p-values

How saddlepoint replaced...

DAS Fraser

Statistical Sciences

U Toronto

2016-10-2

WHOA-PSI

Washington Univ St L

www.utstat.toronto.edu/dfraser/documents/WUSL2016.pdf

1. p-values
2. Likelihood
3. Likelihood to densities
4. density to p-values
5. For interest parameter $\psi(\theta)$
6. Bootstrap
7. Bayes

1) <u>p-values</u>: What are they?   Fisher? ASA?

Start simple: Scalar case $f(y;\theta)$ y°

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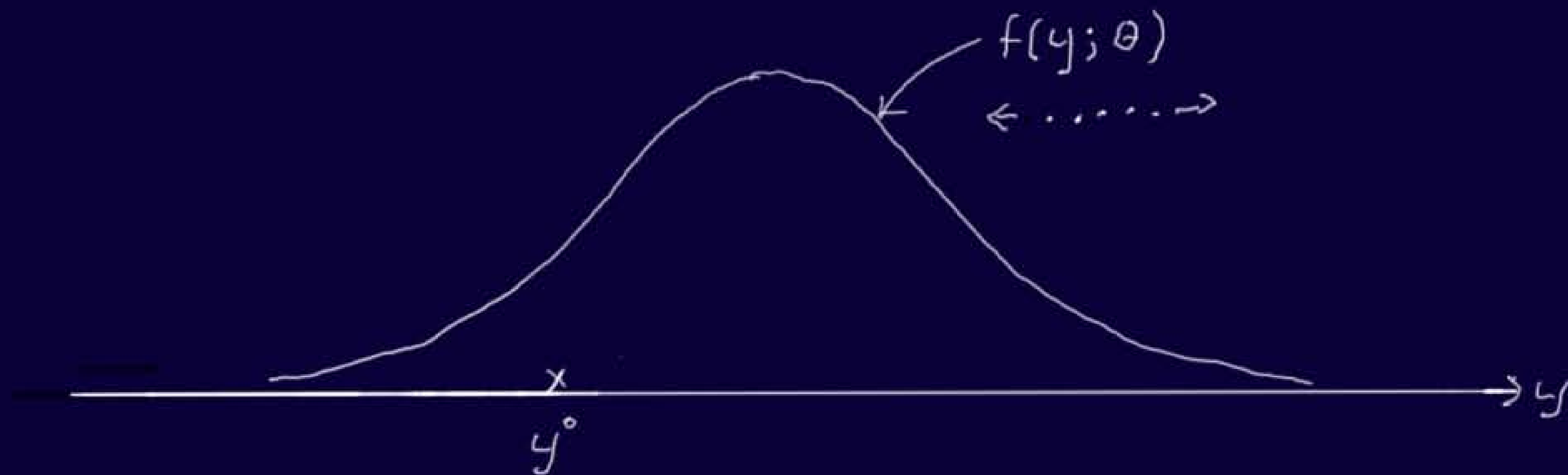

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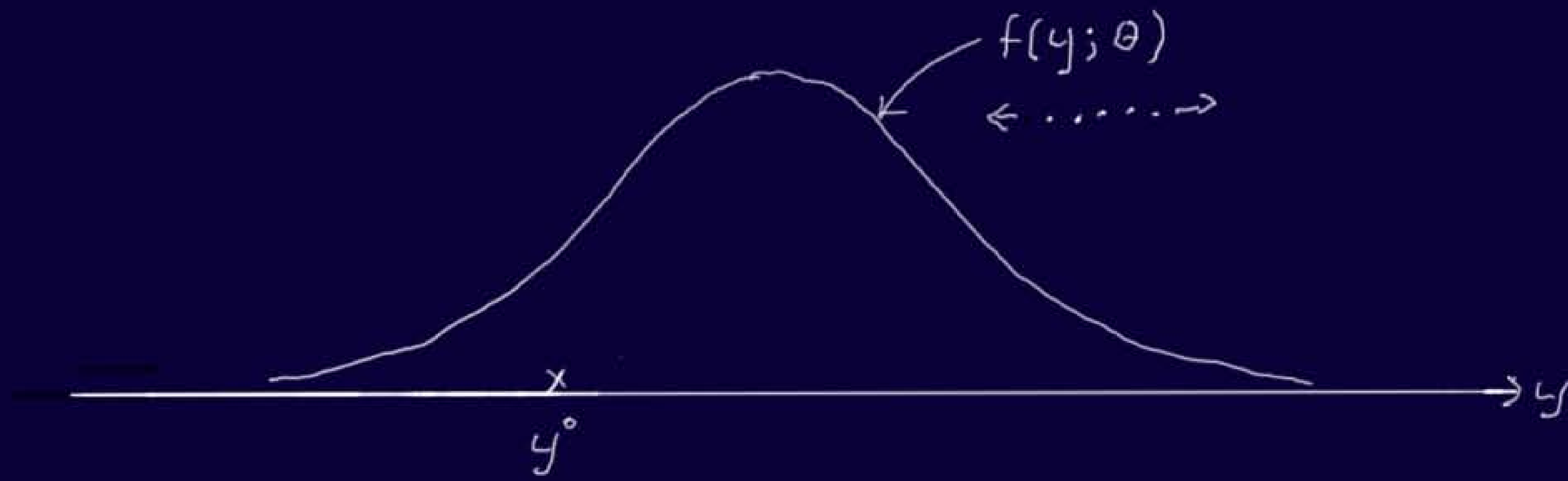

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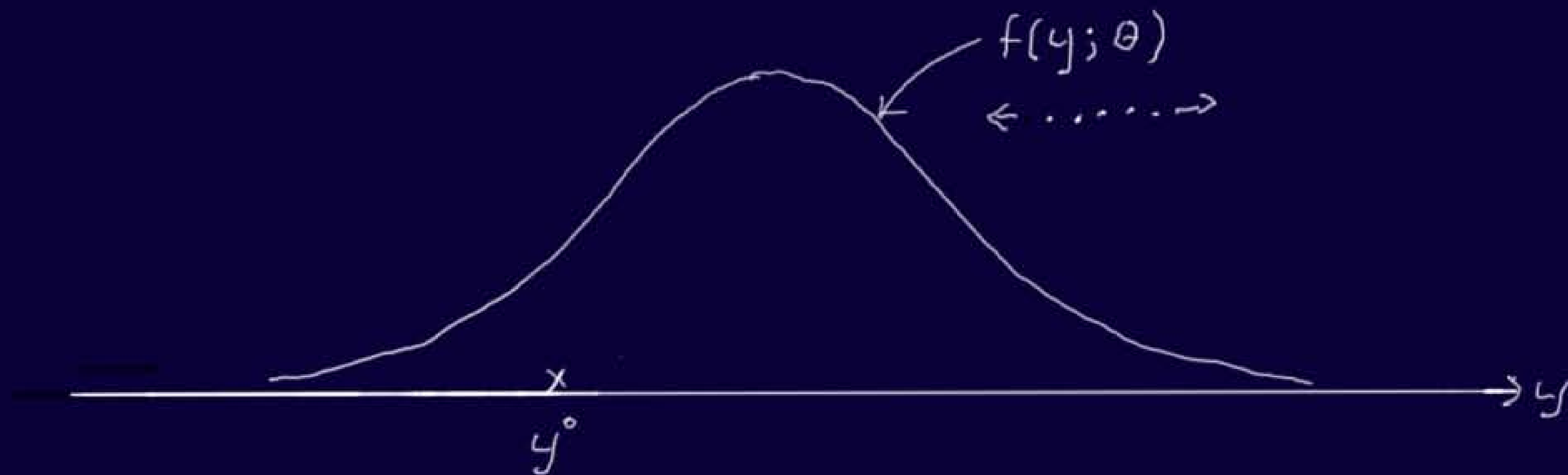

