

- generalized linear models GLM  
GLMM  
(par)

- generalized additive models  
GAM  
(Semi par)

- generalized est'g eqn GEE (s-p)

- generalized linear mixed models  
GLMM



I. Principles of App Stat Cox & D.

II. Davison - Stat Models

Ch 9 Designed Exp

III. ← case studies / examples 10 GLM etc.  
+ other refs

~~GLM~~

~~GLM~~

~~GLM~~

~~GLM~~

Ch 9 - 9.3

next week

Now some more stuff for small

point one

point 2

↓

- if covariate measured  
after randomization,  
or randomization fails,  
you can adjust for it  
using analysis of  
Covariance

model: response

= treatment + covariate eff.

+ error

(§ 9.3)

Model

$$y_j = \mu + \delta + \varepsilon_j$$
$$\mu - \delta + \varepsilon_j$$

treat  
control

Std lin  
model

$$\text{Rand}^n: \exists y_j \quad j=1, \dots, n \quad \text{s.t.} \quad y_j = x_j + \delta$$

under treat,

$$y_j = x_j - \delta, \quad \text{under cont}$$
$$j=1, \dots, 2m=n$$

$$y_j = T_j(x_j + \delta), \quad T_j = 1$$

$$= (1 - T_j)(x_j - \delta), \quad T_j = 0$$

$$T_j = \begin{cases} 1 & \text{w.p. } \frac{1}{2} \\ 0 & \text{w.p. } \frac{1}{2} \end{cases}$$

$$t = (\bar{y}_1 - \bar{y}_0) / \sqrt{\tilde{\sigma}^2}$$

← t under

$$E_R T_j = \frac{1}{2} \quad E_R T_j T_k = \frac{m-1}{2(2m-1)} \quad j \neq k$$

$$E_R \left( \frac{2s^2}{m} \right) =$$

$$E_R (\bar{y}_1 - \bar{y}_0) = 2\delta$$
$$\text{var}_R (\bar{y}_1 - \bar{y}_0) = 2 \sum (x_j - \bar{x})^2 / m(2m-1)$$

← ←

Applied in D to 1-sample paired test

Can be used much more generally

Cox & Reid (D of  $\epsilon$ ), Kempthorne & H

But,  $\gamma_j \pm \delta$ , unit-tmt additivity,  
indep. of tmt alloc to other units

More gen'ly  $y_j = \gamma_j + \alpha_i$ , if tmt  $i$

assigned to unit  $j$

not testable by data

Strengthens case for causality

- physical: "if  $x \uparrow$ ,  $y \uparrow$  by factor of  $b_0$  Hooke's law

- observational - the effect of  $x$  on  $y$  can't  
be explained by dependence

on other <sup>allowable</sup> variables

- expt'l: "we used rand" & a unit-tmt add. ass"

& this ensures sample mean is unbiased est  
of the true mean diff.

"Counterfactual" Concluding caus.  
in obs'l studies

Bradford Hill's guidelines for causality

(i)  
§9.2 - one-way layout / CRD / 1-way anova

- k-group comparison

$$y_{tr} = \beta_t + \varepsilon_{tr}, \quad \varepsilon_{tr} \sim (0, \sigma^2), \quad t=1, \dots, T \\ r=1, \dots, R$$

$$\stackrel{or}{=} \mu + \alpha_t + \varepsilon_{tr} \quad (\text{over-param'd})$$

$$n = TR$$

(ii) - 2 way layout / anova, RB design

$$y_{tr} = \mu + \alpha_t + \beta_r + \varepsilon_{tr} \quad t=1, \dots, T; \quad r=1, \dots, R$$

$$n = TR$$

tmt	38.7	5
blk	15.2	4

x	x	x	x
:			

err	3.7	?
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incomplete block design - can't have each tmt

in each blk - no room; no time; or

- lose orthogonality - we don't know if

tmt diff due only to tmt, or something else

Ex 9.4 - 6 tmts; 2 units/block

1:	C	T <sub>1</sub>	} balance
2:	C	T <sub>2</sub>	
3:	C	T <sub>3</sub>	

T<sub>1</sub> T<sub>2</sub>

T<sub>2</sub> ; T<sub>3</sub>

analysis is less straight forward

- Latin square

- factorial designs (factorial tmt str)

