Components of variance

- Example: 1-way layout/k-group comparison/ 1-way ANOVA
- $y_{tr} = \mu + \alpha_t + \epsilon_{tr}, \quad r = 1, \ldots, R; t = 1, \ldots, T, \quad \epsilon_{tr} \sim (0, \sigma^2)$
- $\hat{\alpha}_t = \bar{y}_{t.} - \bar{y}_{..}, \quad$ under constraint $\sum \alpha_t = 0$
- $\text{var}(\bar{y}_{t.} - \bar{y}_{s.}) = \frac{2\sigma^2}{R}$
- $\hat{\sigma}^2 = s^2 = \sum_{r,t} (y_{tr} - \bar{y}_{t.})^2 / T(R-1)$
- ANOVA:

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- $\hat{\sigma}^2 = s^2 = \sum_{r,t}(y_{tr} - \bar{y}_{t.})^2 / T(R-1)$
- ANOVA:

Source	df	SS	E(MS)
betw	$T-1$	$\sum_{t,r}(\bar{y}_{t.} - \bar{y}_{..})^2$	$\sigma^2 + *$
with	$T(R-1)$	$\sum_{t,r}(y_{tr} - \bar{y}_{t.})^2$	σ^2

$R\sum \alpha_t^2/(T-1)$ $*$ (ntbc)

Change model parameterization

- $y_{tr} = \mu + b_t + \epsilon_{tr}, \quad \epsilon_{tr} \sim (0, \sigma^2), \quad b_t \sim (0, \sigma_b^2)$ (ind.)
- $SS_b =$
- ANOVA unchanged
- $E(MS_b) =$

$\mathrm{cor}(y_{tr}, y_{ts})$

Change model parameterization

$\sum (X_i - \bar{X})^2 \sim \chi^2_{n-1}$

- $y_{tr} = \mu + b_t + \epsilon_{tr}, \quad \epsilon_{tr} \sim (0, \sigma^2), \quad b_t \sim (0, \sigma_b^2)$
- $SS_b = \sum_t \sum_r (\bar{y}_{t\cdot} - \bar{y}_{\cdot\cdot})^2 = \sum_t$

$\sim (\sigma^2 + R\sigma_b^2)\chi^2_{T-1}$

- ANOVA unchanged
- $E(MS_b) =$

$\mathrm{var}\,\bar{y}_{t\cdot} = \mathrm{var}\,\frac{\sum y_{tr}}{R^2}$

$= \frac{\mathrm{var}\sum(b_t + \epsilon_{tr})}{R^2}$

$= \ldots =$

$\mathrm{cor}(y_{tr}, y_{ts})$

Change model parameterization

- $y_{tr} = \mu + b_t + \epsilon_{tr}, \quad \epsilon_{tr} \sim (0, \sigma^2), \quad b_t \sim (0, \sigma_b^2)$

- $SS_b =$

- ANOVA unchanged
- $E(MS_b) =$

$\mathrm{cor}(y_{tr}, y_{ts})$

Change model parameterization

$-\frac{1}{T-1} \le \rho \le 1$

- $y_{tr} = \mu + b_t + \epsilon_{tr}, \quad \epsilon_{tr} \sim (0, \sigma^2), \quad b_t \sim (0, \sigma_b^2)$

Intraclass correlation

$\frac{R \sum \alpha_t^2}{T-1}$

- $SS_b =$
- ANOVA unchanged
- $E(MS_b) =$

E(MS)

betw $T-1$ $\sigma^2 + R\sigma_b^2$

with $T(R-1)$ σ^2

$+\epsilon_{ts})$

$\frac{\sigma_b^2}{\sigma_b^2 + \sigma^2} = \rho$

cor(y_{tr}, y_{ts}) $= \frac{\operatorname{cov}(y_{tr}, y_{ts})}{\sqrt{v_1}\sqrt{v_2}} = \frac{\operatorname{cov}(b_t + \epsilon_{tr}, b_t}{}$

Inference

- Under $H_0 : \sigma_b^2 = 0$:

F test

$y \sim N(\mu, \Sigma)$

$-\frac{1}{T-1} \leq \rho \leq 1$

Σ

- Estimation of σ^2 and σ_b^2
- Estimation of σ_b^2/σ^2 using F distribution
- Estimation of μ:
- See Example 9.14: Exercise: verify CI for ratio

Inference

- Under $H_0 : \sigma_b^2 = 0$:

$E(MS_w) = \sigma^2 \quad \hat{\sigma}^2 = MS_w$

- Estimation of σ^2 and σ_b^2

$E(MS_b) = \sigma^2 + R\sigma_b^2$

m.o.m.