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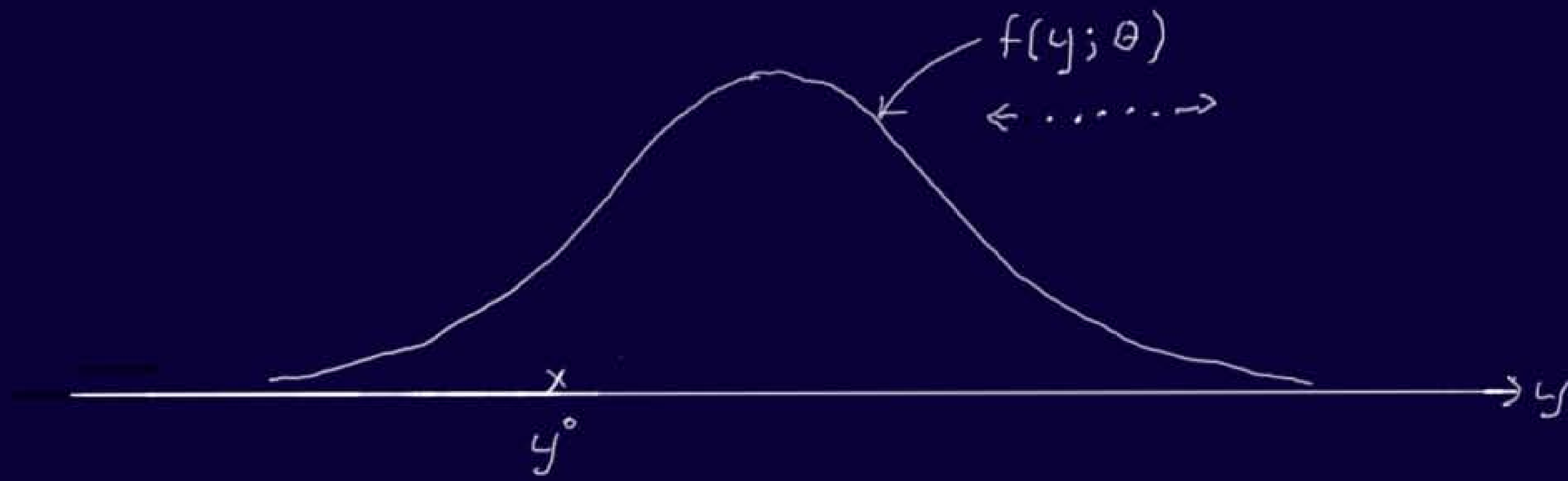

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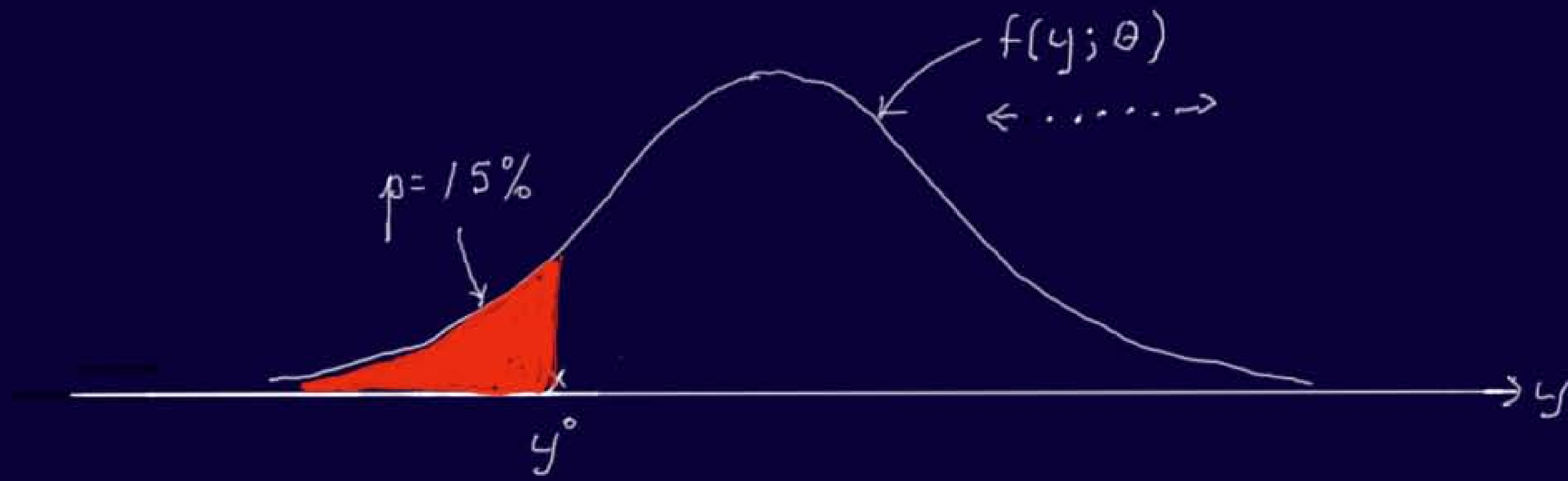

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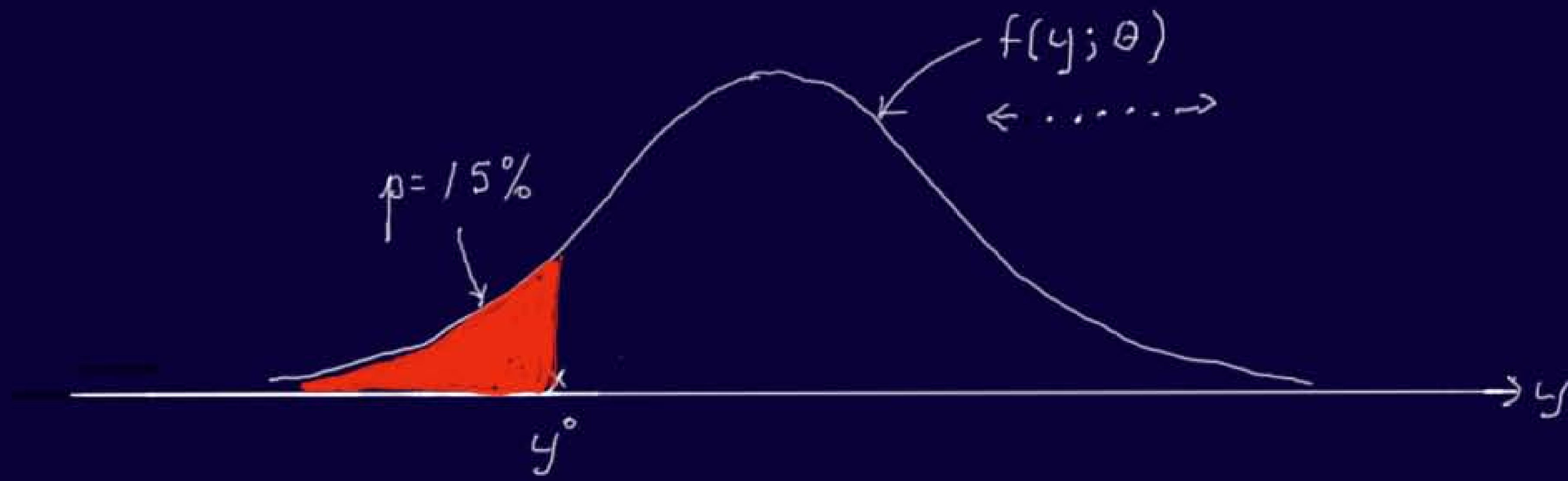

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- All confidence bounds, intervals (Invert; quantile fn)
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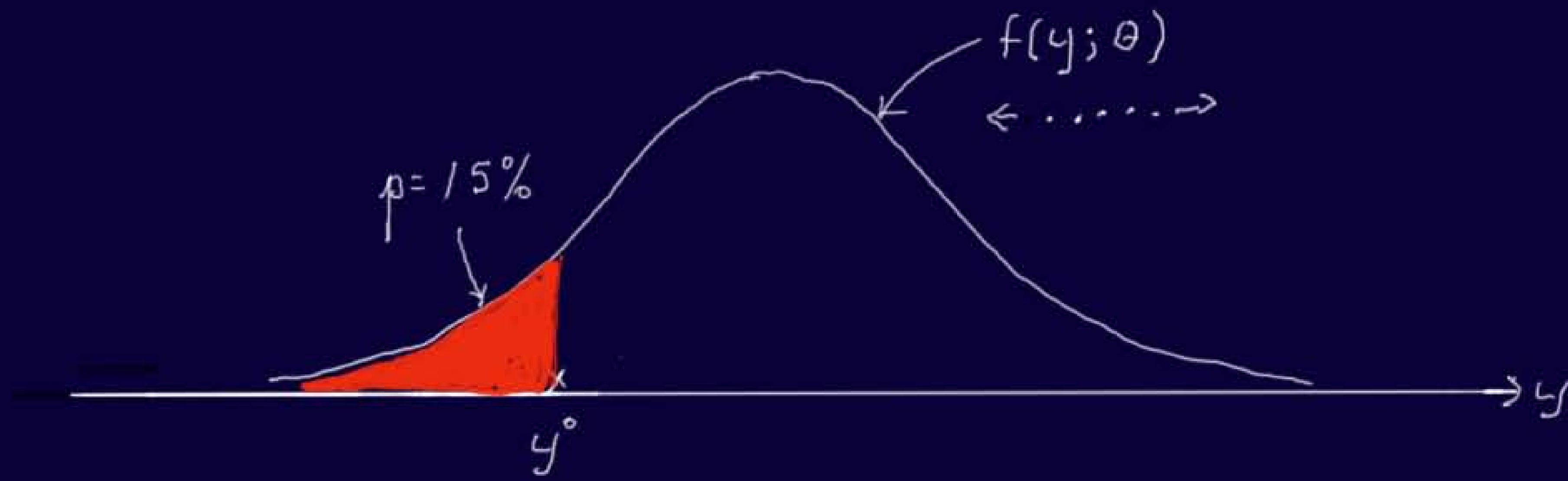

