n}{2}\log(\sigma^2) - \frac{1}{2}\log|V| - \frac{1}{2\sigma^2}(y - X\beta)^{\mathrm{T}} V^{-1}(y - X\beta)$$
- V may have one or more unknown parameters
- Example 9.16: $\gamma \sim N_3(0, \sigma_b^2 I), \quad \epsilon \sim N(0, \sigma^2 I)$
- $$\Lambda + ZDZ^{\mathrm{T}} = \begin{pmatrix} 1+\sigma_b^2/\sigma^2 & \sigma_b^2/\sigma^2 & 0 & 0 & 0 \\ \sigma_b^2/\sigma^2 & 1+\sigma_b^2/\sigma^2 & 0 & 0 & 0 \\ 0 & 0 & 1+\sigma_b^2/\sigma^2 & 0 & 0 \\ 0 & 0 & 0 & 1+\sigma_b^2/\sigma^2 & \sigma_b^2/\sigma^2 \\ 0 & 0 & 0 & \sigma_b^2/\sigma^2 & 1+\sigma_b^2/\sigma^2 \end{pmatrix}$$
- $\hat{\beta}_\psi = (X^{\mathrm{T}} V^{-1} X)^{-1} X^{\mathrm{T}} V^{-1} y$

... estimation

- $\hat{\beta}_\psi = (X^{\mathrm{T}} V^{-1} X)^{-1} X^{\mathrm{T}} V^{-1} y$

- profile log-likelihood

$$\ell_{\mathrm{p}}(\sigma^2, \psi) = -\frac{n}{2}\log\sigma^2 - \frac{1}{2}\log|V_\psi| - \frac{1}{2\sigma^2}(y - X\hat{\beta}_\psi)^{\mathrm{T}} V_\psi^{-1}(y - X\hat{\beta}_\psi)$$

- to get better divisors — properly adjust for degrees of freedom
- modified profile log-likelihood

also called restricted profile log-likelihood

$$\begin{aligned}\ell_{\mathrm{mp}}(\sigma^2, \psi) &= -\frac{n-p}{2}\log\sigma^2 - \frac{1}{2}\log|V_\psi| - \frac{1}{2}\log|X^{\mathrm{T}} V_\psi^{-1} X| \\ &\quad -\frac{1}{2\sigma^2}(y - X\hat{\beta}_\psi)^{\mathrm{T}} V_\psi^{-1}(y - X\hat{\beta}_\psi)\end{aligned}$$

- estimation of σ^2, and ψ (parameters in V) available in most software by the name REML

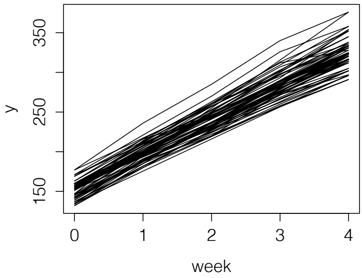
Example: Growth Data

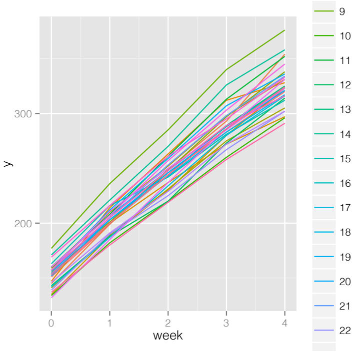
SM Example 9.18

	Week						Week				
	1	2	3	4	5		1	2	3	4	5
1	151	199	246	283	320	16	160	207	248	288	324
2	145	199	249	293	354	17	142	187	234	280	316
3	147	214	263	312	328	18	156	203	243	283	317
4	155	200	237	272	297	19	157	212	259	307	336
5	135	188	230	280	323	20	152	203	246	286	321
6	159	210	252	298	331	21	154	205	253	298	334
7	141	189	231	275	305	22	139	190	225	267	302
8	159	201	248	297	338	23	146	191	229	272	302
9	177	236	285	340	376	24	157	211	250	285	323
10	134	182	220	260	296	25	132	185	237	286	331
11	160	208	261	313	352	26	160	207	257	303	345
12	143	188	220	273	314	27	169	216	261	295	333
13	154	200	244	289	325	28	157	205	248	289	316
14	171	221	270	326	358	29	137	180	219	258	291
15	163	216	242	281	312	30	153	200	244	286	324

Table 9.27 Weights (units unknown) of 30 young rats over a five-week period (Gelfand *et al.*, 1990).