$\frac{1}{R}(MS_b - MS_w) = \tilde{\sigma}_b^2$

- Estimation of σ_b^2/σ^2 using F distribution

- Estimation of μ:

- See Example 9.14: Exercise: verify CI for ratio

Inference

- Under $H_0 : \sigma_b^2 = 0$:

$$\frac{MS_b / (\sigma^2 + K\sigma_b^2)}{MS_w / \sigma^2} \sim F_{T-1,\, T(K-1)}$$

- Estimation of σ^2 and σ_b^2

- Estimation of σ_b^2/σ^2 using F distribution

p. 450

90% CI

$(-0.01, 1.34)$

- Estimation of μ:

- See Example 9.14: Exercise: verify CI for ratio

Inference

- Under $H_0 : \sigma_b^2 = 0$:
- Estimation of σ^2 and σ_b^2
- Estimation of σ_b^2/σ^2 using F distribution

$n \not{b} c$

- Estimation of μ: $E(\bar{y}_{..}) = \mu \quad \text{var}\, \bar{y}_{..} = \dfrac{\sigma^2 + R\sigma_b^2}{RT}$
- See Example 9.14: Exercise: verify CI for ratio

Inference

- Under $H_0 : \sigma_b^2 = 0$:

- Estimation of σ^2 and σ_b^2

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Fixed or random effects?

- depends on context
- one rule of thumb:
- which is of interest: mean or variance?
- nested factors are often modelled with random effects
- are levels of factor in one group same as levels of factors in another group?

Fixed or random effects?

- depends on context
- one rule of thumb: if factor levels are a random sample of all possible levels
- which is of interest: mean or variance?
- nested factors are often modelled with random effects
- are levels of factor in one group same as levels of factors in another group?

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Fixed or random effects?

- depends on context
- one rule of thumb:

sch 1 2 3 4 5

cl 1 2 3 4

0

- which is of interest: mean or variance?
- nested factors are often modelled with random effects
- are levels of factor in one group same as levels of factors in another group?

Example: several nested levels of variation p.450

- response: success of a surgical procedure ("measured on ...")
- patients surgeons hospitals
- $y_{hsp} =$

Example: several nested levels of variation p.450

- response: success of a surgical procedure ("measured on ...")
- patients surgeons hospitals

$p: 1, \ldots, P$ $s: 1 \ldots S$ $h: 1 \ldots H$

- $y_{hsp} =$

Example: several nested levels of variation p.450

- response: success of a surgical procedure ("measured on ...")
- patients surgeons hospitals
- $y_{hsp} =$ $\mu + b_h + e_{hs} + \varepsilon_{hsp}$

$E(y_{hsp}) = \mu$ all random

$\mu + b_h$ if $e, \varepsilon \sim (\ \ )$, $\sim$

$\mu + b_h + e_{hs}$ $\varepsilon \sim$

... fixed or random?

- $y_{hsp} = \mu + b_h + e_{hs} + \epsilon_{hsp}$

- $E(y_{hsp}) =$

- See Table 9.23 and columns of expected mean squares

... fixed or random?

- $y_{hsp} = \mu + b_h + e_{hs} + \epsilon_{hsp}$

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- See Table 9.23 and columns of expected mean squares

... fixed or random?

- $y_{hsp} = \mu + b_h + e_{hs} + \epsilon_{hsp}$

$p = 1, \ldots, P$

$s = \ldots$

$h =$

- $E(y_{hsp}) =$

est σ^2, σ_e^2, σ_b^2 under suitable

ss

$$\tilde{\sigma}^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} \qquad \text{var}(\tilde{\sigma}^2) = \frac{2\sigma^4}{n-1}$$

- See Table 9.23 and columns of expected mean squares

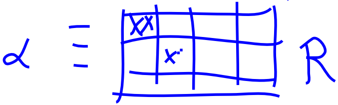
How well can we estimate a variance?