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Scalar parameter:
Widely available
uniqueness
use Saddlepoint

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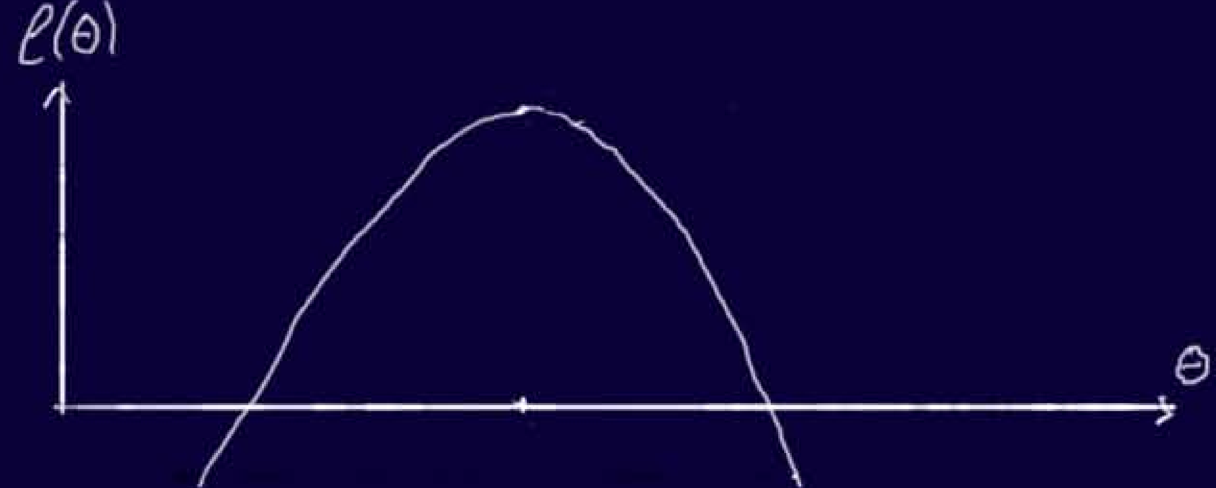

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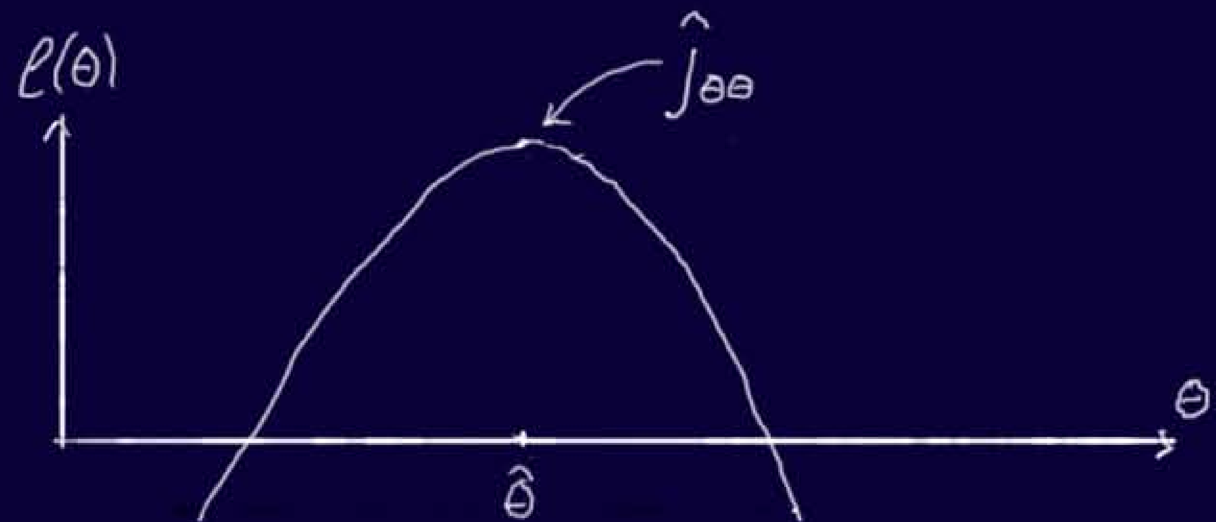

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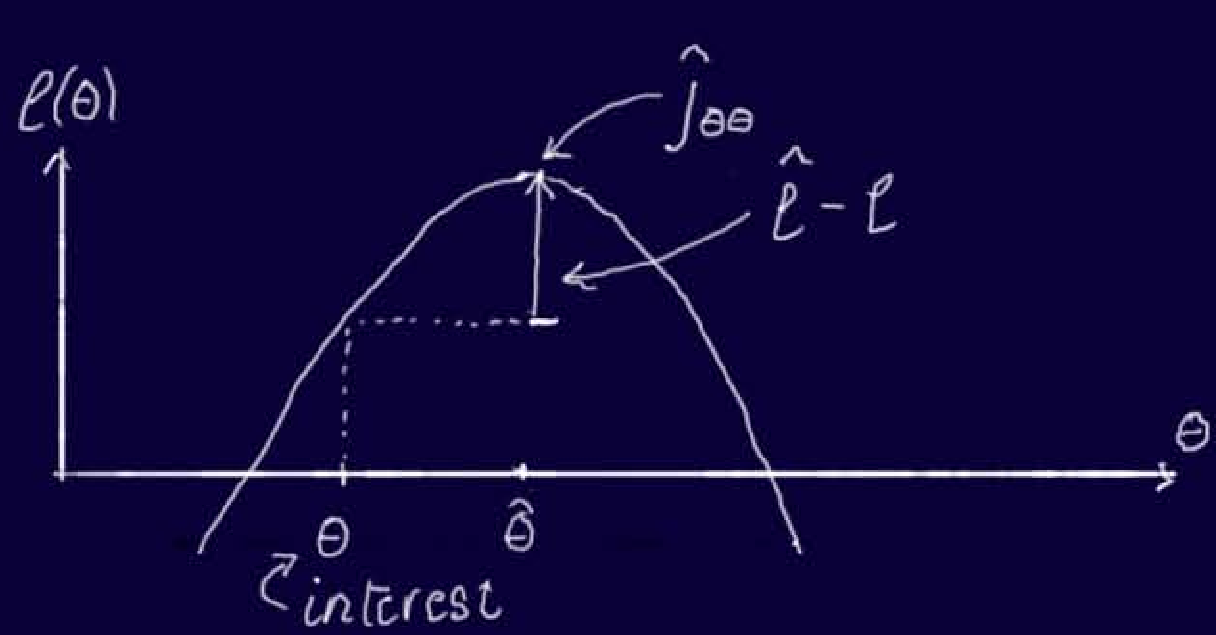

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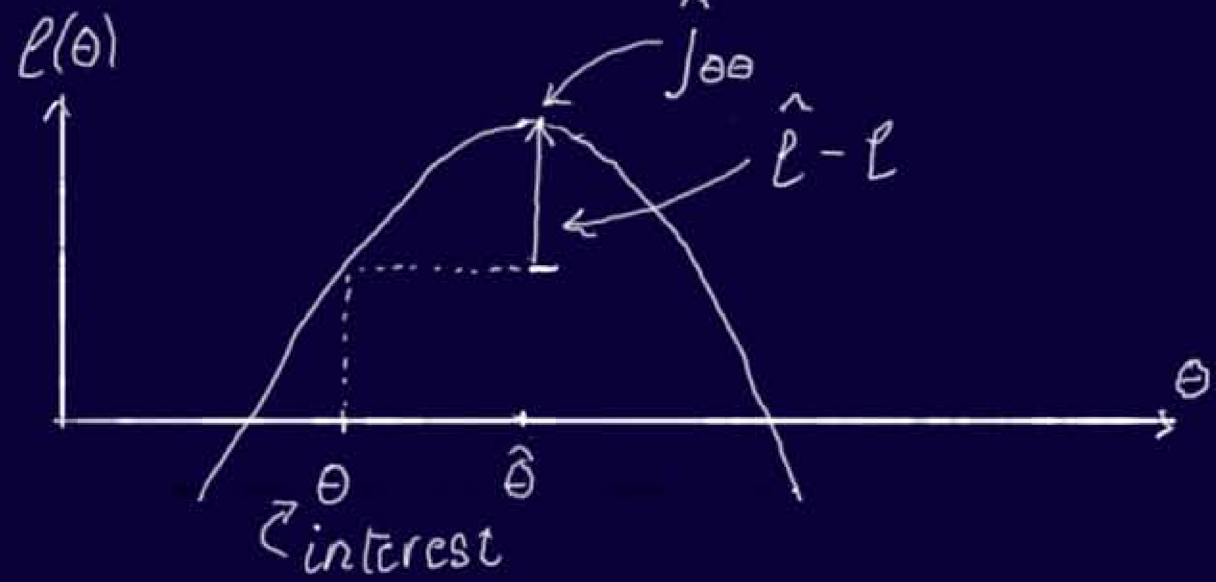