... growth data






```
data(rat.growth, library="SMPracticals")}

with(rat.growth, plot(week, y, type="l", col = levels(rat)))

qplot(week, y, data = rat.growth, geom = "path", colour = rat)
last_plot() + theme(legend.position = "none")
```

Example 9.18

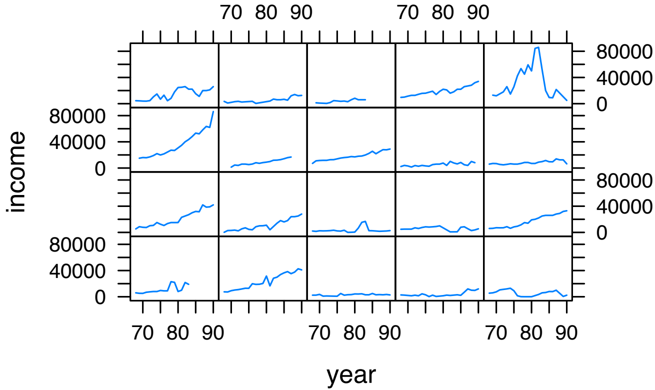
- repeated measurements on the 30 individuals, at 5 time points
- fixed effects model: $y_{jt} = \mu + \gamma_j + \beta_1 x_{jt} + \epsilon_{jt}, \quad t = 1, \ldots, 5$
- $x_{jt} = x_t$ takes values $0, 1, 2, 3, 4$ for $t = 1, 2, 3, 4, 5$
- or even $y_{jt} = \mu + \gamma_j + \alpha_t + \epsilon_{jt}$ rats as blocks, time as 'treatment'
- random effects model
  $y_{jt} = \beta_0 + \gamma_j^0 + (\beta_1 + \gamma_j^1) x_{jt} + \epsilon_{jt}, \quad t = 1, \ldots, 5$
- $(\gamma_j^0, \gamma_j^1) \dot{\sim} N_2(0, \sigma^2 D), \quad \epsilon_{jt} \dot{\sim} N(0, \sigma^2)$ independent
- two fixed parameters β_0, β_1
- four variance/covariance parameters: $\sigma_{g0}^2, \sigma_{g1}^2, \sigma_{g01}, \sigma^2$

... Example 9.18

- maximum likelihood estimates of fixed effects: $\hat{\beta}_0 = 156.05(2.16), \hat{\beta}_1 = 43.27(0.73)$
- weight in week 1 is estimated to be about 156 units, and average increase per week estimated to be 43.27
- there is large variability between rats: estimated standard deviation of 10.93 for intercept, 3.53 for slope
- there is little correlation between the intercepts and slopes
- ```
  separate.lm = lm(y ~ week + factor(rat)+ week:factor(rat),
  data = rat.growth)
  # fit separate linear models to each set of 5 observations
  library(lme4)
  rat.mixed = lmer(y ~ week + (week|rat), data = rat.growth)
  # REML is the default
  summary(rat.mixed)  #
  Fixed effects:
              Estimate Std. Error t value
  (Intercept) 156.0533     2.1590   72.28
  week         43.2667     0.7275   59.47
  ```
  ```

... Example 9.18

- there is large variability between rats
- estimated standard deviation of 10.93 for intercept, 3.53 for slope
- there is little correlation between the intercepts and slopes

- ```
  summary(rat.mixed)   #
  Random effects:
   Groups   Name        Variance Std.Dev. Corr
   rat      (Intercept) 119.53   10.933
            week         12.49    3.535   0.18
   Residual              33.84    5.817
  Number of obs: 150, groups:  rat, 30
  ```

$\text{var}(\gamma_j^0) \approx 119.53 = 10.933^2$; $\text{var}(\gamma_j^1) \approx 12.49 = 3.53^2$
$\tilde{\sigma}^2 = 33.84$
  ```

