Today

- HW 1: due February 7, 2 pm.
- Data preparation: reading and neuroscience study
- Generalized linear models: over-dispersion, examples
- In the News: a serendipitous experiment

"Short- and long- term effects of a novel on connectivity in the brain", Berns, G.S. et al. (2013) Brain Connectivity **3**, 590–600

Reading material

Each participant was subject to 19 consecutive days (July 18, 2011-August 5, 2011) of resting-state scans that consisted of a total appointment time of less than 30 min at the same time each day. The first 5 days and last 5 days were "washin" and "washout" sessions, respectively. Each of the middle 9 scans was preceded by reading approximately 1/9th of the novel (Pompeii: A Novel, by Robert Harris, Fawcett, 2003). This novel was chosen because it was based on true events but written as historical fiction and conveyed in a classic narrative arc (Freytag, 1900). During the "washin" and "washout" sessions, the participants did not perform any other tasks except for the resting-state scan (Fig. 1). For each of the other 9 days, the story days, the participants performed the resting-state scan after taking a quiz and self-report about the effect of the material presented in the portion of the novel that was assigned for the previous night and included a five-point rating scale of how arousing the reading was (see Supplementary Data for quizzes; Supplementary Data are available online at www.liebertpub.com/brain). Through repeated scans, each participant served as his or her own control to measure changes in resting-state connec-

STA 2201: Applied Statistics II January 31 trivity after the consumption of the novel.

"Short- and long- term effects of a novel on connectivity in the brain", Berns, G.S. et al. (2013) Brain Connectivity **3**, 590–600

Preprocessing

All of the preprocessing was performed using the 1000 Functional Connectomes Scripts available from NITRC (www.nitrc.org). The only modification to these scripts was the addition of an iterative loop to cycle through the 19days of data. The scripts performed the following preprocessing procedures using FSL (Analysis Group, FMRIB) and AFNI (NIMH). First, the anatomical image was deobliqued and reorientated to the coordinate space that is compatible with FSL. Next, the image was skull stripped.

"Short- and long- term effects of a novel on connectivity in the brain", Berns, G.S. et al. (2013) Brain Connectivity **3**, 590–600

The resting-state functional images were preprocessed through a multi-step procedure. The images were deobl qued and reoriented similarly to the anatomical imag mean functional image was computed to serve as a target for motion correction. Using 3dvolreg, the functional images were then aligned to the mean image using two-pass Fourier interpolation. To decrease edge artifacts from Fourier interpolation, a zero pad of four voxels was added around the edges and stripped off after motion correction. The images were then skull stripped to create a mask that was then applied to the motion-corrected data. To allow for full magnetization and settling on any startle responses from the onset of the scanning, the eighth volume was used for registration to the anatomical image. Spatial smoothing was performed using a 6-mm Gaussian kernel. Grand mean scaling was performed with an intensity normalization to 10,000. A lowpass filter of 0.1 Hz and a high-pass filter of 0.005 Hz were applied for temporal filtering. The images were detrended by calculating the mean of the temporally filtered image and detrending with the addition of Legendre polynomials of an order up to and including two. An image that was the addition of both the mean and detrended image was

"Short- and long- term effects of a novel on connectivity in the brain", Berns, G.S. et al. (2013) Brain Connectivity **3**, 590–600

Three separate registration alignments were performed. The functional (using the eighth image acquisition as a template) to anatomical alignment was produced using a trilinear interpolation (six degrees of freedom). The anatomical to standard brain (MNI152_T1) was created again using a trilinear interpolation (12 degrees of freedom). The transformation matrices of both of these steps were saved. A third matrix, for the ability to transform between functional to standard, was created by concatenating the matrices of the previous two steps. The inverse of each of these matrices

"Short- and long- term effects of a novel on connectivity in the brain", Berns, G.S. et al. (2013) Brain Connectivity **3**, 590–600

Segmentation was performed to create individual images for each tissue type and individual probability maps. The tissue types recognized as cerebrospinal fluid (CSF) and white matter (WM) were masked. These masks were used to control for nuisance signals. We utilized the global signal, WM and CSF segmentation masks, and the six motion parameters to adjust the functional signals for the effects of physiological noise and motion (Yan et al., 2013). Although adjustment for global signals is controversial, we opted to take a conservative approach and control for physiological noise (Fox et al., 2009). This approach may protect against false positives but may introduce spurious negative correlations (Murphy et al., 2013; Weissenbacher et al., 2009), so our analysis focused only on changes in positive correlations.