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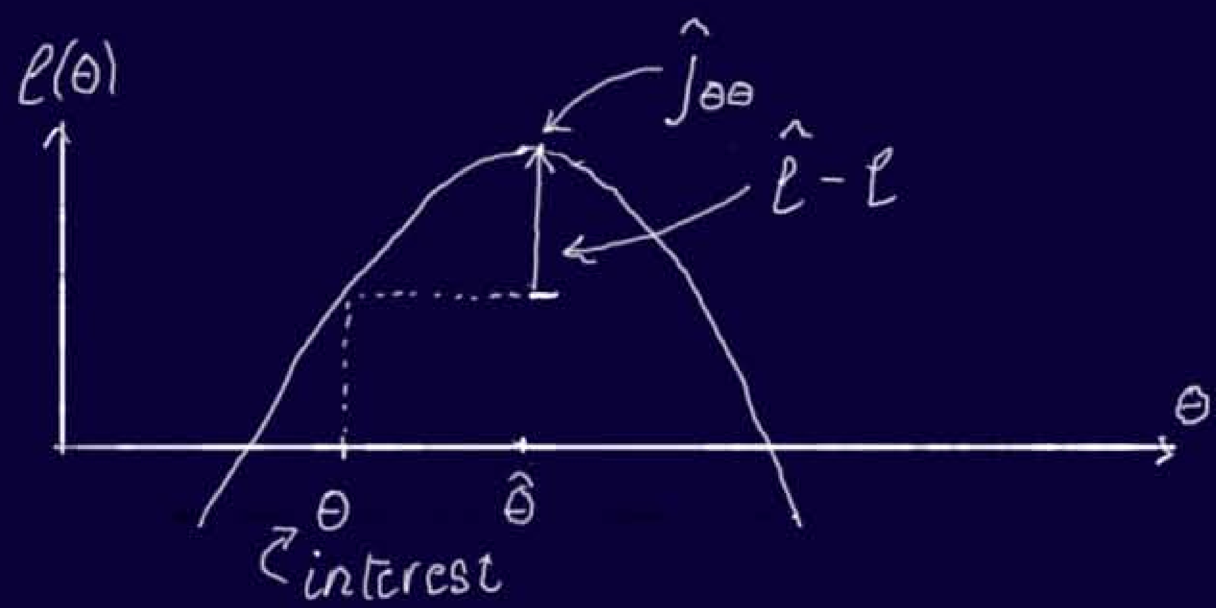

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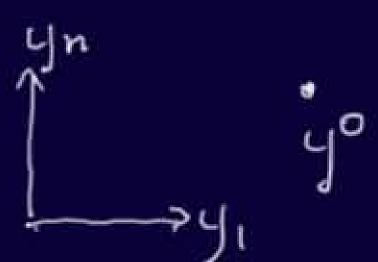

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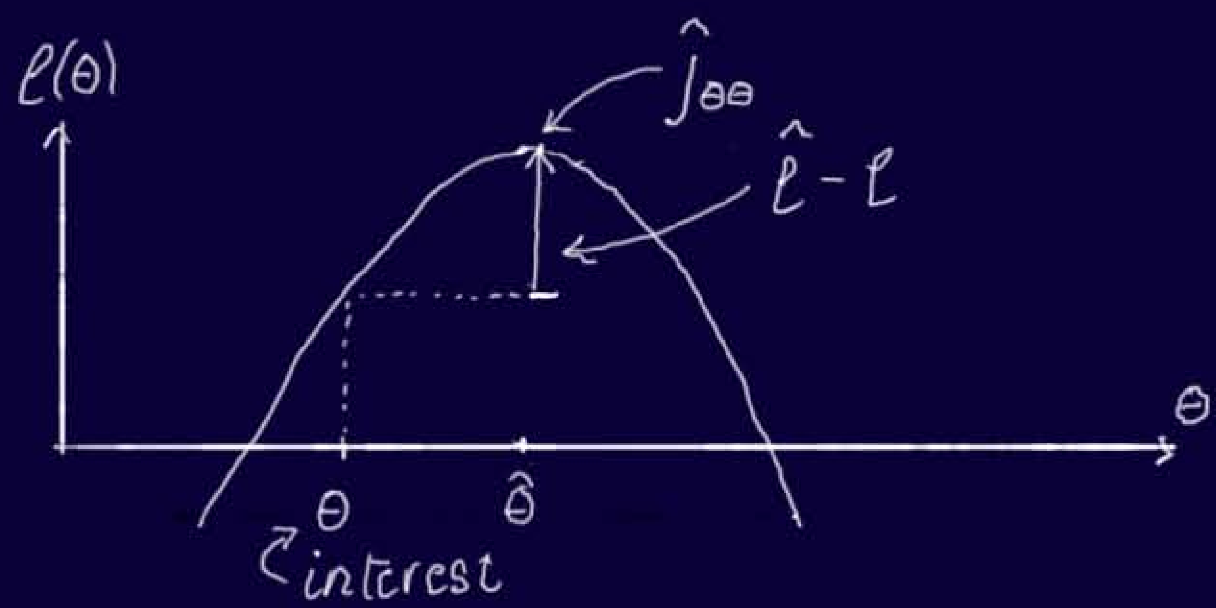

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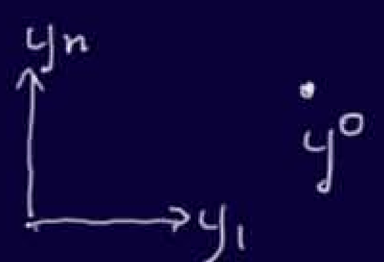

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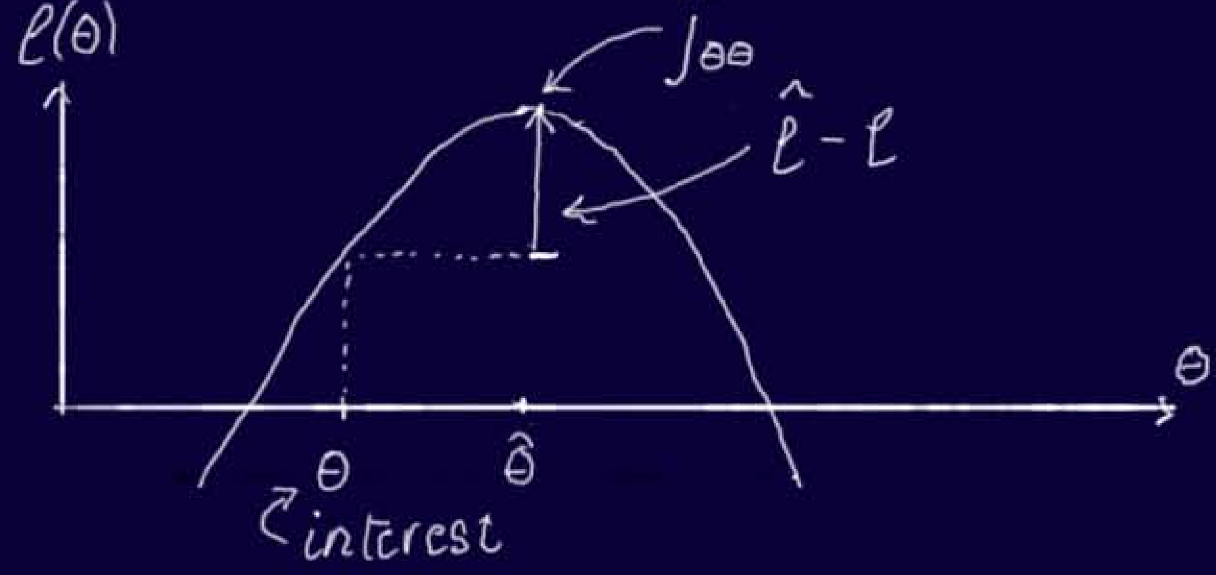

