STA 2004F Final Homework: Sketch of solutions

1. An experiment was conducted to study the effects of drying time in hours and temperature in degrees Centigrade on the content of undesirable compounds in a resin. The response is measured in parts per million.

   (a) Ignoring for this part the coded values, fit a linear regression model $y_i = \beta_0 + \beta_1 time_i + \beta_2 temp_i + \epsilon_i$ under the usual second moment assumptions, and report estimates of β_1, β_2 and their standard error. Construct an analysis of variance table showing the sum of squares due to each of time and temperature and residual.

parameter	estimate	(estimated standard error)
β_0	121.67	(14.57)
β_1	-1.815	(0.548)
β_2	-0.418	(0.075)

Analysis of variance

Source	df	SS	MS	F
time	1	424.78	424.78	10.95
temp	1	1259.60	1259.60	32.48
residuals	10	387.76	38.78	

   (b) Expand the model fit in part (a) to include the three quadratic terms in $time^2$, $temp^2$, and $time * temp$, and show the corresponding analysis of variance table. Next separate from the residual sum of squares the variation among the center points: i.e. $\sum_{i=5}^{9}(y_i - (1/5)\sum_{i=5}^{9} y_i)^2$.

Source	df	SS	MS	F
time	1	424.78	424.78	55.25
temp	1	1259.60	1259.60	163.82
time^2	1	29.08	29.08	3.8
temp^2	1	248.61	248.61	32.3
time×temp	1	56.25	56.25	7.3
residuals	7	53.82	7.69	
pure error	4	38.99	9.75	
lack of fit	3	14.83	4.71	

   The F-tests in the final column above were computed using the full residual SS on 7 degrees of freedom. They are slightly changed if we use 'pure error'. This is called 'pure error' because it is the variation among responses taken at exactly the same treatment combinations. The lack-of-fit SS includes the higher order interactions between time and temperature.

(c) Now construct a different analysis of variance table, using the coded values as factor variables. Fit a model with main effects and interactions. How does this differ from the models fit above?

Source	df	SS	MS	F
$x1$	4	477.93	119.48	12.26
$x2$	3	1498.98	499.66	51.26
$x1 \times x2$	1	56.25	56.25	5.77
residual	4	38.99	9.75	

In this model the coded variables are treated as factors with 5 levels each. The design is incomplete in that not all combinations of factors appear an equal number of times. The SS due to $x1$ can be subdivided into linear, quadratic, and remaining terms, as can the SS due to $x2$. The linear SS is (almost) equal to the time and temp SS found in part (a) and (b), and the interaction SS can be seen to be identical. Since all higher order interactions are now included in the treatment lines of the anova table, the residuals term is the pure error term found in part (b).

2. (DV 15.12): Suppose you are asked to design an experiment for 6 treatment factors each having two levels. Only 64 observations can be taken in total, and these should be divided into 8 blocks of size 8. Suppose that you decide to confound the interaction contrasts ABD, DEF and $ACDF$.

(a) Can all the other interaction contrasts be estimated?

No, the generalized interactions between these are also confounded with blocks, namely ABEF, BCF, ACE and BCDE.

(b) What does the statement "ABD is confounded" mean? It means that the contrast used to estimate the ABD interaction consists of treatments that appear in different blocks, so that the ABD interaction SS is one component of the block SS and it is impossible to distinguish the ABD effect from the block effect.

(c) How would you obtain the 8 blocks? Write out two blocks as an example. This can be done using linear combinations L_1, L_2 and L_3, or it can be done by partitioning the 64 treatments into two sets, determined by the ABD contrast, then partitioning one of these into two sets determined by the DEF contrast, and finally dividing this into two sets determined by the $ACDF$ contrast. The full set of 8 blocks is as follows, although you were only asked to find two of them:

Block	treatments							
1	(1)	abc	ade	bcde	bdf	acdf	abef	cef
2	d	abcd	ae	bce	bf	acf	abdef	cdef
3	a	bc	de	abcde	abdf	cdf	bef	acef
4	b	ac	abde	cde	df	abcdf	aef	bcef
5	c	ab	acde	bde	bcdf	adf	abcef	ef
6	e	abce	ad	bcd	bdef	acdef	abf	cf
7	f	abcf	adef	bcdef	bd	acd	abe	cd
8	af	bcf	def	abcdef	abd	cd	be	ace

(d) Suppose the budget is cut before the experiment can take place, and only 8 observations can be taken in total. How would you decide which 8 observations to take? What can be estimated?

Any one of the 8 blocks listed above could be used. By taking the alias set $I = ABD = DEF = ACDF = ABEF = BCF = ACE = BCDE$ and multiplying through by A, B, ... F, you can verify that no main effects are aliased with each other. So we can estimate 6 main effects and 1 two-factor interaction (which is however aliased with several other two factor interactions).

3. In the *Applied Statistics* article handed out in the last class, "Modifying a central composite design..." by Kowalski et al, the motivating example is discussed in Sections 2 and 5. Referring to the experimental design laid out in Table 1 on p.624, consider just the first four whole plots.

(a) What is the treatment structure applied to the whole plots?

This is a 2^2 factorial in the two factors chamber temperature and chamber pressure.

(b) What is the treatment structure applied to the sub plots?

This is a 2^2 factorial in the two factors processing time and argon/nitrogen ratio.

(c) How do the sixteen runs given in this part of the table differ from a completely randomized 2^4 factorial experiment?

The 16 treatments were not assigned at random to each of 16 experimental units.

(d) Why was this particular design chosen for this experiment? This design was chosen because it was difficult and took considerable time to change the whole plot factors, and because there were four experimental units (wafers) available for each run at a fixed setting of the whole plot factors. Thus it was possible to consider a 2^2 factorial run within each setting of the whole plot factors.

4. A client has come to the consulting service to discuss designing an experiment on peoples' ability to respond to visual stimuli in the presence of various distractions. She wants to know what the difference is between a randomized block design, a Latin square design, and a balanced incomplete block design. She also wants to know which design is best for her experiment. Describe in one or two paragraphs how you would go about helping her design this experiment.

The three designs are all block designs, meaning that the experimental units are grouped into homogeneous sets, called blocks, and treatments are assigned at random to units within a block. A RB design is the simplest: each block has one unit assigned to at random to each of the v treatments. A LS design has two blocking factors, conventionally called rows and columns, and each treatment appears once in each row and column. A BIBD has fewer experimental units per block than the number of treatments, but subject to this restriction the treatments are assigned to blocks at random, and balanced in the sense that each pair of treatments appears the same number of times (usually once) in each block.

We need to know what the blocks, treatments, and experimental units are, and how many levels each has. We also need to have an idea what resources, either time or money or both, are available. Assuming that subjects will serve as blocks, an experimental unit would be a subject/time combination. If it is feasible to have each subject exposed to each treatment in one experimental session, then a RB design might be possible. If the number of treatments is equal to the number of blocks, and if another blocking factor, such as time of day, or laboratory, or whatever, seems important, then a LS design might be useful, although it may need to be replicated. If there are two many treatments for a subject to be exposed to in one session, a BIBD might be considered. It will be important to know if there may be carryover effects of treatments, and if so how to either eliminate them from the design, by using washout periods, or eliminate them from the analysis in some way.

[This should give a rough idea what I was looking for, but there are many variations and several students submitted better solutions than this.]

5. Bonus question: Analyse the data given in Table 1 of Kowalski et al referred to in Question 3, and provide an executive summary of your main conclusions. (Ignore the discussion in the paper of the "10-step algorithm".)