"Short- and long- term effects of a novel on connectivity in the brain", Berns, G.S. et al. (2013) Brain Connectivity 3, 590-600



599

Generalized linear models: recap

$$f(\mathbf{y}_j; \mu_j, \phi_j) = \exp\{\frac{\mathbf{y}_j \theta_j - \mathbf{b}(\theta_j)}{\phi_j} + \mathbf{c}(\mathbf{y}_j; \phi_j)\}$$

• $E(y_j | x_j) = b'(\theta_j) = \mu_j$ defines μ_j as a function of θ_j

g(µ_j) = x_j^Tβ = η_j links the *n* observations together via covariates

• $g(\cdot)$ is the link function; η_j is the linear predictor

•
$$\operatorname{Var}(y_j \mid x_j) = \phi_j b''(\theta_j) = \phi_j V(\mu_j)$$

- ... recap
 - in most cases, either φ_j is known, or φ_j = φa_j, where a_i is known
 - ► Normal distribution, $\phi = \sigma^2, a_j = 1$
 - Binomial distribution $\phi = 1, a_j = 1/m_j$
 - Gamma distribution, $\phi = 1/\nu, a_j = 1$
 - Poisson distribution, $\phi = 1, a_j = 1$

$$\hat{\phi} = \frac{1}{n-p} \sum_{j=1}^{n} \frac{(y_j - \hat{\mu}_j)^2}{a_j V(\hat{\mu}_j)}$$

►

... recap

•
$$\ell(\beta) = \sum_{j} \ell_{j} \{\eta_{j}(\beta); y_{j}, \phi_{j}\}$$

• $\frac{\partial \ell(\beta)}{\partial \beta} = X^{\mathrm{T}} u(\beta), \quad X = \frac{\partial \eta}{\partial \beta^{\mathrm{T}}}, \quad u_{j} = \frac{(y_{j} - \mu_{j})}{\phi_{j} g'(\mu_{j}) V(\mu_{j})}$
• $\hat{\beta} = (X^{T} \widehat{W} X)^{-1} X^{T} \widehat{W} (X \hat{\beta} + \widehat{W}^{-1} \hat{u})$
• $w_{j} = w_{j}(\beta) = 1/\{g'(\mu_{j})^{2} \phi_{j} V(\mu_{j})\}, \quad W = \mathrm{diag}(w_{j})$
• $z = X\beta + W^{-1} u$
• $z_{j} = x_{j}^{\mathrm{T}} \beta + w_{j}^{-1} u_{j} = x_{j}^{\mathrm{T}} \beta + g'(\mu_{j})(y_{j} - \mu_{j})$
• $l(\beta) = \mathrm{E}\{-\partial^{2} \ell(\beta)/\partial \beta \partial \beta^{\mathrm{T}}\} = X^{\mathrm{T}} W X$
• $\mathrm{a.var}(\hat{\beta}) \doteq (X^{\mathrm{T}} \widehat{W} X)^{-1}$

)