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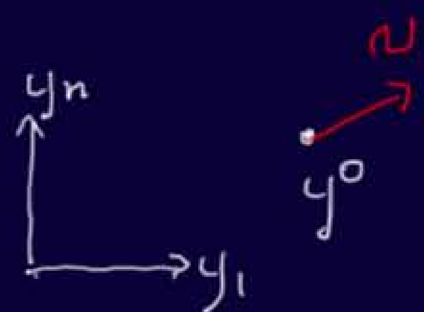

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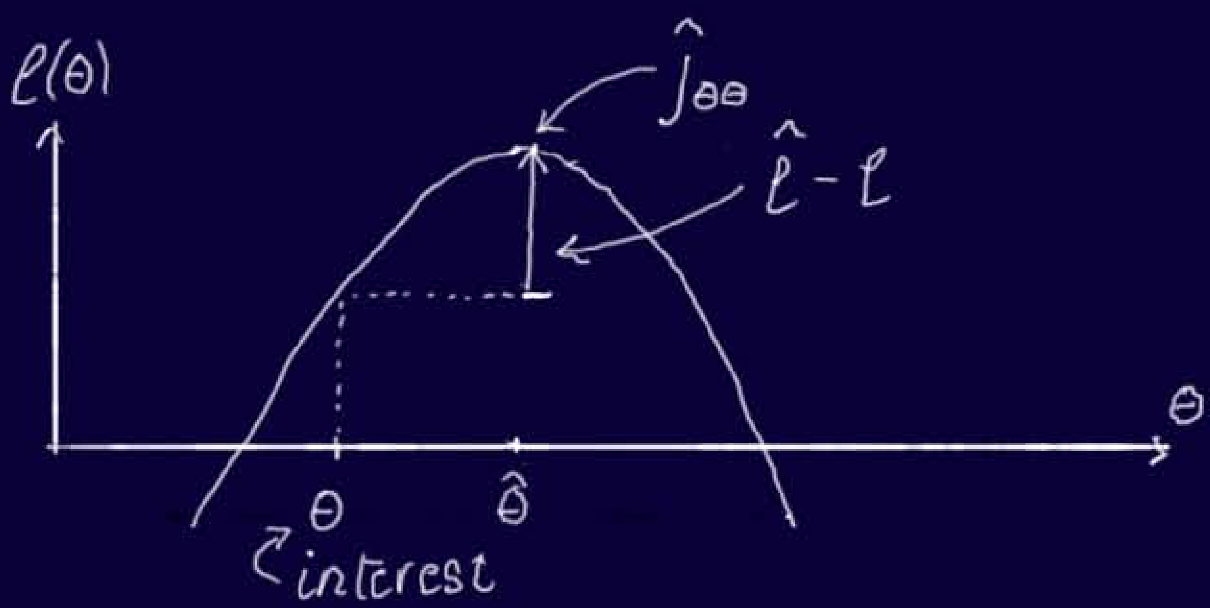

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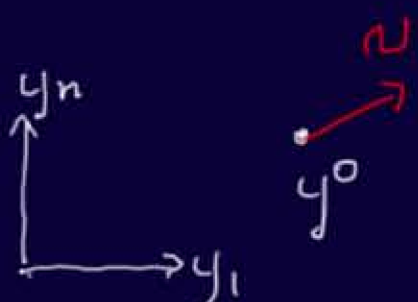

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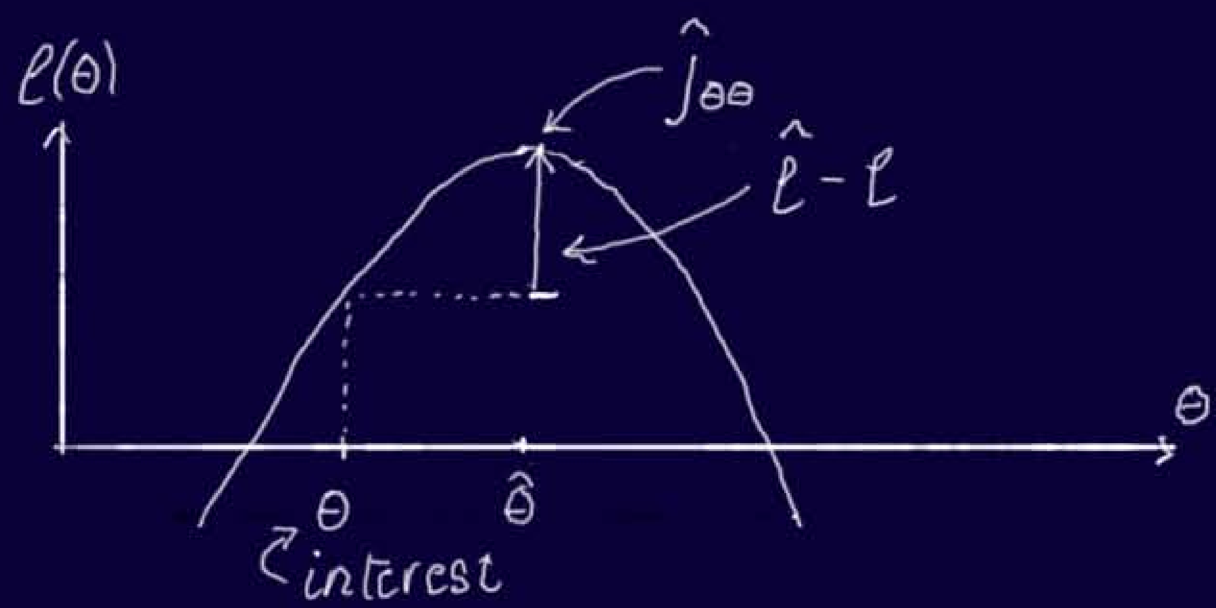

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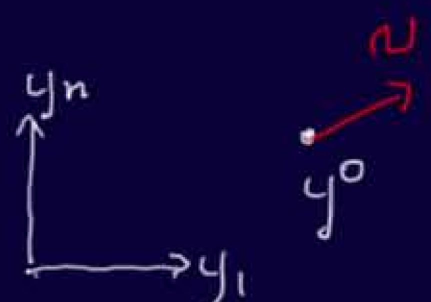

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There is a conditional model thru y°,
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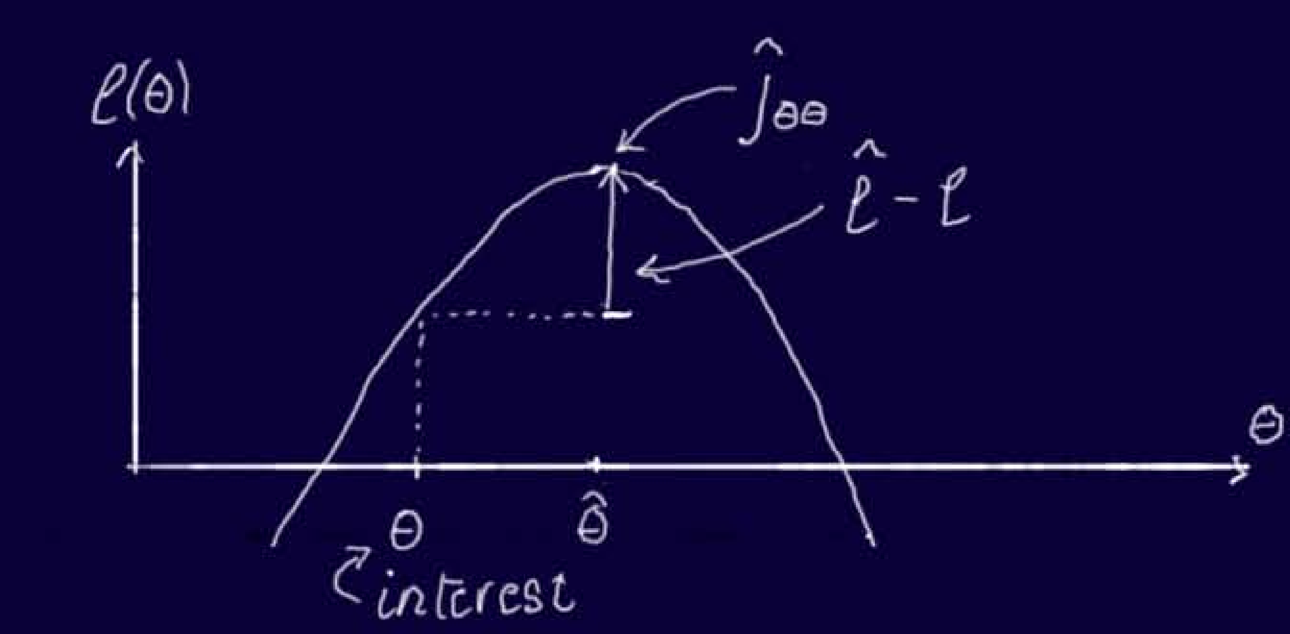