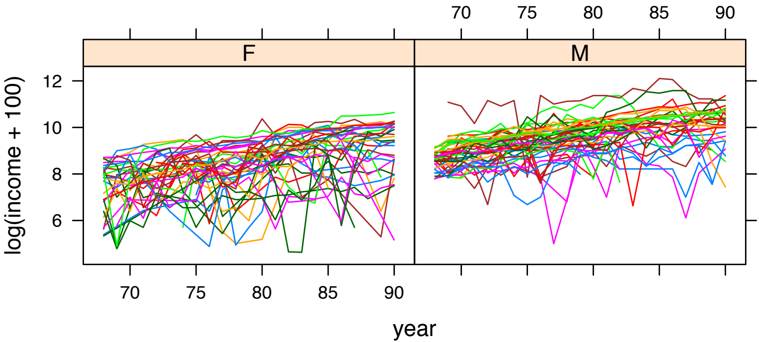
Example: Panel Study of Income Dynamics

ELM, §9.1




```
library(lattice)
xyplot(income ~ year | person, data = psid,
type="l", subset = (person < 21), strip = F)
```

... PSID




```
psid$cyear = psid$year - 1978
head(psid)
  age educ sex income year person cyear
1  31   12   M   6000   68      1   -10
2  31   12   M   5300   69      1    -9
3  31   12   M   5200   70      1    -8
4  31   12   M   6900   71      1    -7
5  31   12   M   7500   72      1    -6
```

... PSID

```
> mmod = lmer(log(income) ~ cyear*sex + age + educ +
+ (cyear | person), data=psid)
```

$$
\begin{aligned}
\text{log(income)}_{ij} &= \mu + \gamma_j^0 + \alpha\,\text{year}_i + \gamma_j^1\text{year}_i + \\
&\quad \beta\,\text{sex}_j + \alpha\beta\,(\text{year}_i \times \text{sex}_j) + \beta_2\text{educ}_j + \beta_3\text{age}_j + \epsilon_{ij}, \\
&\quad \epsilon_{ij} \sim N(0, \sigma^2), \quad \gamma_j \sim N_2(0, \sigma^2 D)
\end{aligned}
$$

- we could fit separate lines for each subject

  as with rat growth data
- this would give us 85 slopes and 85 intercepts
- we could compare these slopes and intercepts between genders

  two-sample test
- analysis of derived responses is often simple, but sometimes limited

  see p.188

... PSID – using `lmer`

compare random effects model to fixed effects model:

```
> mmod = lmer(log(income) ~ cyear*sex + age + educ +
+ (cyear | person), data=psid)

Fixed effects:
             Estimate Std. Error t value
(Intercept)  6.67420    0.54332  12.284
cyear        0.08531    0.00900   9.480
sexM         1.15031    0.12129   9.484
age          0.01093    0.01352   0.808
educ         0.10421    0.02144   4.861
cyear:sexM  -0.02631    0.01224  -2.150

> lmod = lm(log(income) ~ cyear*sex + age + educ, data = paid)
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  6.737201   0.206490  32.627   <2e-16 ***
cyear        0.082049   0.005304  15.470   <2e-16 ***
sexM         1.130826   0.045554  24.824   <2e-16 ***
age          0.009401   0.005061   1.858   0.0634 .
educ         0.106934   0.008184  13.066   <2e-16 ***
cyear:sexM  -0.017716   0.007088  -2.499   0.0125 *

Residual standard error: 0.9126 on 1655 degrees of freedom
```

- coefficients the same; standard errors for `lm` much smaller
- 1655 degrees of freedom?
- all observations treated as independent