... recap

•
$$\ell(\beta) = \sum_{j} \ell_{j} \{\eta_{j}(\beta); \mathbf{y}_{j}\}$$

• $\frac{\partial \ell(\beta)}{\partial \beta} = X^{\mathrm{T}} u(\beta), \quad X = \frac{\partial \eta}{\partial \beta^{\mathrm{T}}}, \quad u_{j} = \frac{(\mathbf{y}_{j} - \mu_{j})}{\phi_{j} \mathbf{g}'(\mu_{j}) \mathbf{V}(\mu_{j})}$
• $\frac{\partial^{2} \ell(\beta)}{\partial \beta \partial \beta^{\mathrm{T}}} = X^{\mathrm{T}} \frac{\partial u(\beta)}{\partial \beta^{\mathrm{T}}}$

$$\frac{\partial u_j(\beta)}{\partial \beta_b} = \frac{-(\partial \mu_j/\partial \beta_b)\phi_j g'(\mu_j) V(\mu_j) + (y_j - \mu_j)\phi_j \{g''(\mu_j) V(\mu_j) + g'(\mu_j) V'(\mu_j)\}}{\{\phi_j g'(\mu_j) V(\mu_j)\}^2}$$

$$\triangleright \ \partial \mu_j / \partial \beta_b = x_{jb} / g'(\mu_j)$$

$$I(\beta) = \mathsf{E}\{-\partial^2 \ell(\beta)/\partial \beta \partial \beta^{\mathrm{T}}\} = X^{\mathrm{T}}WX$$

Over-dispersion §10.6

- over-dispersion means Var(Y) is larger than expected under the Poisson or Binomial model
- which specify $Var(Y) = \mu$, or $v(\mu) = \mu(1 \mu)/m$
- where does over-dispersion come from? possibly multiplicative "noise", see p. 511 for Poisson, (10.34) for Binomial
- ► likelihood analysis computes marginal density, averaged over noise – e.g. Poisson → Negative Binomial (Ex. 10.26)
- alternative analysis based on "quasi-likelihood" uses analogy with least squares
- recall that if E(Y) = Xβ, Var(Y) = σ²I, then β̂ is best linear unbiased estimator of β, even if Y is not normally distributed (Gauss-Markov theorem)
- there could be better nonlinear estimators of β

... overdispersion

- If E(Y) = Xβ and Var(Y) = V, then β̂ = unbiased for β
- ► $\operatorname{Var}(\hat{\beta}) =$ (8.19)
- if we knew V, replace β̂ by weighted least squares estimator; otherwise, use β̂ and adjust confidence intervals by some estimate of V, see p.377

... overdispersion

- estimation of β in a generalized linear model depends only on the specification of the mean function
- and the variance function
- suggests using the same estimating equation for β, but allow inflation of the variance function by an unknown dispersion parameter

► e.g.
$$E(y_j) = \mu_j$$
, $Var(y_j) = \phi \mu_j$ -

• e.g.
$$E(y_j) = \mu_j$$
, $Var(y_j) = \phi \pi_j (1 - \pi_j)/m$ -

• estimating equation for β is unchanged

... overdispersion

$$\sum_{j=1}^n x_j \frac{y_j - \mu_j}{g'(\mu_j) V(\mu_j)} = 0$$

this is an unbiased estimating function g(y; β); satisfies
 E{g(Y; β)} = 0

- under some regularity conditions the solution of g(y; β) = 0 is consistent, asymptotically normal
- ► a. $\operatorname{Var}(\tilde{\beta}) = \phi(X^T \tilde{W} X)^{-1}$; \tilde{W} is diagonal with the same w_j , but **without** the ϕ_i
- from general theory on unbiased estimating functions

$$E\left\{-\frac{\partial g(Y;\beta)}{\partial \beta}\right\}^{-1} \operatorname{Var}\left\{g(Y;\beta)\right\} E\left\{-\frac{\partial g(Y;\beta)}{\partial \beta}\right\}^{-1}$$