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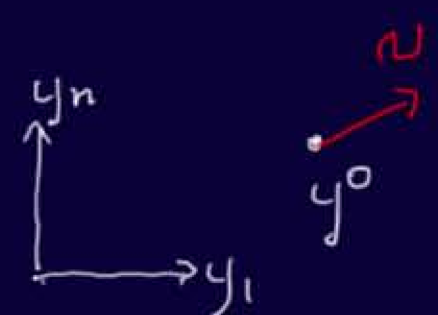

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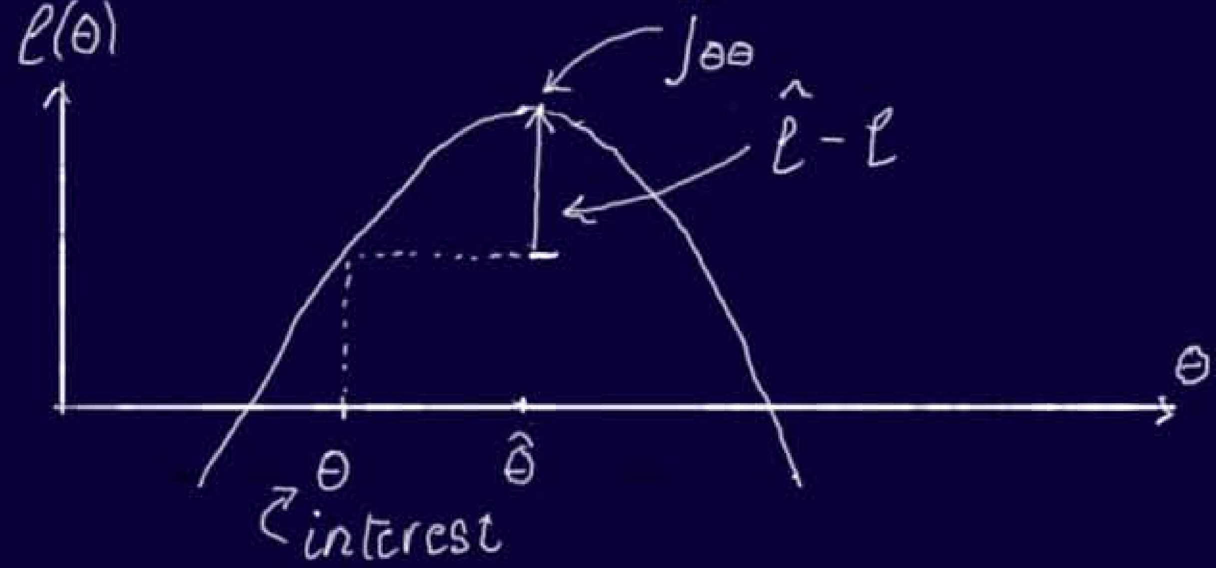

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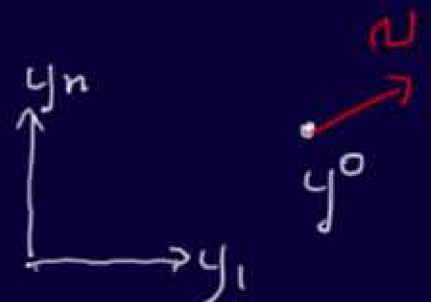

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F et al Bernoulli 2010 pp1208+

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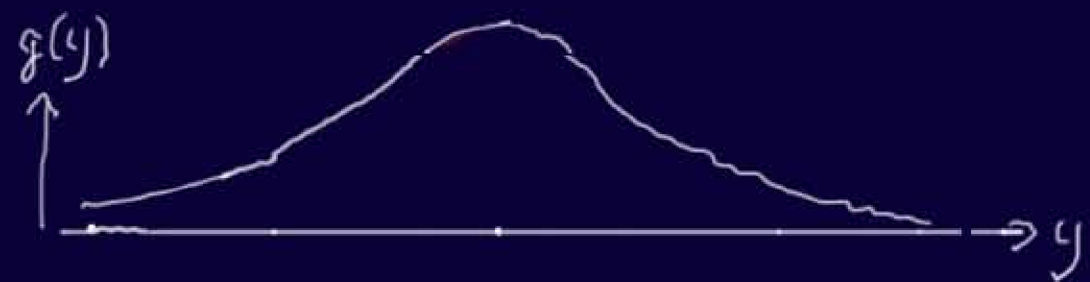

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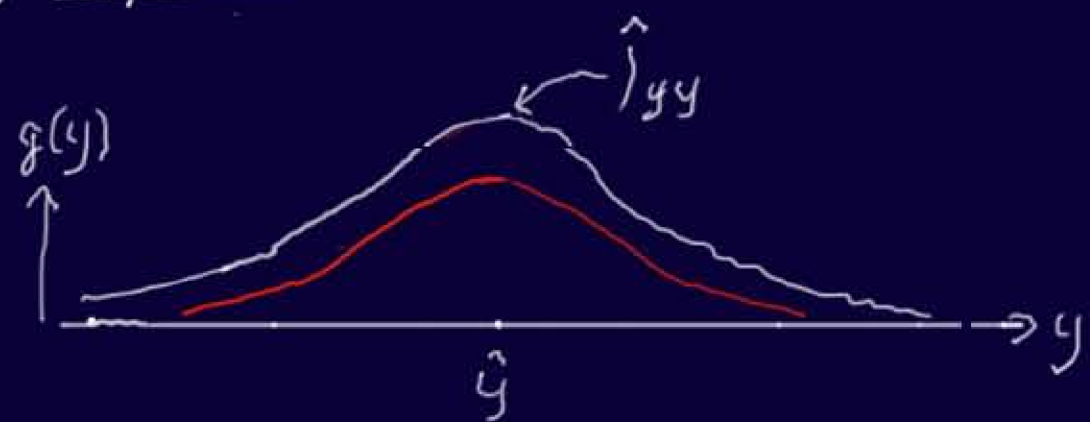

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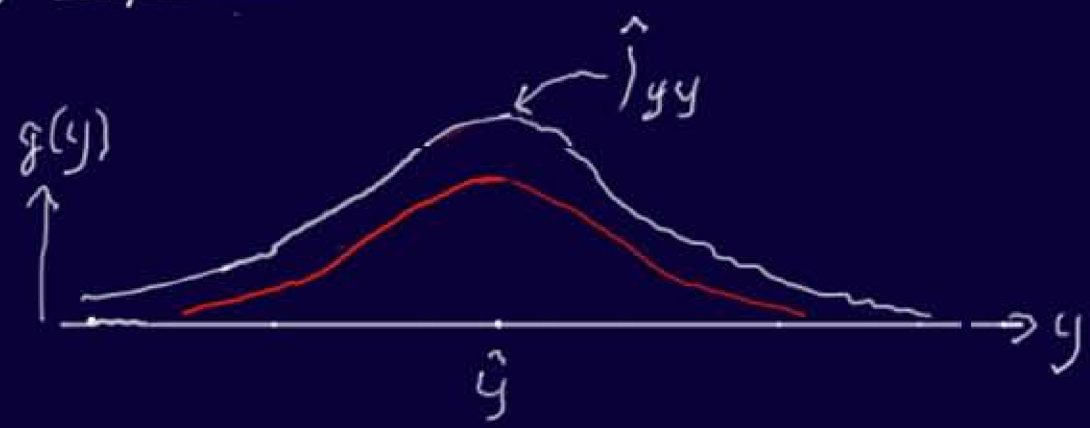

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$$g(y) \approx \underbrace{\cdot \frac{\hat{j}^{1/2}}{(2\pi)^{1/2}} e^{-\frac{1}{2}(\cdot\cdot)} \cdot}_{\text{Normal}} \qquad dy$$