Inference for fixed effects

$$
\begin{aligned}
\log(\text{income})_{ij} &= \mu + \gamma_j^0 + \alpha\,\text{year}_i + \gamma_j^1\text{year}_i + \\
&\quad \beta\,\text{sex}_j + \alpha\beta\,(\text{year}_i \times \text{sex}_j) + \beta_2\text{educ}_j + \beta_3\text{age}_j + \epsilon_{ij}, \\
&\quad \epsilon_{ij} \sim N(0, \sigma^2), \quad \gamma_j \sim N_2(0, \sigma^2 D)
\end{aligned}
$$

- $\hat{\beta} = (X^{\mathrm{T}}\hat{V}^{-1}X)^{-1}X^{\mathrm{T}}\hat{V}^{-1}y, \quad \tilde{\sigma}^2$ by REML
- s.e.$(\hat{\beta}_j) = \surd\{\tilde{\sigma}^2(X^{\mathrm{T}}\hat{V}^{-1}X)^{-1}_{jj}\}$

- `educ` coefficient estimate 0.1042, $e^{0.1042} = 1.11$, 11% increase in income per year of education
- `sexM` coefficient estimate 1.15, $e^{1.15} = 3.16$, $3\times$ higher at baseline for males
- slope for females approximately 9% per year; for males approximately 6% per year

... PSID – using `lme(nlme)` glmm faq

```
> mmod2 = lme(log(income) ~ cyear*sex + age + educ ,
random = ~ 1 +  cyear | person, data=psid)

Fixed effects: log(income) ~ cyear * sex + age + educ
                 Value Std.Error   DF   t-value p-value
(Intercept)   6.674204 0.5433252 1574 12.283995  0.0000
cyear         0.085312 0.0089996 1574  9.479521  0.0000
sexM          1.150313 0.1212925   81  9.483790  0.0000
age           0.010932 0.0135238   81  0.808342  0.4213
educ          0.104210 0.0214366   81  4.861287  0.0000
cyear:sexM   -0.026307 0.0122378 1574 -2.149607  0.0317

Random effects:
 Formula: ~1 + cyear | person
 Structure: General positive-definite, Log-Cholesky parametrization
            StdDev     Corr
(Intercept) 0.53071321 (Intr)
cyear       0.04898952 0.187
Residual    0.68357323

> summary(mmod) # using lmer

Fixed effects:
            Estimate Std. Error t value
(Intercept)  6.67420    0.54332  12.284
cyear        0.08531    0.00900   9.480
sexM         1.15031    0.12129   9.484
age          0.01093    0.01352   0.808
educ         0.10421    0.02144   4.861
cyear:sexM  -0.02631    0.01224  -2.150
```

Inference for random effects

```
Random effects: # using lmer
 Groups   Name        Variance Std.Dev. Corr
 person   (Intercept) 0.2817   0.53071
          cyear       0.0024   0.04899  0.19
 Residual             0.4673   0.68357
Number of obs: 1661, groups:  person, 85

Random effects: # using lme
 Formula: ~1 + cyear | person
 Structure: General positive-definite, Log-Cholesky parametriz
            StdDev     Corr
(Intercept) 0.53071321 (Intr)
cyear       0.04898952 0.187
Residual    0.68357323
```

- standard deviation of slopes estimated to be 0.049
- variation within subjects $(0.68)^2$ larger than between subjects $(0.53)^2$

Random effects

- estimates (predictions) of b_{0i}, b_{1i} available
- $Y = X\beta + Zb + \epsilon; \quad b \sim N(0, \sigma^2\Omega_b), \epsilon \sim N(0, \sigma^2\Omega_j)$
- $Y \sim N(X\beta, (\Omega + Z\Omega_b Z^{\mathrm{T}}))$
- $\tilde{b} = (Z^{\mathrm{T}}\hat{\Omega}^{-1}Z + \hat{\Omega}_b^{-1})^{-1}Z^{\mathrm{T}}\Omega^{-1}(y - X\beta)$