Example 10.29

516

10 - Nonlinear Regression Models

City	Rain	r/m	City	Rain	r/m	City	Rain	r/m	City	Rain	r/m	Table 10.19 Toxoplamosis data: rainfall (mm) and the
1	1735	2/4	11	2050	7/24	21	1756	2/12	31	1780	8/13	numbers of people testing positive for
2	1936	3/10	12	1830	0/1	22	1650	0/1	32	1900	3/10	toxoplasmosis, r, our of m
3	2000	1/5	13	1650	15/30	23	2250	8/11	33	1976	1/6	people tested, for 34 cities
4	1973	3/10	14	2200	4/22	24	1796	41/77	34	2292	23/37	in El Salvador (Efron, 1086)
5	1750	2/2	15	2000	0/1	25	1890	24/51				1900).
6	1800	3/5	16	1770	6/11	26	1871	7/16				
7	1750	2/8	17	1920	0/1	27	2063	46/82				
8	2077	7/19	18	1770	33/54	28	2100	9/13				
9	1920	3/6	19	2240	4/9	29	1918	23/43				
10	1800	8/10	20	1620	5/18	30	1834	53/75				

Terms	df	Deviance		
Constant	33	74.21		
Linear	32	74.09		
Quadratic	31	74.09		
Cubic	30	62.63		

Table 10.20 Analysis of deviance for polynomial logistic models fitted to the toxoplasmosis data.

- incidence of toxoplasmosis as a function of rainfall
- residual deviances approximately twice the degrees of freedom

... example 10.29

```
> data(toxo)
 rain m r
1 1620 18 5
2 1650 30 15
3 1650 1 0
4 1735 4 2
> toxo.glm0 = glm(cbind(r,m-r) ~ rain + I(rain^2) + I(rain^3), data = toxo,
family = binomial)
> anova(toxo.glm0)
         Df Deviance Resid. Df Resid. Dev
NULT.T.
                           33
                                74.212
       1 0.1244
                         32
                                74.087
rain
                               74.087
I(rain^2) 1 0.0000
                         31
I(rain^3) 1 11.4529
                         30
                                 62.635
> toxo.glm1 = glm(cbind(r,m-r) ~ poly(rain,3), data = toxo, family = binomial)
> summarv(toxo.glm1)
Coefficients:
                      Estimate Std. Error z value Pr(>|z|)
(Intercept)
                      0.02427 0.07693 0.315 0.752401
poly(rain, degree = 3)1 -0.08606 0.45870 -0.188 0.851172
poly(rain, degree = 3)2 -0.19269 0.46739 -0.412 0.680141
poly(rain, degree = 3)3 1.37875 0.41150 3.351 0.000806 ***
Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 `' 1
(Dispersion parameter for binomial family taken to be 1)
```

Mull domionoo. 74 919 on 22 dogroop of freedom

... example 10.29

```
> toxo.guasi2 <- glm(cbind(r,m-r) ~ rain +I(rain^2)+I(rain^3),</pre>
+ data = toxo, family = guasibinomial)
> summary(toxo.guasi2)
Call·
qlm(formula = cbind(r, m - r) ~ rain + I(rain^2) + I(rain^3),
   family = quasibinomial, data = toxo)
Deviance Residuals:
   Min 10 Median 30 Max
-2.7620 -1.2166 -0.5079 0.3538 2.6204
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -2.902e+02 1.215e+02 -2.388 0.0234 *
rain 4.500e-01 1.876e-01 2.398 0.0229 *
I(rain^2) -2.311e-04 9.616e-05 -2.404 0.0226 *
I(rain^3) 3.932e-08 1.635e-08 2.405 0.0225 *
Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 `' 1
(Dispersion parameter for quasibinomial family taken to be 1.940446)
   Null deviance: 74.212 on 33 degrees of freedom
Residual deviance: 62.635 on 30 degrees of freedom
> (74.212-62.635)/3/1.940446
[1] 1.988718
> pf(1.988718,3,30,lower=F)
[1] 0.1368842
```

... inference

$$\hat{\phi} = \frac{1}{n-p} \sum_{j=1}^{n} \frac{(y_j - \hat{\mu}_j)^2}{V(\hat{\mu}_j)}$$
$$\hat{Var}(\hat{\beta}_j) = \hat{\phi} \ \widehat{Var}(\hat{\beta}_j), \quad \widehat{Var}(\hat{\beta}_j) \text{ from glm fit}$$

