STA 2004F Homework 3 Solutions (Sketch).

1. In part (a) the estimated main effects of A and B are 2.6 and 3.5, respectively. These are significantly different from zero, as the standard error of an effect estimate is $2\sigma^2/2 = 1$ and $\sigma = 1$. The estimated AB interaction is not significantly different from zero. In part (b) the estimates are A: 0.65, B: 0.85, and AB: 0.15. These are all within 1 standard error of zero.

   In part (c) the estimates are

I	A	B	AB	C	AC	BC	ABC
4.40	-1.35	0.45	0.00	2.10	-0.95	0.65	-2.10

   and the standard error of an estimated effect is $1/\sqrt{2}$. Thus any estimates more than $2\sqrt{2} = 1.41$ from zero are significantly different from zero at level 0.05. Only the C and ABC effects are significant at this level, with signs in the opposite direction. The interpretation of the large ABC interaction is difficult.

   In part (d) the estimates are

I	A	B	AB	C	AC	BC	ABC
10.44	4.18	4.98	1.88	2.33	-0.08	-0.18	-0.18

   In this case the main effects are all significantly different from zero, and there is just one significant interaction, AB.

2. Executive Summary: Both the mix and the method of application have a significant effect on the degree of pigment dispersion in paint. Mix three gave the highest percentage reflectance, as did application method three. Mixes 1, 2 and 4 were not significantly different from each other; the three different methods were significantly different from each other, all comparisons at level 0.05. The recommended combination for the maximum reflectance is mix 3 and method 3.

   Design Summary: This experiment was a split plot experiment; the whole plot treatment is mix and the subplot treatment is method. The whole plot experiment is a randomized block experiment with days as blocks; the interaction between mixes and blocks gives the estimated mean square error for assessing the differences between the mixes. The analysis of variance table is:

Source	SS	df	MS
day	2.04	2	1.02
mix	307.5	3	102.5
day x mix (error 1)	4.5	6	0.8
method	222.1	2	111.0
mix x method	10.0	6	1.7
error 2	10.7	16	0.7

   The F-test for comparing mixes is based on the ratio 102.5/0.8; the F-test for comparing methods, and for assessing the interaction between mix and method is based on comparing the respective mean squares to the mean squared error in the second part of the anova table. The table of treatment means is

	mix				
method	1	2	3	4	Mean
1	65.3	65.9	73.4	66.3	67.7
2	68.8	69.6	74.6	69.0	70.5
3	70.8	73.4	79.1	72.1	73.8
Mean	68.3	69.6	75.7	69.1	70.7

The estimated standard error for comparing method means (given in the final column) is $0.33 = \surd(2 \times 0.672/12)$, and for comparing mix means (given in the final row) is$0.41 = \surd(2 \times 0.755/9)$. As expected the precision for comparing mixes is less than that for comparing methods.

3. The contrast subgroup is $\{I, ABCD, CDE, ABE\}$, and the corresponding treatment subgroups are $\{(1), ab, ace, ade, bce, bde, cd, abcd\}$, $\{a, b, ce, de, abce, abde, acd, bcd\}$, $\{c, abc, ae, acde, be$ $\{ac, bc, e, cde, abe, abcde, ad, bd\}$. The alias sets are
$I = ABCD = CDE = ABE$
$A = BCD = ACDE = BE$
$B = ACD = BCDE = AE$
$C = ABD = DE = ABCE$
$D = ABC = CE = ABDE$
$E = ABCDE = CD = AB$
$AC = BD = ADE = BCE$
$AD = BC = ACE = BDE$

The fractional factorial is determined by any one of the four treatment subgroups.

