Components of variance

- Example: 1-way layout/k-group comparison/ 1-way ANOVA
- $y_{tr} = \mu + \alpha_t + \epsilon_{tr}, \quad r = 1, \ldots, R; t = 1, \ldots, T, \quad \epsilon_{tr} \sim (0, \sigma^2)$
- $\hat{\alpha}_t = \bar{y}_{t.} - \bar{y}_{..}, \quad$ under constraint $\sum \alpha_t = 0$
- $\text{var}(\bar{y}_{t.} - \bar{y}_{s.}) = \frac{2\sigma^2}{R}$
- $\hat{\sigma}^2 = s^2 = \sum_{r,t}(y_{tr} - \bar{y}_{t.})^2 / T(R-1)$
- ANOVA:

Change model parameterization

- $y_{tr} = \mu + b_t + \epsilon_{tr}, \quad \epsilon_{tr} \sim (0, \sigma^2), \quad b_t \sim (0, \sigma_b^2)$
- $SS_b =$
- ANOVA unchanged
- $E(MS_b) =$

$\text{cor}(y_{tr}, y_{ts})$

Inference

- Under $H_0 : \sigma_b^2 = 0$:
- Estimation of σ^2 and σ_b^2
- Estimation of σ_b^2/σ^2 using F distribution
- Estimation of μ:
- See Example 9.14: Exercise: verify CI for ratio

Inference

- Under $H_0 : \sigma_b^2 = 0$:
- Estimation of σ^2 and σ_b^2
- Estimation of σ_b^2/σ^2 using F distribution
- Estimation of μ:
- See Example 9.14: Exercise: verify CI for ratio

Fixed or random effects?

- depends on context
- one rule of thumb:

- which is of interest: mean or variance?

- nested factors are often modelled with random effects

- are levels of factor in one group same as levels of factors in another group?

Example: several nested levels of variation p.450

- response: success of a surgical procedure ("measured on ...")
- patients surgeons hospitals
- $y_{hsp} =$

... fixed or random?

- $y_{hsp} = \mu + b_h + e_{hs} + \epsilon_{hsp}$
- $E(y_{hsp}) =$
- See Table 9.23 and columns of expected mean squares

How well can we estimate a variance?

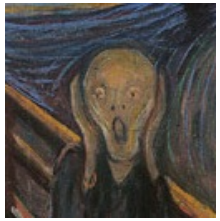
-

Example: randomized blocks with replications

- $y_{tbr} = \mu + \alpha_t + \beta_b + (\alpha\beta)tb + \epsilon_{tbr}$

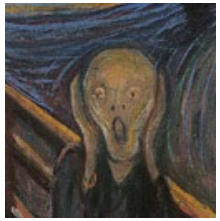
- ANOVA:

Split plot experiments



One design, often RB, at ‘whole plot’ level
Second design, often with random effects, at subplot level
Example 9.15

Split plot experiments



One design, often RB, at ‘whole plot’ level
Second design, often with random effects, at subplot level
Example 9.15

Linear mixed effects models

- $y = X\beta + Zb + \epsilon$
- Assumptions:
- $y \mid b \sim$
- $y \sim$
- See Example 9.16 – note imbalance

... Example 9.16

Example 9.16 (Longitudinal data) A short longitudinal study has one individual allocated to the treatment and two to the control, with observations

$$y_{1j} = \beta_0 + b_1 + \varepsilon_{1j}, \quad y_{21} = \beta_0 + b_2 + \varepsilon_{21}, \quad y_{3j} = \beta_0 + \beta_1 + b_3 + \varepsilon_{3j}, \quad j = 1, 2.$$

Thus there are two measurements on the first and third individuals, and just one on the second. The b_j represent variation among individuals and the ε_{ij} variation between measures on the same individuals. If the b's and ε's are all mutually independent with variances σ_b^2 and σ^2, then

$$\begin{pmatrix} y_{11} \\ y_{12} \\ y_{21} \\ y_{31} \\ y_{32} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{21} \\ \varepsilon_{31} \\ \varepsilon_{32} \end{pmatrix},$$

and this fits into formulation (9.12) with $\Omega_b = \sigma_b^2 I_3$ and $\Omega = \sigma^2 I_5$. Here ψ comprises the scalar σ_b^2/σ^2, and hence the variance matrix

$$\Omega + Z\Omega_b Z^{\mathrm{T}} = \begin{pmatrix} \sigma_b^2 + \sigma^2 & \sigma_b^2 & 0 & 0 & 0 \\ \sigma_b^2 & \sigma_b^2 + \sigma^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_b^2 + \sigma^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_b^2 + \sigma^2 & \sigma_b^2 \\ 0 & 0 & 0 & \sigma_b^2 & \sigma_b^2 + \sigma^2 \end{pmatrix}$$

may be written as

$$\begin{pmatrix} 1 + \psi & \psi & 0 & 0 & 0 \end{pmatrix}$$

... Example 9.16

Inference

- $y \sim N(X\beta, Z\Omega_b Z^T + \Omega) = N(X\beta, \sigma^2 \Upsilon^{-1})$
- log-likelihood function
- constrained m.l.e.'s
- REML