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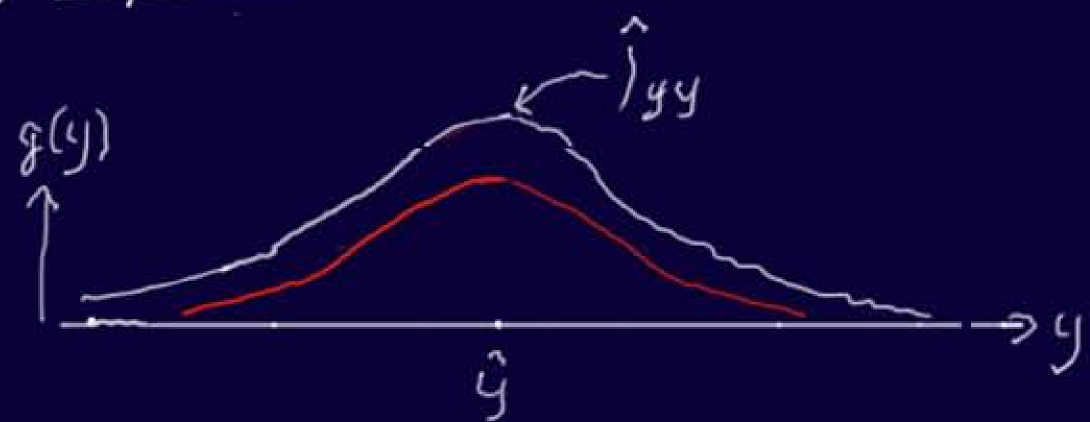

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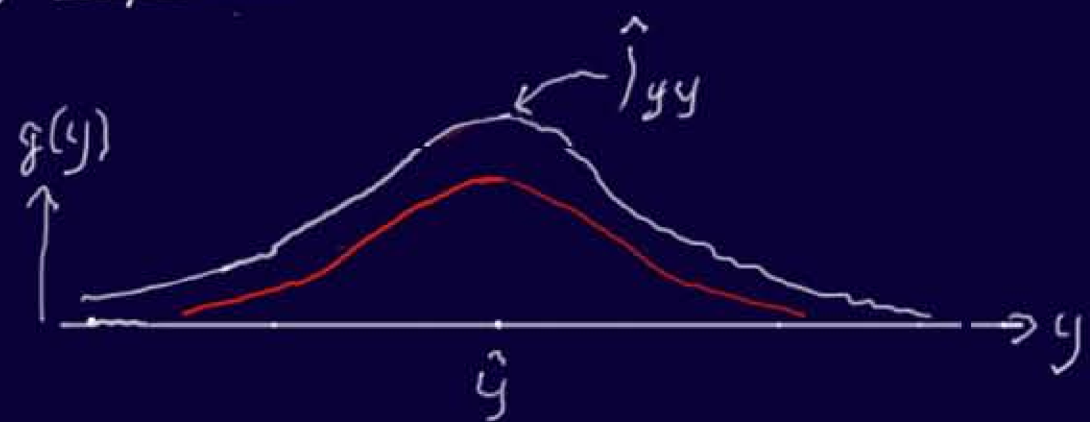

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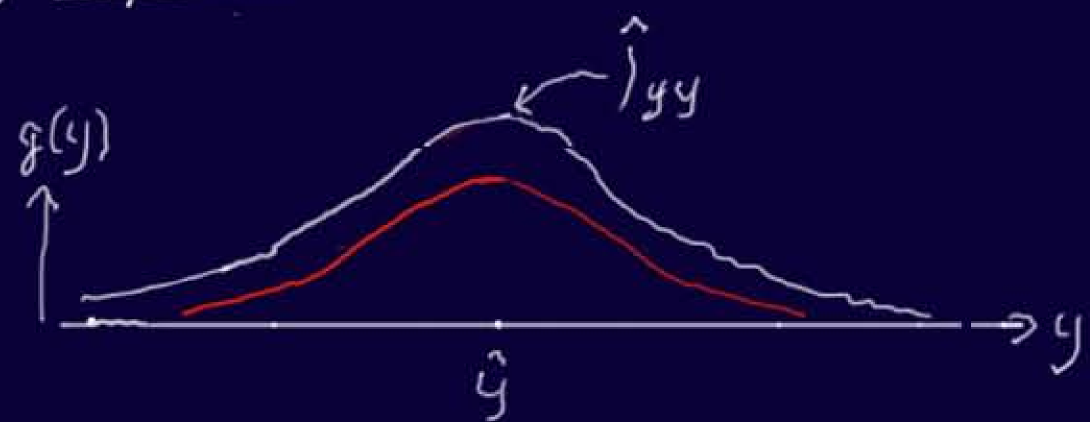

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3rd

High Acc.

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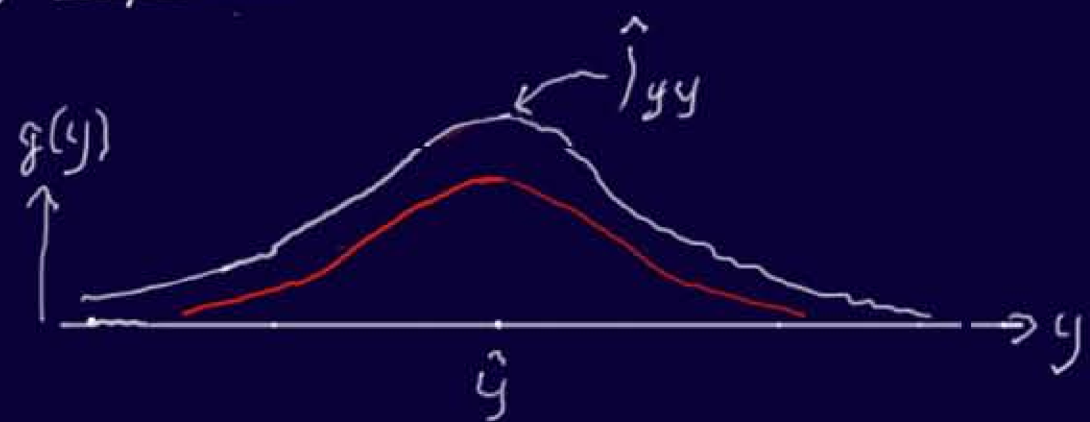

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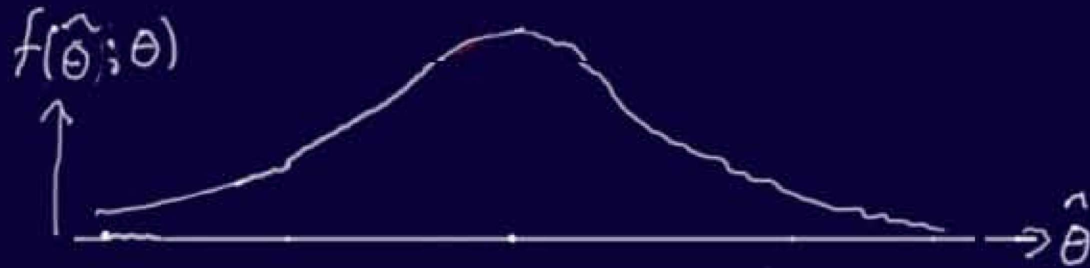

$g(\hat{\theta};\theta)\,d\hat{\theta} =$

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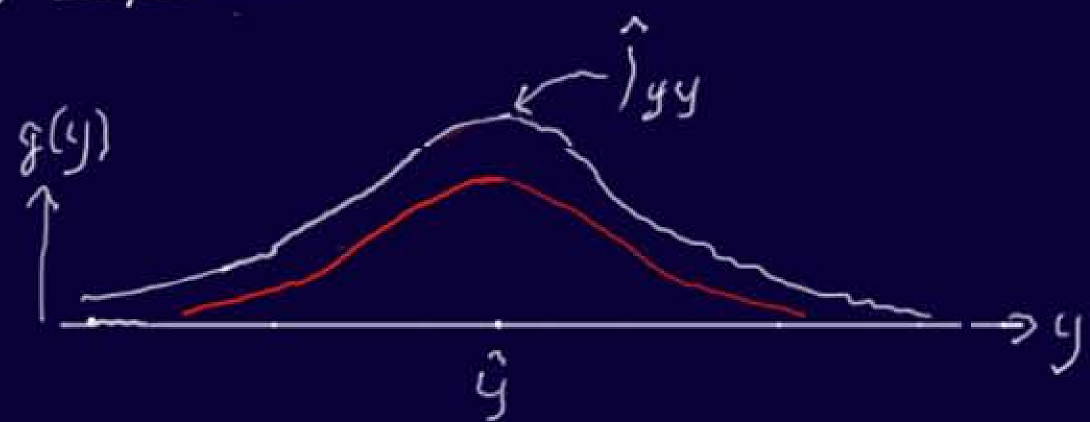

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High Acc.