- ▶ comparison of models: $A \subset B D_A D_B \sim \chi^2_{p_B p_A}$
- changes to

$$rac{(D_{A}-D_{B})/(
ho_{B}-
ho_{A})}{\hat{\phi}} \sim F_{
ho_{B}-
ho_{A},
ho_{B}}$$

• $\hat{\phi}$ estimated under the larger model, *B*

In the News

Scien	се мал	.ORG FEEDBACK	HELP LIBRAR	All Science Journals					
		NS SCIENCE J	OURNALS CA	REERS MULTIME	DIA COLLI	NANCY REID	ALERTS ACCESS R		
Science	The World's	Leading Journal	of Original Scient	tific Research, Glob	al News, and (Commentary.	J. The		
Science Home	Current Issue	Previous Issues	Science Express	Science Products	My Science	About the Jou	ırnal		
Home > <u>Science Ma</u>	gazine > 17 Jan	uary 2014 > Taubr	man <i>et al.</i> , 343 (61	.68): 263–268					
Article Views	Publi	shed Online Janua	ry 2 2014			< Prev T	able of Contents Next		
Abstract	Vol. 3	Science 17 January 2014: Provide 1 Vol. 343 no. 6168 pp. 263–268 Provide 2 DDI: 10.1126/science 1246183 Provide 2							
Full Text	DOI.	10.1120/3000000	1240105						
Full Text (PDF)	RESE	ARCH ARTICLE							
Supplementary Materials	Mee	dicaid Incre gon's Healt	ases Emerg th Insuranc	gency-Depar e Experimen	tment Us t	e: Eviden	ce from		
VERSION HISTOR	Y Saral	ı L. Taubman ^{1,*} , I	Heidi L. Allen ² , B	ill J. Wright ³ , Kathe	rine Baicker ^{1,}	⁴ , Amy N. Fink	elstein <u>1,5</u>		
> 343/6168/26	3 <u>+</u> Au	thor Affiliations							
> science.12461	83v1 <u>⊣</u> *Co	rresponding auth	or. E-mail: <u>staub(</u>	@nber.org					
	AB	STRACT EDITO	R'S SUMMARY						
Article Tools									

... in the News

RESEARCH ARTICLE

Medicaid Increases Emergency-Department Use: Evidence from Oregon's Health Insurance Experiment

Sarah L. Taubman,¹* Heidi L. Allen,² Bill J. Wright,³ Katherine Baicker,^{1,4} Amy N. Finkelstein^{1,5}

In 2008, Oregon initiated a limited expansion of a Medicaid program for uninsured, low-income adults, drawing names from a waiting list by lottery. This lottery created a rare opportunity to study the effects of Medicaid coverage by using a randomized controlled design. By using the randomization provided by the lottery and emergency-department records from Portland-area hospitals, we studied the emergency department use of about 25,000 lottery participants over about 18 months after the lottery. We found that Medicaid coverage significantly increases overall emergency use by 0.41 visits per person, or 40% relative to an average of 1.02 visits per person in the control group. We found increases in emergency-department visits across a broad range of types of visits, conditions, and subgroups, including increases in visits for conditions that may be most readily treatable in primary have less than \$2000 in assets. OHP Standard provides relatively comprehensive medical benefits (including prescription drug coverage) with no consumer cost sharing and low monthly premiums (between \$0 and \$20, based on income), provided mostly through managed care organizations.

Oregon conducted eight lottery drawings from a waiting list for this Medicaid program between March and September 2008. Among the individuals randomly selected by lottery, those who completed the application process and met the eligibility criteria were enrolled (fig. S1). The lottery process and the insurance program are described in more detail elsewhere (11). Multiple institutional review boards have approved the Oregon Health Insurance Experiment research.

Our prior work on the Oregon Health Insurance Experiment used the random assignment of the lottery to study the impacts of the first 2 years of Medicaid coverage (11–13). We found that



Happy Chinese New Year!