4. (a) In the first part of this question just the mean response of each run is used as a single observation. After estimating the 15 main effects and 2-factor interactions there are no degrees of freedom left for estimating error. The usual approach is then to pool the smallest interaction terms to use as an estimate of error. For this experiment the AD interaction turns out to be exactly 0, which is probably just chance. It's not that clear which interactions to pool for estimating error, but certainly AD, CE, AB and CD are candidates. If just these four are used to estimate error the MSE is 1618 on four degrees of freedom. The estimated A and B main effects are significantly different from zero, as are several two-factor interactions which are harder to interpret. Note that 1618 estimates the variance of one 'observation', i.e. one ybar.

   (b) In this analysis we use all the observations, giving an analysis of variance table with 32 degrees of freedom for error. These come from pooling the s^2 in each run. The MSE from this analysis is 9516 on 32 degrees of freedom. THis estimates the variance of one single response, so we'd expect it to be 3 times larger than the estimate in (a), but it is about twice as large as that: the estimate in (a) may well be too small. In this analysis A and B main effects are again significantly different from zero, and again there are several significant two-factor interactions. The estimated effects are identical to those in (a); note that the SS in the anova table are all multiplied by 3 compared to the anova table of (a).

(c) The analysis here is based on $\log(s_i^2)$, where i indexes the 16 runs. Since $s_i^2 = 0$ for runs three, four and twelve we have to make some adjustment before we can do this analysis. A common solution is to use $\log(s_i^2 + 0.5)$, another is to replace $\log(s_i^2)$ for $i = 3, 4, 12$ with an arbitrary number; some people used 0, others -100. Using $\log(s_i^2+0.5)$ the largest effects on the sample variability are the main effects of A and C, whose high levels are associated with a decrease in variability, and D, whose high level is associated with an increase in variability.

(d) (Thanks to Tim and Xiao-Hong): Setting factor A (button diameter) to its high level has a significant effect on strength, and the strength further increases with A at its high level as C (holding time) is increased from low to high. Setting B (holding time) to the high level also increases the tensile strength. Results from the analysis of $\log s^2$ shows that the variability decreases at high levels of A and C, and increases when D (electrode force) is at its high level. Thus the recommendation is for A, B and C to be set at their high level, and D, and E to their low level.

It is possible that machine type is a 'noise' factor not usually under the control of the operator, in which case we might want to consider which factor combinations give consistent results regardless of level of factor E; I did not carry out the details for this though.