-

Example: randomized blocks with replications

- $y_{tbr} = \mu + \alpha_t + \beta_b + (\alpha\beta)_{tb} + \epsilon_{tbr}$

(crossed r.e.) β



- ANOVA:

Example: randomized blocks with replications

- $y_{tbr} = \mu + \alpha_t + \beta_b + (\alpha\beta)tb + \epsilon_{tbr}$

$\bar{\tau}$ blocks

$E(MS)$

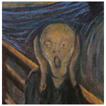
- ANOVA:

		fixed ①	rand.
trt	$T-1$	$\sigma^2 + \sum\alpha_t^2 \frac{BR}{T-1}$	$\sigma^2 + R\sigma^2_{AB} + ①$
blk	$B-1$	$\sigma^2 + \sum\beta_b^2 \frac{TR}{B-1}$	$\sigma^2 + RT\sigma^2_B$
$t \times b$	$(T-1)(B-1)$	$\sigma^2 + \sum_{t,b}(\alpha\beta)^2_{tb} \cdot \frac{R}{(T-1)(B-1)}$	$\sigma^2 + R\sigma^2_{AB}$
within cell	$TB(R-1)$	σ^2	σ^2

$\beta \sim (0, \sigma^2_B)$ $(\alpha\beta) \sim (0, \sigma^2_{AB})$

MS_t / MS_{AB}

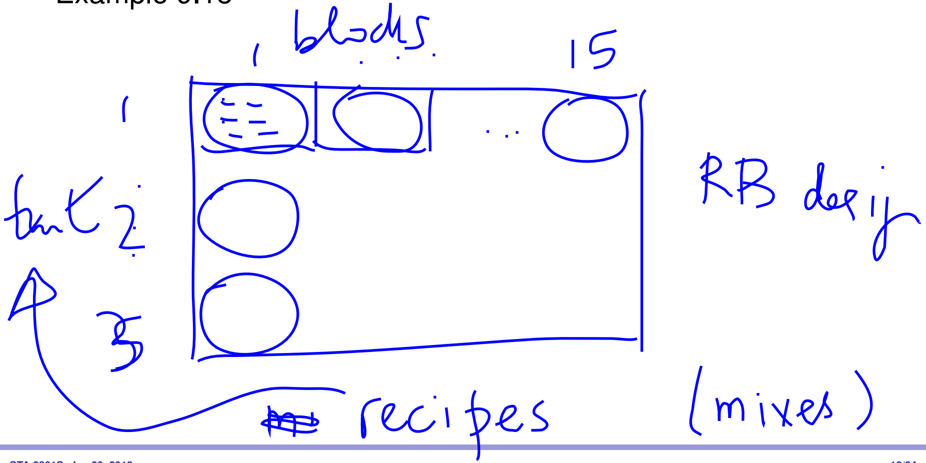
Split plot experiments



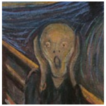
One design, often RB, at ‘whole plot’ level

Second design, often with random effects, at subplot level

Example 9.15



Split plot experiments



p 455

One design, often RB, at 'whole plot' level

Second design, often with random effects, at subplot level

Example 9.15

whole plots is 1 expt

Sub plots is another

> 2 levels of var =

Anova	①	F	②
rec	MS_r	MS_r / MS_{mr}	temp
mix	MS_m		temp×rec
r×mix	MS_{mr}		residual

Linear mixed effects models v. general

- $y = X\beta + Zb + \epsilon$

  y: $n \times 1$ $\quad$ X: $n \times p$ $\quad$ β: $p \times 1$ fixed eff.

  $Z_{n \times q}$ ← design

  $b_{q \times 1}$ ← random

- Assumptions:

- $y \mid b \sim$

- $y \sim$

- See Example 9.16 – note imbalance

Linear mixed effects models

- $y = X\beta + Zb + \epsilon$
  $n \times 1$
  $\underset{\sim}{\epsilon} \perp \underset{\sim}{b}$

- Assumptions: $\underset{\sim}{\epsilon} \sim N(0, \Omega)$; $\underset{\sim}{b} \sim N_q(0, \Omega_b)$
  $(\sim N(0, \sigma^2 I))$

- $y \mid b \sim$

- $y \sim$

- See Example 9.16 – note imbalance

Linear mixed effects models

- $y = X\beta + Zb + \epsilon$

app ?