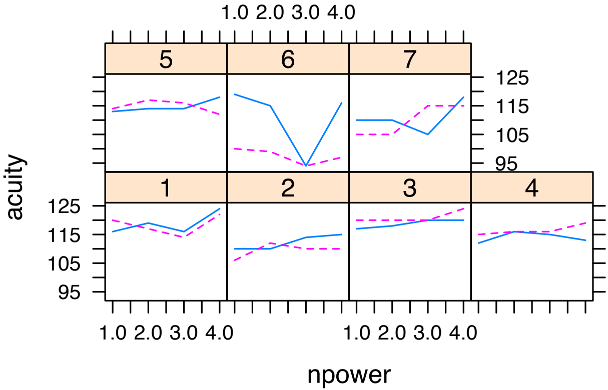
$$
\begin{aligned}
y - X\hat{\beta} &= Z\tilde{b} + y - X\hat{\beta} - Z\tilde{b} \\
&= Z\tilde{b} + \underbrace{\{I_n - Z(Z^{\mathrm{T}}\hat{\Omega}^{-1}Z + \hat{\Omega}_b^{-1})^{-1}Z^{\mathrm{T}}\hat{\Omega}^{-1}\}(y - X\hat{\beta})}_{\text{new residual}}
\end{aligned}
$$

pieces of `lmer`

```
> methods(class="merMod")
 [1] anova.merMod*        as.function.merMod*  coef.merMod*
 [4] confint.merMod       deviance.merMod*     drop1.merMod*
 [7] extractAIC.merMod*   family.merMod*       fitted.merMod*
[10] fixef.merMod*        formula.merMod*      isGLMM.merMod*
[13] isLMM.merMod*        isNLMM.merMod*       isREML.merMod*
[16] logLik.merMod*       model.frame.merMod*  model.matrix.merMod*
[19] nobs.merMod*         plot.merMod*         predict.merMod*
[22] print.merMod*        profile.merMod*      ranef.merMod*
[25] refit.merMod*        refitML.merMod*      residuals.merMod*
[28] sigma.merMod*        simulate.merMod*     summary.merMod*
[31] terms.merMod*        update.merMod*       VarCorr.merMod*
[34] vcov.merMod          weights.merMod*

> ranef(mmod)
$person
    (Intercept)         cyear
1  -0.029975590  0.0161575447
2   0.015961618  0.0198586106
3  -0.122972629 -0.0449473569
4   0.109534933 -0.0074016139
5  -0.572308284 -0.1108678330
6   0.218592408  0.0263156155

> length(residuals(mmod))
[1] 1661
```

Example: Acuity of Vision

ELM, §9.2




```
> xyplot(acuity ~ npower | subject, data=vision,
+ type="l", groups=eye, lty=1:2, layout = c(4,2))
```

... vision

```
> head(vision)
  acuity power   eye subject npower
1    116   6/6  left       1      1
2    119  6/18  left       1      2
3    116  6/36  left       1      3
4    124  6/60  left       1      4
5    120   6/6 right       1      1
6    117  6/18 right       1      2
> eyemod <- lmer(acuity ~ power + (1 | subject) +
+   (1 | subject:eye), data = vision)
```

$$y_{ijk} = \mu + p_j + s_i + e_{ik} + \epsilon_{ijk}, \quad i = 1, \dots, 7; j = 1, \dots 4; k = 1, 2$$

$$s_i \sim N(0, \sigma_s^2), \quad e_{ik} \sim N(0, \sigma_e^2), \quad \epsilon_{ijk} \sim N(0, \sigma^2)$$

... vision

```
> summary(eyemod)
Linear mixed model fit by REML ['lmerMod']
Formula: acuity ~ power + (1 | subject) + (1 | subject:eye)
   Data: vision

REML criterion at convergence: 328.7098

Random effects:
 Groups      Name        Variance Std.Dev.
 subject:eye (Intercept) 10.27    3.205
 subject     (Intercept) 21.53    4.640
 Residual                16.60    4.075
Number of obs: 56, groups: subject:eye, 14; subject, 7

Fixed effects:
            Estimate Std. Error t value
(Intercept) 112.6429     2.2349   50.40
power6/18     0.7857     1.5400    0.51
power6/36    -1.0000     1.5400   -0.65
power6/60     3.2857     1.5400    2.13
```

