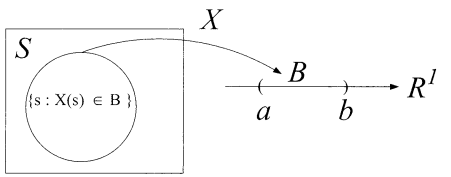

Figure 2.2.1: If $B = (a, b) \subset R^1$, then $\{s \in S : X(s) \in B\}$ is the set of elements such that $a < X(s) < b$.

EXAMPLE 2.2.2 *A Very Simple Distribution*
Consider once again the above random variable, where $S = \{\text{rain, snow, clear}\}$ and where X is defined by $X(\text{rain}) = 3$, $X(\text{snow}) = 6$, and $X(\text{clear}) = -2.7$, and $P(\text{rain}) = 0.4$, $P(\text{snow}) = 0.15$, and $P(\text{clear}) = 0.45$. What is the distribution of X? Well, if B is any subset of the real numbers, then $P(X \in B)$ should count 0.4 if $3 \in B$, plus 0.15 if $6 \in B$, plus 0.45 if $-2.7 \in B$. We can formally write all this information at once by saying that

$$P(X \in B) = 0.4\, I_B(3) + 0.15\, I_B(6) + 0.45\, I_B(-2.7),$$

where again $I_B(x) = 1$ if $x \in B$, and $I_B(x) = 0$ if $x \notin B$. ∎

EXAMPLE 2.2.3 *An Almost-As-Simple Distribution*
Consider once again the above setting, with $S = \{\text{rain, snow, clear}\}$, and $P(\text{rain}) = 0.4$, $P(\text{snow}) = 0.15$, and $P(\text{clear}) = 0.45$. Consider a random variable Y defined by $Y(\text{rain}) = 5$, $Y(\text{snow}) = 7$, and $Y(\text{clear}) = 5$.

What is the distribution of Y? Clearly, $Y = 7$ only when it snows, so that $P(Y = 7) = P(\text{snow}) = 0.15$. However, here $Y = 5$ if it rains *or* if it is clear. Hence, $P(Y = 5) = P(\{\text{rain, clear}\}) = 0.4 + 0.45 = 0.85$. Therefore, if B is any subset of the real numbers, then

$$P(Y \in B) = 0.15\, I_B(7) + 0.85\, I_B(5). \blacksquare$$

While the above examples show that it is possible to keep track of $P(X \in B)$ for all subsets B of the real numbers, they also indicate that it is rather cumbersome to do so. Fortunately, there are simpler functions available to help us keep track of probability distributions, including cumulative distribution functions, probability functions, and density functions. We discuss these next.

Summary of Section 2.2

- The distribution of a random variable X is the collection of probabilities $P(X \in B)$ of X belonging to various sets.
- The probability $P(X \in B)$ is determined by calculating the probability of the set of response values s such that $X(s) \in B$, i.e., $P(X \in B) = P(\{s \in S : X(s) \in B\})$.