- multi level model
- repeated measures
- longitudinal data
- components of var

- Assumptions:

- $y \mid b \sim N(X\beta + Zb, \Omega)$

(ntbc)

$y \sim N(X\beta, Z\Omega_b Z^T + \Omega)$

- See Example 9.16 – note imbalance

Linear mixed effects models

- $y = X\beta + Zb + \epsilon$
- Assumptions:
- $y \mid b \sim$
- $y \sim$
- See Example 9.16 – note imbalance

Linear mixed effects models

- $y = X\beta + Zb + \epsilon$
- Assumptions:
- $y \mid b \sim$
- $y \sim$
- See Example 9.16 – note imbalance

not equal #'s of reps

... Example 9.16

Example 9.16 (Longitudinal data) A short longitudinal study has one individual allocated to the treatment and two to the control, with observations

$$y_{1j} = \beta_0 + b_1 + \varepsilon_{1j}, \quad y_{21} = \beta_0 + b_2 + \varepsilon_{21}, \quad y_{3j} = \beta_0 + \beta_1 + b_3 + \varepsilon_{3j}, \quad j = 1, 2.$$

Thus there are two measurements on the first and third individuals, and just one on the second. The b_j represent variation among individuals and the ε_{ij} variation between measures on the same individuals. If the b's and ε's are all mutually independent with variances σ_b^2 and σ^2, then

$$\begin{pmatrix} y_{11} \\ y_{12} \\ y_{21} \\ y_{31} \\ y_{32} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} + \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \varepsilon_{21} \\ \varepsilon_{31} \\ \varepsilon_{32} \end{pmatrix},$$

and this fits into formulation (9.12) with $\Omega_b = \sigma_b^2 I_3$ and $\Omega = \sigma^2 I_5$. Here ψ comprises the scalar σ_b^2/σ^2, and hence the variance matrix

$$\Omega + Z\Omega_b Z^{\mathrm{T}} = \begin{pmatrix} \sigma_b^2 + \sigma^2 & \sigma_b^2 & 0 & 0 & 0 \\ \sigma_b^2 & \sigma_b^2 + \sigma^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_b^2 + \sigma^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_b^2 + \sigma^2 & \sigma_b^2 \\ 0 & 0 & 0 & \sigma_b^2 & \sigma_b^2 + \sigma^2 \end{pmatrix}$$

may be written as

$$\begin{pmatrix} 1+\psi & \psi & 0 & 0 & 0 \end{pmatrix}$$

... Example 9.16

Inference

- $y \sim N(X\beta, Z\Omega_b Z^T + \Omega) = N(X\beta, \sigma^2\Upsilon^{-1})$
- log-likelihood function
- constrained m.l.e.'s
- REML

Inference

- $y \sim N(X\beta, Z\Omega_b Z^T + \Omega) = N(X\beta, \sigma^2 \Upsilon^{-1})$ ψ par. in Υ

- log-likelihood function

$$\ell(\beta, \sigma^2, \psi) = -\frac{n}{2}\log\sigma^2 + \frac{1}{2}\log|\Upsilon| - \frac{1}{2\sigma^2}(y - X\beta)^T \Upsilon (y - X\beta)$$

- constrained m.l.e.'s

$$\hat\beta_\psi, \hat\sigma^2_\psi : \hat\beta_\psi = (X^T \Upsilon X)^{-1} X^T \Upsilon y \quad \hat\sigma^2_\psi = \frac{1}{n}(y - X\hat\beta_\psi)^T \Upsilon (y - X\hat\beta)$$

- REML

$$\ell(\hat\beta_\psi, \hat\sigma^2_\psi, \psi) =: \ell_p(\psi)$$

Inference

- $y \sim N(X\beta, Z\Omega_b Z^T + \Omega) = N(X\beta, \sigma^2\Upsilon^{-1})$
- log-likelihood function $-\frac{1}{2\sigma^2}(y - X\beta)^T\Upsilon(y - X\beta)$
- constrained m.l.e.'s $-\frac{n}{2}\log\sigma^2 + \frac{1}{2}\log|\Upsilon|$
- REML

$$\ell_{REML}(\beta, \sigma^2, \psi) = \ell(\beta, \sigma^2, \psi) + \frac{p}{2}\log\sigma^2 - \frac{1}{2}\log|X^T\Upsilon X|$$

Inference

- $y \sim N(X\beta, Z\Omega_b Z^T + \Omega) = N(X\beta, \sigma^2\Upsilon^{-1})$

- log-likelihood function

$$\tilde{\sigma}^2_{REML} = \frac{(y - X\hat{\beta})\hat{\Upsilon}(y - X\hat{\beta})}{n-p}$$

- constrained m.l.e.'s

- REML restr'd max. lik.