Part 2 in Rstudio

Non-specific effects

C&D §7.2

- example: a clinical trial involves several or many centres
- an agricultural field trial repeated at a number of different farms, and over a number of different growing seasons
- a sociological study repeated in broadly similar form in a number of countries
- laboratory study uses different sets of analytical apparatus, imperfectly calibrated
- such factors are non-specific
- how do we account for them
  - on an appropriate scale, a parameter represents a shift in outcome
  - more complicated: the primary contrasts of concern vary across centres
  - i.e. treatment-center interaction

... non-specific effects

- suppose no treatment-center interaction
- example:

$$\mathrm{logit}\{\mathrm{pr}(Y_{ci} = 1)\} = \alpha_c + x_{ci}^T \beta$$

- should α_c be ?fixed? or ?random?
- effective use of a random-effects representation will require estimation of the variance component corresponding to the centre effects
- even under the most favourable conditions the precision achieved in that estimate will be at best that from estimating a single variance from a sample of a size equal to the number of centres
- very fragile unless there are at least, say, 10 centres and preferably considerably more

... non-specific effects

- if centres are chosen by an effectively random procedure from a large population of candidates, ... the random-effects representation has an attractive tangible interpretation. This would not apply, for example, to the countries of the EU in a social survey

- some general considerations in linear mixed models:
  - in balanced factorial designs, the analysis of treatment means is unchanged
  - in other cases, estimated effects will typically be 'shrunk', and precision improved
  - representation of the nonspecific effects as random effects involves independence assumptions which certainly need consideration and may need some empirical check

... non-specific effects

- if estimates of effect of important explanatory variables are essentially the same whether nonspecific effects are ignored, or are treated as fixed constants, then random effects model will be unlikely to give a different result
- it is important in applications to understand the circumstances under which different methods give similar or different conclusions
- in particular, if a more elaborate method gives an apparent improvement in precision, what are the assumptions on which that improvement is based, and are they reasonable?

... non-specific effects

- if there is an interaction between an explanatory variable [e.g. treatment] and a nonspecific variable
- i.e. the effects of the explanatory variable change with different levels of the nonspecific factor
- the first step should be to explain this interaction, for example by transforming the scale on which the response variable is measure or by introducing a new explanatory variable
  - example: two medical treatments compared at a number of centres show different treatment effects, as measured by an ratio of event rates
  - possible explanation: the difference of the event rates might be stable across centres
  - possible explanation: the ratio depends on some characteristic of the patient population, e.g. socio-economic status
- an important special application of random-effect models for interactions is in connection with overviews, that is, assembling of information from different studies of essentially the same effect

In the News

- Globe & Mail, March 3: “U of T investigates instructor over anti-vaccine course materials”
- Globe & Mail, Feb 18: “Health experts criticize government approval of homeopathic ‘vaccines’”
- British Homeopathic Association: “In line with the Department of Health’s advice, the BHA recommends that immunization should be carried out in the normal way using the conventional tested and approved vaccines”
- Faculty of Homeopathy: randomized controlled trials in homeopathy

Generalized linear mixed models

- $$f(y_j \mid \theta_j, \phi) = \exp\{\frac{y_j\theta_j - b(\theta_j)}{\phi a_j} + c(y_j; \phi a_j)\}$$
- $$b'(\theta_j) = \mu_j$$
- random effects
$$g(\mu_j) = x_j^{\mathrm{T}}\beta + z_j^{\mathrm{T}}b, \quad b \sim N(0, \Omega_b)$$
- likelihood
$$L(\beta, \phi; y) = \prod_{j=1}^{n} \int f(y_j \mid \beta, b, \phi) f(b; \Omega_b) db$$

... generalized linear mixed models

- likelihood

$$L(\beta, \phi; y) = \prod_{j=1}^{n} \int f(y_j \mid \beta, b, \phi) f(b; \Omega_b) db$$