Some notes on R

Here is some code that I used for Question 4. There are lots of equivalent ways to get the same results.

```
 > strength<-scan()
1: 1330 1330 1165 1935 1935 1880 1770 1770 1770 1275 1275 1275 1880 1935
 1880 1385 1440 1495 1220 1165 1440 2155 2100 2100 1715 1715 1660 1385
  1550 1550 1000 1165 1495 1990 1990 1990 1275 1660 1550 1660 1605 1660
   1880 1935 1935 1275 1220 1275

> run<-factor(c(rep(1,3),rep(2,3),rep(3,3),rep(4,3),rep(5,3),rep(6,3),rep(7,3),
rep(8,3),rep(9,3),rep(10,3),rep(11,3),rep(12,3),rep(13,3),rep(14,3),rep(15,3),rep(16,3))

> ybar<-tapply(strength,run,mean)
> s2<-tapply(strength,run,var)
> E<-factor(c(rep(-1,8),rep(1,8)))
> D<-factor(rep(c(rep(-1,4),rep(1,4)),2))
> C<-rep(c(-1,-1,1,1),4)
> C<-factor(C)
> B<-factor(rep(c(-1,1),8))
> A<-scan()
1: -1 1 1 -1 1 -1 -1 1 1 -1 -1 1 -1 1 1 -1
17:
Read 16 items
```

```
> A
 [1] -1  1  1 -1  1 -1 -1  1  1 -1 -1  1 -1  1  1 -1
> A<-factor(A)

> options(contrasts=c("contr.sum","contr.poly"))
> parta <- aov(ybar~A*B*C*D*E)
> coef(parta)
  (Intercept)            A1            B1            C1            D1
 1.605000e+03 -2.635417e+02 -3.666667e+01  2.291667e+00 -2.520833e+01
           E1         A1:B1         A1:C1         B1:C1         A1:D1
 1.604167e+01  1.145833e+01  8.250000e+01  2.062500e+01  4.402352e-14
        B1:D1         C1:D1         A1:E1         B1:E1         C1:E1
-5.270833e+01  1.375000e+01 -4.125000e+01 -2.979167e+01  9.166667e+00
        D1:E1
-3.666667e+01

# these don't give exactly what we want; the effect estimates are -2 times these

> print(coef(parta)[-1]*-2)
        A1         B1         C1         D1         E1      A1:B1
 5.271e+02  7.333e+01 -4.583e+00  5.042e+01 -3.208e+01 -2.292e+01
     A1:C1      B1:C1      A1:D1      B1:D1      C1:D1      A1:E1
-1.650e+02 -4.125e+01 -8.805e-14  1.054e+02 -2.750e+01  8.250e+01
     B1:E1      C1:E1      D1:E1
 5.958e+01 -1.833e+01  7.333e+01

# even here the notation is terrible, but these are the effect estimates

> sort(abs(.Last.value))
    A1:D1        C1     C1:E1     A1:B1     C1:D1        E1     B1:C1
8.805e-14 4.583e+00 1.833e+01 2.292e+01 2.750e+01 3.208e+01 4.125e+01
       D1     B1:E1        B1     D1:E1     A1:E1     B1:D1     A1:C1
5.042e+01 5.958e+01 7.333e+01 7.333e+01 8.250e+01 1.054e+02 1.650e+02
       A1
5.271e+02

# this is the basis for the suggestion to use AD, CE, AB and CD as error

> anova(aov(ybar~A+B+C+D+E+A:C+A:E+B:C+B:D+B:E+D:E))
Analysis of Variance Table

Response: ybar
          Df  Sum Sq Mean Sq F value  Pr(>F)
A          1 1111267 1111267  687.01 1.3e-05 ***
B          1   21511   21511   13.30  0.0218 *
C          1      84      84    0.05  0.8309
D          1   10167   10167    6.29  0.0663 .
E          1    4117    4117    2.55  0.1858
A:C        1  108900  108900   67.32  0.0012 **
A:E        1   27225   27225   16.83  0.0148 *
B:C        1    6806    6806    4.21  0.1095
B:D        1   44451   44451   27.48  0.0063 **
B:E        1   14201   14201    8.78  0.0414 *
D:E        1   21511   21511   13.30  0.0218 *
```

```
Residuals  4    6470    1618

# for part (b) I have to string out the responses:

> junk <- matrix(strength,ncol=3)

> long <-c(junk[,1],junk[,2],junk[,3] )

> rm(junk)

AA<-factor(rep(A,3))  # and so on for the others

> anova(aov(long~AA*BB*CC*DD*EE))
Analysis of Variance Table

Response: long
          Df  Sum Sq Mean Sq F value  Pr(>F)
AA         1 3333802 3333802  350.33 < 2e-16 ***
BB         1   64533   64533    6.78 0.01385 *
CC         1     252     252    0.03 0.87173
DD         1   30502   30502    3.21 0.08287 .
EE         1   12352   12352    1.30 0.26303
AA:BB      1    6302    6302    0.66 0.42178
AA:CC      1  326700  326700   34.33 1.6e-06 ***
BB:CC      1   20419   20419    2.15 0.15273
AA:DD      1 8.7e-27 8.7e-27 9.1e-31 1.00000
BB:DD      1  133352  133352   14.01 0.00072 ***
CC:DD      1    9075    9075    0.95 0.33612
AA:EE      1   81675   81675    8.58 0.00621 **
BB:EE      1   42602   42602    4.48 0.04223 *
CC:EE      1    4033    4033    0.42 0.51967
DD:EE      1   64533   64533    6.78 0.01385 *
Residuals 32  304517    9516

# analysis of log(s2+.5) proceeds as in part (a) and is omitted here
```