fixes the ~~problems~~ divisor in max lik est. of σ^2, & ψ

Inference

- $y \sim N(X\beta, Z\Omega_b Z^T + \Omega) = N(X\beta, \sigma^2 \Upsilon^{-1})$

- prediction:

$$E(b \mid y) = (Z^T \Omega^{-1} Z + \Omega_b^{-1})^{-1} Z^T \Omega^{-1} (y - X\beta)$$

$$\text{var}(b \mid y) = (Z^T \Omega^{-1} Z + \Omega_b^{-1})^{-1}$$

- $\tilde{b} =$

Inference

- $y \sim N(X\beta, Z\Omega_b Z^T + \Omega) = N(X\beta, \sigma^2 \Upsilon^{-1})$

$\hat{\beta} \quad \hat{\sigma}^2$

$\hat{\Omega} \quad \hat{\Omega}_b \quad \ldots$

- prediction:

$$E(b \mid y) = (Z^T \Omega^{-1} Z + \Omega_b^{-1})^{-1} Z^T \Omega^{-1}(y - X\beta)$$

$$\text{var}(b \mid y) = (Z^T \Omega^{-1} Z + \Omega_b^{-1})^{-1}$$

- $\tilde{b} = (Z^T \hat{\Omega}^{-1} Z + \hat{\Omega}_b^{\cdot\cdot})^{-1} Z^T \hat{\Omega}^{-1}(y - X\hat{\beta})$

Inference

- $y \sim N(X\beta, Z\Omega_b Z^T + \Omega) = N(X\beta, \sigma^2 \Upsilon^{-1})$

- prediction:

$$E(b \mid y) = (Z^T \Omega^{-1} Z + \Omega_b^{-1})^{-1} Z^T \Omega^{-1} (y - X\beta)$$

$$\text{var}(b \mid y) = (Z^T \Omega^{-1} Z + \Omega_b^{-1})^{-1}$$

- $\tilde{b} =$

Example 9.17

- $y_{ij} = \mu + b_i + \epsilon_{ij}, \quad j = 1, \ldots \qquad ; i = 1, \ldots, q$

- $\Omega = \qquad \Omega_b =$

- $X = \qquad Z =$

- $\tilde{b}_i = \frac{\bar{y}_{i.} - \bar{y}_{..}}{} \qquad \text{var}(\tilde{b}_i) =$

Example 9.17

- $y_{ij} = \mu + b_i + \epsilon_{ij}, \quad j = 1, \dots \qquad ; i = 1, \dots, q$

- $\Omega =$ $\qquad \Omega_b =$

- $X =$ $\qquad Z =$

- $\tilde{b}_i = \dfrac{\bar{y}_{i.} - \bar{y}_{..}}{}$ $\qquad \mathrm{var}(\tilde{b}_i) =$

Example 9.17

- $y_{ij} = \mu + b_i + \epsilon_{ij}, \quad j = 1, \ldots \qquad ; i = 1, \ldots, q$
- $\Omega = \qquad \Omega_b =$
- $X = \qquad Z =$
- $\tilde{b}_i = \frac{\bar{y}_{i.} - \bar{y}_{..}}{\qquad} \qquad \text{var}(\tilde{b}_i) =$

Example 9.17

$\sim (0, \sigma_b^2)$

- $y_{ij} = \mu + b_i + \epsilon_{ij}, \quad j = 1, \ldots n_i \;; i = 1, \ldots, q$ 1-way

- $\Omega = \sigma^2 I_n \quad \Omega_b = \sigma_b^2 \cdot I_q$ $\quad \underset{n\times q}{Z} \underset{q\times 1}{\tilde{b}}$

- $X = \begin{pmatrix} 1 \\ \vdots \\ \vdots \\ 1 \end{pmatrix}$ $\quad p = 1$ $\quad Z = \begin{array}{c} n_1 \left\{ \begin{array}{c} \\ \\ \\ \end{array} \right. \\ \\ \\ \end{array} \begin{pmatrix} 1 & 0 & 0 \\ 1 & & \\ \vdots & & \\ 1 & & \\ 0 & 1 & 0 \\ & 1 & \\ & \vdots & 1 \\ & & \vdots \\ & & 1 \end{pmatrix}$

- $\tilde{b}_i = \dfrac{\bar{y}_{i.} - \bar{y}_{..}}{\left(1 + \dfrac{\hat{\sigma}^2}{n_i \hat{\sigma}_b^2}\right)} \qquad \widehat{\operatorname{var}}(\tilde{b}_i) = \dfrac{1}{\dfrac{1}{\hat{\sigma}_b^2} + \dfrac{n_i}{\hat{\sigma}^2}}$

