

... inference

```
> head(shuttle2)
  m r temperature pressure
1 6 0          66       50
2 6 1          70       50
3 6 0          69       50
4 6 0          68       50
5 6 0          67       50
6 6 0          72       50
```

```
> logitmodcorrect2 <- glm(cbind(r,m-r) ~ temperature + pressure, family = binomial, data = shuttle2)
```

```
> summary(logitmodcorrect2)
```

family = bin, data = shuttle2

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	2.520195	3.486784	0.723	0.4698
temperature	-0.098297	0.044890	-2.190	0.0285 *
pressure	0.008484	0.007677	1.105	0.2691

$\hat{\beta}_2 / \text{s.e.}(\hat{\beta})$

Null deviance: 24.230 on 22 degrees of freedom
Residual deviance: 16.546 on 20 degrees of freedom
AIC: 36.106

Number of Fisher Scoring iterations: 5

$2 \{ \ell(\hat{\beta}_A) - \ell(\hat{\beta}_B) \}$

```
> anova(logitmodcorrect, logitmodcorrect2)
Analysis of Deviance Table
```

Model	Resid. Df	Resid. Dev	Df	Deviance
Model 1: cbind(r, m - r) ~ temperature	21	18.086		
Model 2: cbind(r, m - r) ~ temperature + pressure	20	16.546	1	1.5407

... inference

- ▶ Model A: $\text{logit}(p_i) = \beta_0 + \beta_1 \text{temp}_i + \beta_2 \text{pressure}_i$
- ▶ Model B: $\text{logit}(p_i) = \beta_0 + \beta_1 \text{temp}_i$
- ▶ **nested**: Model B is obtained by setting $\beta_2 = 0$
- ▶ Under Model B, the **change in deviance** is (approximately) an observation from a χ_1^2
- ▶ $\Pr(\chi_1^2 \geq 1.5407) = 0.22$
this is a p -value for testing $H_0 : \beta_2 = 0$
- ▶ so is $1 - \Phi\left\{\frac{\hat{\beta}_2}{\widehat{\text{s.e.}}(\hat{\beta}_2)}\right\} = 1 - \Phi(1.105) = 0.27$

... inference

- ▶ confidence intervals for β_1

? logitmodcorrect(2)?

- ▶ based on normal approximation: $\hat{\beta}_1 \pm \widehat{\text{s.e.}}(\hat{\beta}_1) * 1.96$
- ▶ $(-0.208, -0.023)$

- ▶ based on profile log-likelihood

$\ell_p(\beta_1)$, details to follow

- ▶ `confint(logitmodcorrect):`
`(-0.2122262, -0.0244701)`

ELM p. 31

... inference

- ▶ confidence intervals for β_1
- ▶ based on normal approximation: $\hat{\beta}_1 \pm \widehat{\text{s.e.}}(\hat{\beta}_1) * 1.96$
- ▶ $(-0.208, -0.023)$

- ▶ based on profile log-likelihood

$\ell_p(\beta_1)$, details to follow

- ▶ `confint(logitmodcorrect)` :
 $(-0.2122262, -0.0244701)$

ELM p. 31

knitr (Sweave) || markdown

Rstudio

Special to the binomial

and Poisson

- ▶ likelihood ratio test for logistic model

$$p_i = p_i(\beta) = \text{expit}(x_i^T \beta), \quad \hat{p}_i = p_i(\hat{\beta})$$

- ▶ this model is nested in the saturated model $\tilde{p}_i = y_i/n_i$

- ▶ residual deviance compares fitted model to saturated model

$$\text{fitted } p_i(\hat{\beta}) \text{ to } \tilde{p} = y_i/n_i$$

- ▶ under the fitted model, approximately χ^2_{n-q} distributed as χ^2_{n-q} if n_i 's $\rightarrow \infty$

```
> summary(logitmodcorrect)
```

```
...  
Null deviance: 24.230 on 22 degrees of freedom  
Residual deviance: 18.086 on 21 degrees of freedom  
AIC: 35.647  
Number of Fisher Scoring iterations: 5
```

$$p(\chi^2_{21} \geq 18.086) = .75?$$

Actually, null model ($\beta_1 = 0$) also fits: `pchisq(24.23, 22, lower.tail = F) = 0.33`, but improvement is statistically significant