- doesn't simplify unless $f(y_j \mid b)$ is normal
- solutions proposed include
  - numerical integration, e.g. by quadrature
  - integration by MCMC
  - Laplace approximation to the integral – penalized quasi-likelihood
- reference: `MASS` library and book (§10.4):
  `glmmNQ`, `GLMMGibbs`, `glmmPQL`, all in `library(MASS)`
  `glmer` in `library(lme4)`

Example: Balance experiment

Faraway, 10.1

- effects of surface and vision on balance; 2 levels of surface; 3 levels of vision
- surface: normal or foam
- vision: normal, eyes closed, domed
- 20 males and 20 females tested for balance, twice at each of 6 combinations of treatments
- auxiliary variables age, height, weight

Steele 1998, OzDASL

- linear predictor: `Sex + Age + Weight + Height + Surface + Vision + Subject(?)`
- response measured on a 4 point scale; converted by Faraway to binary (stable/not stable)
- analysed using linear models at OzDASL

... balance

```
> balance <- glmer(stable ~ Sex + Age + Height + Weight + Surface + Vision +
+  (1|Subject), family = binomial, data = ctsib)

# Subject effect is random

> summary(balance)
Generalized linear mixed model fit by maximum likelihood ['glmerMod']

 ...

Random effects:
 Groups  Name        Variance Std.Dev.
 Subject (Intercept) 8.197    2.863
Number of obs: 480, groups: Subject, 40

Fixed effects:
             Estimate Std. Error z value Pr(>|z|)
(Intercept)  9.920750  13.358013   0.743    0.458
Sexmale      2.825305   1.762383   1.603    0.109
Age         -0.003644   0.080928  -0.045    0.964
Height      -0.151012   0.092174  -1.638    0.101
Weight       0.058927   0.061958   0.951    0.342
Surfacenorm  7.524423   0.888827   8.466  < 2e-16 ***
Visiondome   0.683931   0.530654   1.289    0.197
Visionopen   6.321098   0.839469   7.530 5.08e-14 ***
---
```

... balance

```
> library(MASS)

> balance2 <- glmmPQL(stable ~ Sex + Age + Height + Weight + Surface + Vision,
+ random = ~1 | Subject, family = binomial, data = ctsib)
> summary(balance2)

Random effects:
 Formula: ~1 | Subject
        (Intercept)  Residual
StdDev:    3.060712 0.5906232

Variance function:
 Structure: fixed weights
 Formula: ~invwt

Fixed effects: stable ~ Sex + Age + Height + Weight + Surface + Vision
                Value Std.Error  DF   t-value p-value
(Intercept) 15.571494 13.498304 437  1.153589  0.2493
Sexmale      3.355340  1.752614  35  1.914478  0.0638
Age         -0.006638  0.081959  35 -0.080992  0.9359
Height      -0.190819  0.092023  35 -2.073601  0.0455
Weight       0.069467  0.062857  35  1.105155  0.2766
Surfacenorm  7.724078  0.573578 437 13.466492  0.0000
Visiondome   0.726464  0.325933 437  2.228873  0.0263
Visionopen   6.485257  0.543980 437 11.921876  0.0000
```

... balance

```
> balance4 <- glmer(stable ~ Sex + Age + Height + Weight + Surface + Vision +
+ (1|Subject), family = binomial, data = ctsib, nAGQ = 9)
> summary(balance4)

Random effects:
 Groups  Name        Variance Std.Dev.
 Subject (Intercept) 7.8      2.793
Number of obs: 480, groups: Subject, 40

Fixed effects:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) 13.551847  13.067369   1.037   0.2997
Sexmale      3.109307   1.724797   1.803   0.0714 .
Age         -0.001804   0.079161  -0.023   0.9818
Height      -0.175061   0.090239  -1.940   0.0524 .
Weight       0.065742   0.060606   1.085   0.2780
Surfacenorm  7.428046   0.872416   8.514  < 2e-16 ***
Visiondome   0.682509   0.527836   1.293   0.1960
Visionopen   6.210825   0.822012   7.556 4.17e-14 ***
```