Each tmt is composed of 2 or more factors

each at 2 or more levels

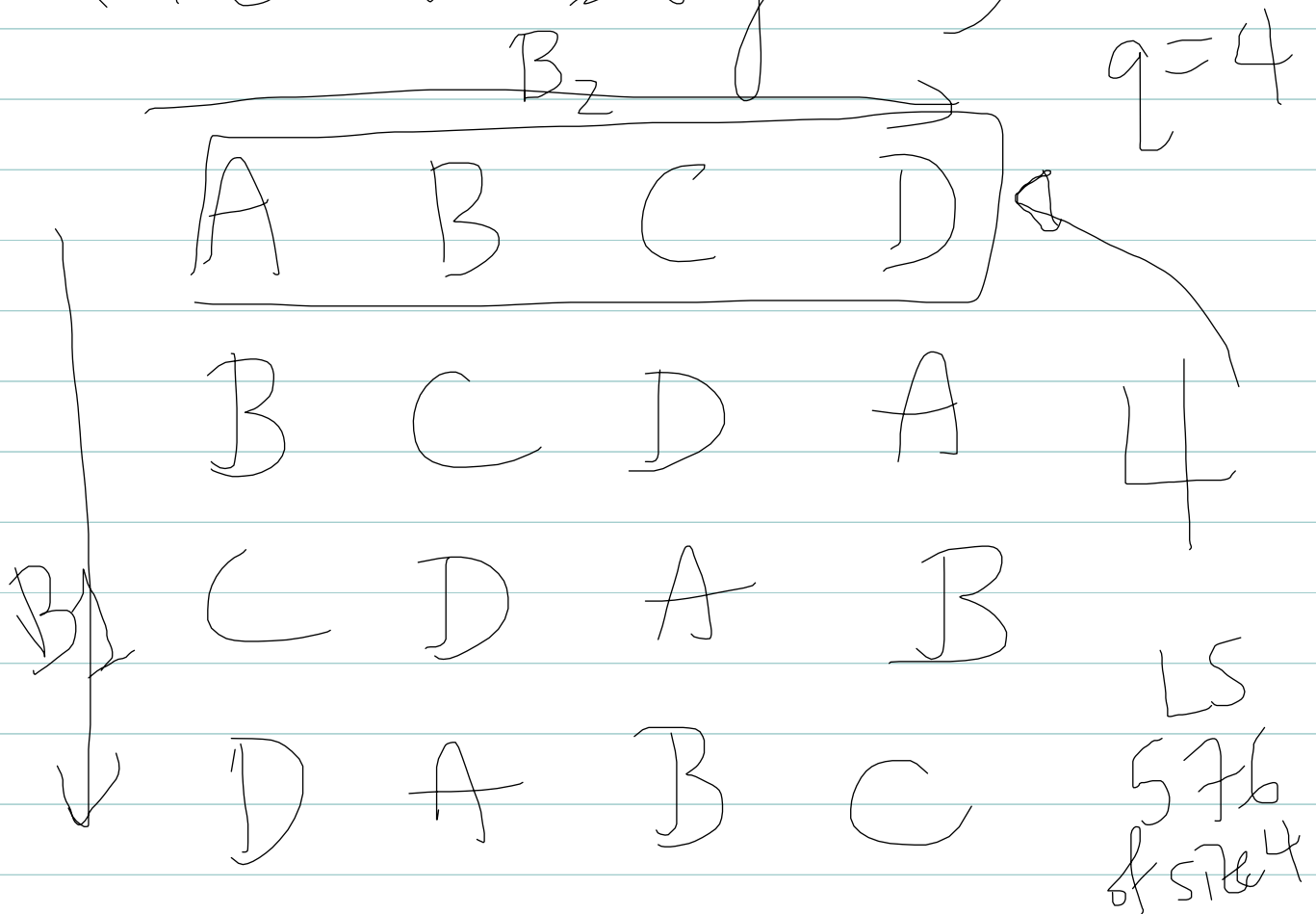
Ex 9.6  $3 \times 4$  2 tmts 3 levels A  
4 levels B

$i = 1, \dots, 3$

12 combinations (tmts)

	1	2	3	4
1	$x \times x$ $y_{11i}$	$x \times x$ $y_{12i}$	$x \times x$	
2				
3				

2 blocking factors  
 1 treatment factor  
 each have  $q$  levels  
 (IBlock Design)



B $\gamma$  D $\alpha$  C $\beta$  A $\delta$

C $\delta$  A $\beta$  B $\alpha$  D $\gamma$

A $\alpha$  C $\gamma$  D $\delta$  B $\beta$

D $\beta$  B $\delta$  A $\gamma$  C $\alpha$

Græco - L 59.

pair of  $\perp$  (orthog)

LS site 4

no GL sq size 6

? 10 ?

1730 ~~m~~ = 2 mod 4

~~X~~ GL squares

X

1960 10 x 10 pair  
of L.S.