Logistic regression

- ▶ read §2.4 for one motivation of logistic regression model
- ▶ read §2.5 (and AS I) for interpretation of parameters in terms of **log odds**
- ▶ see Example `mdl` in §2.5 for logistic regression with **qualitative** covariates
- ▶ what is the algebraic form of the model? how are the dummy covariates coded?
- ▶ in other words, what is x_i^T ?

$$\begin{aligned} \text{logit}(p_i) &= \beta_0 \\ &+ \beta_1 \text{---} + \beta_2 \text{---} \\ &+ \dots \end{aligned}$$

“What are the principles of applied statistics?”

CD Ch. 1

- ▶ “formulation and clarification of focused research questions of subject-matter importance
- ▶ design of individual investigations and sequences of investigations that produce secure answers and open up new possibilities
- ▶ production of effective and reliable measurement procedures
- ▶ development of simple, and where appropriate, not-so-simple methods of analysis, with suitable software, that address the primary research questions, often through a skilful choice of statistical model, and give some assessment of uncertainty
- ▶ effective presentation of conclusions
- ▶ structuring of analyses to facilitate their interpretation in subject matter terms and their relationship to the knowledge base of the field.”

“What are the principles of applied statistics?”

CD Ch. 1

- ▶ “formulation and clarification of focused research questions of subject-matter importance
- ▶ design of individual investigations and sequences of investigations that produce secure answers and open up new possibilities
- ▶ production of effective and reliable measurement procedures
- ▶ development of simple, and where appropriate, not-so-simple methods of analysis, with suitable software, that address the primary research questions, often through a skilful choice of statistical model, and give some assessment of uncertainty
- ▶ effective presentation of conclusions
- ▶ structuring of analyses to facilitate their interpretation in subject matter terms and their relationship to the knowledge base of the field.”

“What are the principles of applied statistics?”

CD Ch. 1

- ▶ “formulation and clarification of focused research questions of subject-matter importance
- ▶ design of individual investigations and sequences of investigations that produce secure answers and open up new possibilities
- ▶ production of effective and reliable measurement procedures
- ▶ development of simple, and where appropriate, not-so-simple methods of analysis, with suitable software, that address the primary research questions, often through a skilful choice of statistical model, and give some assessment of uncertainty
- ▶ effective presentation of conclusions
- ▶ structuring of analyses to facilitate their interpretation in subject matter terms and their relationship to the knowledge base of the field.”

“What are the principles of applied statistics?”

CD Ch. 1

- ▶ “formulation and clarification of focused research questions of subject-matter importance
- ▶ design of individual investigations and sequences of investigations that produce secure answers and open up new possibilities
- ▶ production of effective and reliable measurement procedures
- ▶ development of simple, and where appropriate, not-so-simple methods of analysis, with suitable software, that address the primary research questions, often through a skilful choice of statistical model, and give some assessment of uncertainty
- ▶ effective presentation of conclusions
- ▶ structuring of analyses to facilitate their interpretation in subject matter terms and their relationship to the knowledge base of the field.”

“What are the principles of applied statistics?”

CD Ch. 1

- ▶ “formulation and clarification of focused research questions of subject-matter importance
- ▶ design of individual investigations and sequences of investigations that produce secure answers and open up new possibilities
- ▶ production of effective and reliable measurement procedures
- ▶ development of simple, and where appropriate, not-so-simple methods of analysis, with suitable software, that address the primary research questions, often through a skilful choice of statistical model, and give some assessment of uncertainty
- ▶ effective presentation of conclusions
- ▶ structuring of analyses to facilitate their interpretation in subject matter terms and their relationship to the knowledge base of the field.”