Example 9.18

- repeated measurements on the 30 individuals, at 5 time points
- might expect that regression relationship against time is similar for each individual, subject to random variation
- model $y_{jt} = \beta_0 + b_{j0} + (\beta_1 + b_{j1})x_{jt} + \epsilon_{jt}, \quad t = 1, \ldots, 5$
- x_{jt} takes values $0, 1, 2, 3, 4$ for $t = 1, 2, 3, 4, 5$
- same for each j
- `data(rat.growth, library="SMPracticals")`
- $(b_{j0}, b_{j1}) \dot\sim N_2(0, \Omega_b), \quad \epsilon_{jt} \dot\sim N(0, \sigma^2)$ independent
- two fixed parameters β_0, β_1
- four variance/covariance parameters: $\sigma^2_{b0}, \sigma^2_{b1}, \text{cov}(b_0, b_1), \sigma^2$

Example 9.18

- repeated measurements on the 30 individuals, at 5 time points
- might expect that regression relationship against time is similar for each individual, subject to random variation
- model $y_{jt} = \beta_0 + b_{j0} + (\beta_1 + b_{j1})x_{jt} + \epsilon_{jt}, \quad t = 1, \ldots, 5$
- x_{jt} takes values $0, 1, 2, 3, 4$ for $t = 1, 2, 3, 4, 5$
- same for each j
- `data(rat.growth, library="SMPracticals")`
- $(b_{j0}, b_{j1}) \dot\sim N_2(0, \Omega_b), \quad \epsilon_{jt} \dot\sim N(0, \sigma^2)$ independent
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... Example 9.18

- maximum likelihood estimates of fixed effects: $\hat{\beta}_0 = 156.05(2.16), \hat{\beta}_1 = 43.27(0.73)$
- weight in week 1 is estimated to be about 156 units, and average increase per week estimated to be 43.27
- there is large variability between rats: estimated standard deviation of 10.93 for intercept, 3.53 for slope
- there is little correlation between the intercepts and slopes
-
```
 library(MASS)  # this is included the standard R distribution
library(SMPracticals) # this has various data sets from Davison's book
library(ellipse) # but I got an error the first time and had to download an additional p
library(SMPracticals) # and now it works
data(rat.growth) # for Example 9.18
rat.growth[1:10,]  # to see what it looks like, and to see variable names
with(rat.growth, plot( y ~ week , type="l"))
separate.lm = lm(y ~ week + factor(rat)+ week:factor(rat), data = rat.growth) # fit sepa
rat.mixed = lmer(y ~ week + (week|rat), data = rat.growth) # REML is the default
summary(rat.mixed)  # compare Table 9.28
```

Principles (C&D, §7.2 "Non-specific effects")

- "aspects of the system under study that may well correspond to systematic differences in the variables being studies, but which are of no, or limited, direct concern"
- examples: clinical trial carried out at several centres; agricultural field trials at a number of different farms; sociological study in a number of different countries; laboratory experiments with different sets of apparatus
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C&D, §7.2.2 "Stable treatment effect"

- model:
$$E(Y_{tci}) = \alpha_c + x_{ci}^T\beta + \delta_t$$
- no treatment / centre interaction
- should α_c be ?fixed? or ?random?
- "effective use of a random-effects representation will require estimation of the variance component corresponding to the centre effects"
- "even under the most favourable conditions the precision achieved in that estimate will be at best that from estimating a single variance from a sample of a size equal to the number of centres"
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- some general considerations in linear mixed models:
  - in balanced factorial designs, the analysis of treatment means is unchanged
  - in other cases, estimated effects will typically be “shrunk”, and precision improved
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... C& D, §7.2.3 "Unstable treatment effect"

- " if there is an interaction between an explanatory variable [e.g. treatment] and a nonspecific variable"
- i.e. the effects of the explanatory variable change with different levels of the nonspecific factor
- "the first step should be to explain this interaction, for example by transforming the scale on which the response variable is measure or by introducing a new explanatory variable"
- example: two medical treatments compared at a number of centres show different treatment effects, as measured by an ratio of event rates
- possible explanation: the difference of the event rates might be stable across centres
- possible explanation: the ratio depends on some characteristic of the patient population, e.g. socio-economic status
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This week's study

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No clear evidence Tamiflu works, study finds

CARLY WEEKS

From Thursday's Globe and Mail
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Neuraminidase inhibitors for preventing and treating influenza in healthy adults and children

Tom Jefferson[1,*], Mark A Jones[2], Peter Doshi[3], Chris B Del Mar[4], Carl J Heneghan[5], Rokuro Hama[6], Matthew J Thompson[5]

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