STA 2004F Homework 3.

1. In the following highly idealized sets of data from two-level experiments, each value given is the mean of observations for the relevant factor combinations. The means can be treated as independent, with standard error 1. Without much detailed calculation summarize the conclusions from each set:

   (a) A 2^2 system with responses
   (1): 5.6, a: 7.8, b: 8.7, ab: 11.7

   (b) Another 2^2 system
   (1): 1.7, a: 2.2, b: 2.4, ab: 3.2

   (c) A 2^3 system
   (1):4.7, a: 2.2, b: 2.4, ab: 4.1, c: 5.0, ac: 4.8, bc: 8.2, abc: 3.8

   (d) Another 2^3 system
   (1): 5.6, a: 7.8, b: 8.7, ab: 15.0, c: 8.0, ac: 10.4, bc: 11.1, abc: 16.9

2. An experiment is designed to study pigment dispersion in paint. Four different mixes of a particular pigment are studied. The procedure consists of preparing a particular mix and then applying that mix to a panel by three application methods (brushing, spraying, and rolling). The response measured is the percentage reflectance of pigment. Three days are required to run the experiment, and the data obtained follow. Present an executive summary of one paragraph describing the effects of mix and method on the percentage reflectance. Provide the details of modelling and analysis in a technical appendix.

Day	Application Method	Mix 1	Mix 2	Mix 3	Mix 4
1	1	64.5	66.3	74.1	66.5
	2	68.3	69.5	73.8	70.0
	3	70.3	73.1	78.0	72.3
2	1	65.2	65.0	73.8	64.8
	2	69.2	70.3	74.5	68.3
	3	71.2	72.8	79.1	71.5
3	1	66.2	66.5	72.3	67.7
	2	69.0	69.0	75.4	68.6
	3	70.8	74.2	80.1	72.4

3. (CR 5.2) Construct a 1/4 fraction of a 2^5 factorial using the generators ABCD and CDE. Write out the sets of aliased effects.

4. (HW 4.27) In a resistance spot welding experiment, five factors were chosen to study their effects on the tensile strength, which is the maximum load a weld can sustain in a tensile test. The five factors are button diameter (A), welding time (B), holding time (C), electrode force (D), and machine type (E), each at two levels. The last factor is qualitative, while the others are quantitative. A 1/2 fraction of a 2^5 design with $I = ABCDE$ was used for the experiment. Each run has three replicates. The data are given below.

   (a) Analyse the mean response over replicates, using a half-normal plot to determine which contrasts to use for an estimate of error.

   (b) Analyse the raw data, using the pooled variance within replicates to estimate error. Do the conclusions change from part (a)?

   (c) In this type of experiment it is usual to analyse $\log s_i^2$, where s_i^2 is the sample variance within replicates $\sum_j (y_{ij} - \bar{y}_{i.})^2/(3-1)$ as a response, in addition to the analysis of the responses y_{ij} or $\bar{y}_{i.}$. This is a means of assessing which factors affect the variability of the response. Carry out this analysis.

   (d) Write a short summary describing which factor settings are recommended for maximizing the tensile strength and minimizing the variability of tensile strength.

Run	A	B	C	D	E	Tensile strength		
1	-	-	-	-	-	1330	1330	1165
2	+	+	-	-	-	1935	1935	1880
3	+	-	+	-	-	1770	1770	1770
4	-	+	+	-	-	1275	1275	1275
5	+	-	-	+	-	1880	1935	1880
6	-	+	-	+	-	1385	1440	1495
7	-	-	+	+	-	1220	1165	1440
8	+	+	+	+	-	2155	2100	2100
9	+	-	-	-	+	1715	1715	1660
10	-	+	-	-	+	1385	1550	1550
11	-	-	+	-	+	1000	1165	1495
12	+	+	+	-	+	1990	1990	1990
13	-	-	-	+	+	1275	1660	1550
14	+	+	-	+	+	1660	1605	1660
15	+	-	+	+	+	1880	1935	1935
16	-	+	+	+	+	1275	1220	1275