Inference

- $y \sim N(X\beta, Z\Omega_b Z^T + \Omega) = N(X\beta, \sigma^2 \Upsilon^{-1})$
- prediction:

$$E(b \mid y) = (Z^T \Omega^{-1} Z + \Omega_b^{-1})^{-1} Z^T \Omega^{-1} (y - X\beta)$$

$$\text{var}(b \mid y) = (Z^T \Omega^{-1} Z + \Omega_b^{-1})^{-1}$$

- $\tilde{b} =$

Example 9.17

- $y_{ij} = \mu + b_i + \epsilon_{ij}, \quad j = 1, \ldots \quad ; i = 1, \ldots, q$
- $\Omega =$ $\Omega_b =$
- $X =$ $Z =$
- $\tilde{b}_i = \underline{\bar{y}_{i.} - \bar{y}_{..}}$ $\text{var}(\tilde{b}_i) =$

Example 9.18

- repeated measurements on the 30 individuals, at 5 time points
- might expect that regression relationship against time is similar for each individual, subject to random variation
- model $y_{jt} = \beta_0 + b_{j0} + (\beta_1 + b_{j1})x_{jt} + \epsilon_{jt}, \quad t = 1, \ldots, 5$
- x_{jt} takes values $0, 1, 2, 3, 4$ for $t = 1, 2, 3, 4, 5$
- same for each j
- `data(rat.growth, library="SMPracticals")`
- $(b_{j0}, b_{j1}) \dot{\sim} N_2(0, \Omega_b), \quad \epsilon_{jt} \dot{\sim} N(0, \sigma^2)$ independent
- two fixed parameters β_0, β_1
- four variance/covariance parameters: $\sigma^2_{b0}, \sigma^2_{b1}, \text{cov}(b_0, b_1), \sigma^2$

... Example 9.18

- maximum likelihood estimates of fixed effects: $\hat{\beta}_0 = 156.05(2.16), \hat{\beta}_1 = 43.27(0.73)$
- weight in week 1 is estimated to be about 156 units, and average increase per week estimated to be 43.27
- there is large variability between rats: estimated standard deviation of 10.93 for intercept, 3.53 for slope
- there is little correlation between the intercepts and slopes
- ```
  library(MASS)  # this is included the standard R distribution
  library(SMPracticals) # this has various data sets from Davison's book
  library(ellipse) # but I got an error the first time and had to download an additional p
  library(SMPracticals) # and now it works
  data(rat.growth) # for Example 9.18
  rat.growth[1:10,]  # to see what it looks like, and to see variable names
  with(rat.growth, plot( y ~ week , type="l"))
  separate.lm = lm(y ~ week + factor(rat)+ week:factor(rat), data = rat.growth) # fit sepa
  rat.mixed = lmer(y ~ week + (week|rat), data = rat.growth) # REML is the default
  summary(rat.mixed)  # compare Table 9.28
  ```
  ```

Principles (C&D, §7.2 "Non-specific effects")

- "aspects of the system under study that may well correspond to systematic differences in the variables being studies, but which are of no, or limited, direct concern"
- examples: clinical trial carried out at several centres; agricultural field trials at a number of different farms; sociological study in a number of different countries; laboratory experiments with different sets of apparatus
- "it may be necessary to take account of such features in one of two different ways..."

C&D, §7.2.2 "Stable treatment effect"

- model:

$$E(Y_{tci}) = \alpha_c + x_{ci}^T \beta + \delta_t$$

- no treatment / centre interaction
- should α_c be ?fixed? or ?random?
- "effective use of a random-effects representation will require estimation of the variance component corresponding to the centre effects"
- "even under the most favourable conditions the precision achieved in that estimate will be at best that from estimating a single variance from a sample of a size equal to the number of centres"
- "... very fragile unless there are at least, say, 10 centres and preferably considerably more"

... C&D, §7.2.2 "Stable treatment effect"

- "if centres are chosen by an effectively random procedure from a large population of candidates, ... the random-effects representation has an attractive tangible interpretation. This would not apply, for example, to the countries of the EU in a social survey."
- some general considerations in linear mixed models:
  - in balanced factorial designs, the analysis of treatment means is unchanged
  - in other cases, estimated effects will typically be 'shrunk', and precision improved
  - "representation of the nonspecific effects as random effects involves independence assumptions which certainly need consideration and may need some empirical check"

... C& D, §7.2.3 “Unstable treatment effect”

- “ if there is an interaction between an explanatory variable [e.g. treatment] and a nonspecific variable”
- i.e. the effects of the explanatory variable change with different levels of the nonspecific factor
- “the first step should be to explain this interaction, for example by transforming the scale on which the response variable is measure or by introducing a new explanatory variable”
- example: two medical treatments compared at a number of centres show different treatment effects, as measured by an ratio of event rates
- possible explanation: the difference of the event rates might be stable across centres
- possible explanation: the ratio depends on some characteristic of the patient population, e.g. socio-economic status
- “an important special application of random-effect models for interactions is in connection with overviews, that is, assembling of information from different studies of essentially the same effect”

This week's study

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No clear evidence Tamiflu works, study finds

CARLY WEEKS

From Thursday's Globe and Mail

Published Wednesday, Jan. 18, 2012 5:41PM EST

Last updated Wednesday, Jan. 18, 2012 5:47PM EST

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