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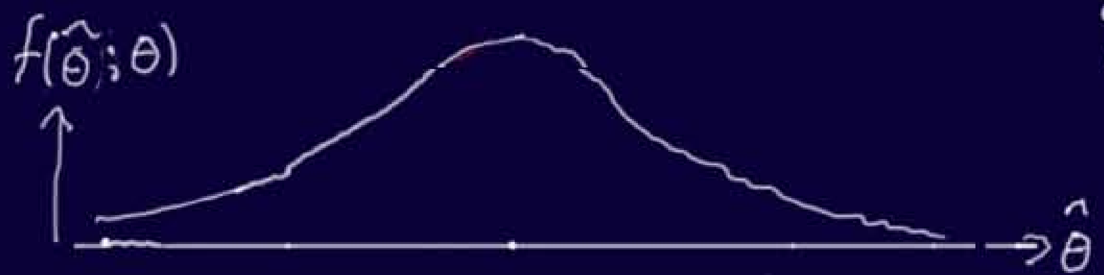


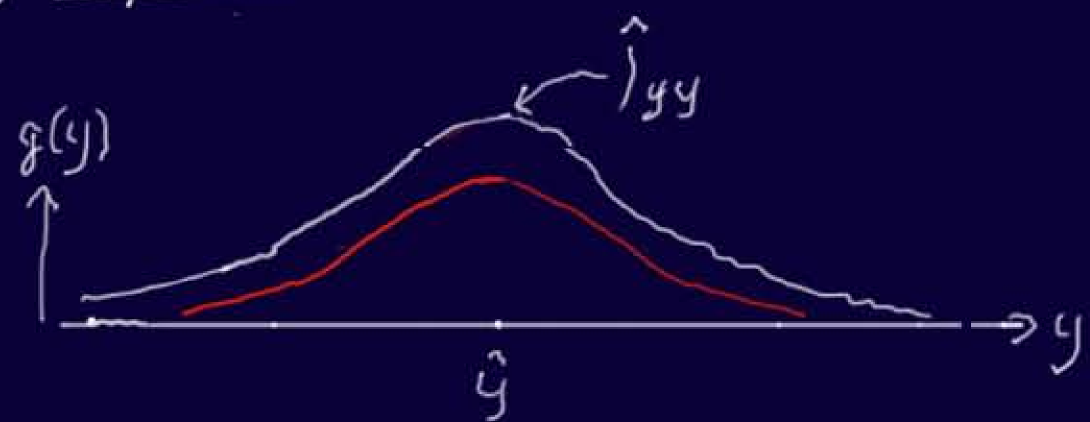
$$g(\hat{\theta};\theta)\, d\hat{\theta} = \qquad e^{-r^2/2} \qquad d\hat{\theta}$$

Get correct Likelihood

3) Likelihood to Densities

a) Laplace: Integrate $g(y)$

$\log g(y)$ is $O(n)$ ....



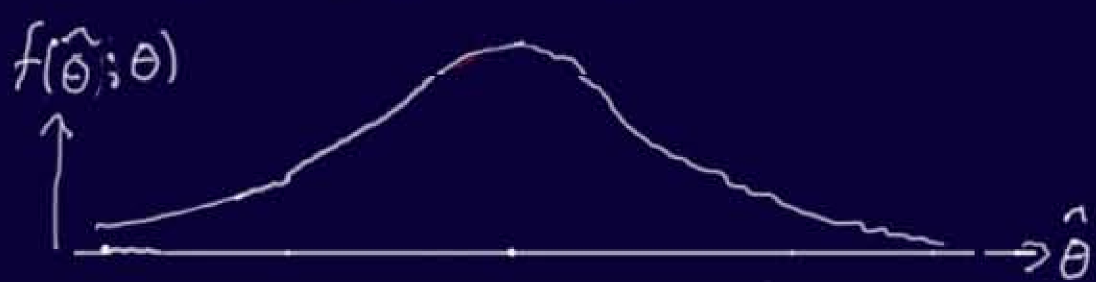
Fit a Normal $N(\hat{y}, \hat{j}^{-1})$

$$g(y) \approx g(\hat{y}) \frac{(2\pi)^{1/2}}{\hat{j}^{1/2}} \cdot \underbrace{\frac{\hat{j}^{1/2}}{(2\pi)^{1/2}} e^{-\frac{1}{2}(\cdot\cdot)}}_{} \cdot e^{\underbrace{a_3 z^3/6n^{3/2} + a_4 z^4/24n}_{\text{cubic, quartic}}} dy$$

$$\int g(y)\,dy = g(\hat{y}) \frac{(2\pi)^{1/2}}{\hat{j}^{1/2}} \cdot 1 \cdot e^{(3a_4 + 5a_3^2)/24n}$$

3rd
High Acc.

b) Barndorff-Nielsen p^*

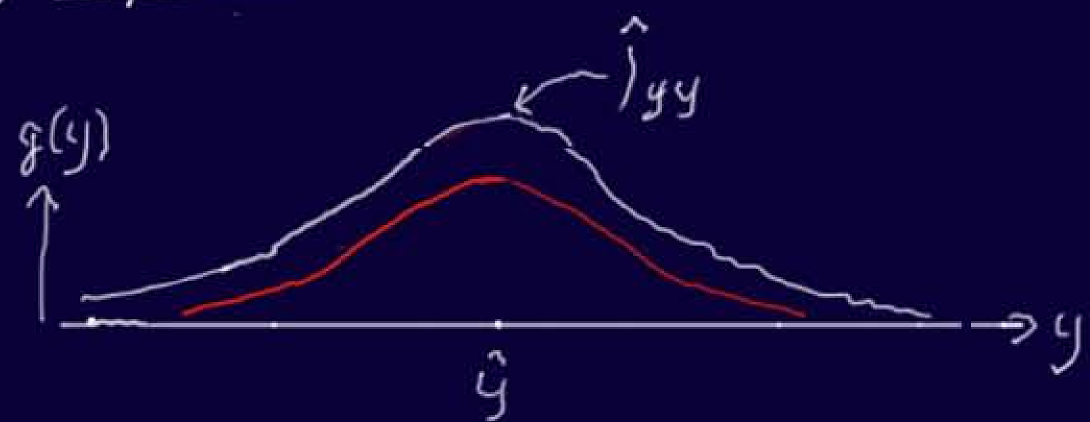


$$g(\hat{\theta};\theta)\,d\hat{\theta} = \frac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} \qquad d\hat{\theta}$$

Anticipate a constant

3) Likelihood to Densities

a) Laplace: Integrate $g(y)$



$\log g(y)$ is $O(n)$ ....

Fit a Normal $N(\hat{y}, \hat{j}^{-1})$

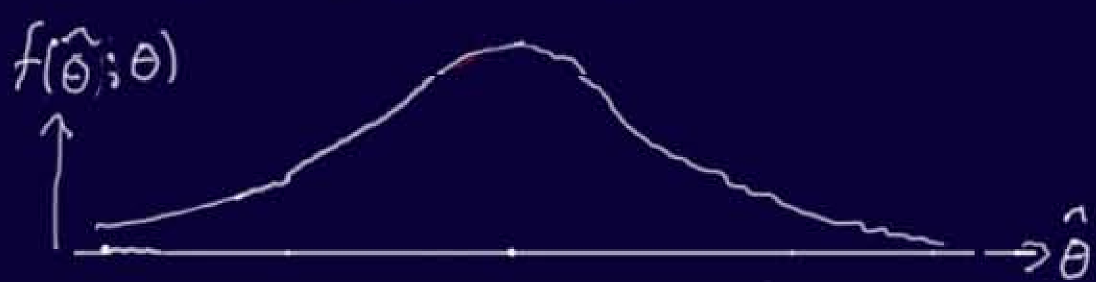
$$g(y) \approx g(\hat{y}) \frac{(2\pi)^{1/2}}{\hat{j}^{1/2}} \cdot \frac{\hat{j}^{1/2}}{(2\pi)^{1/2}} e^{-\frac{1}{2}(..)} \cdot e^{a_3 z^3/6n^{3/2} + a_4 z^4/24n} \, dy$$

cubic, quartic

$$\int g(y)\,dy = g(\hat{y}) \frac{(2\pi)^{1/2}}{\hat{j}^{1/2}} \cdot 1 \cdot e^{(3a_4 + 5a_3^2)/24n}$$

3rd
High Acc.

b) Barndorff-Nielsen p^*

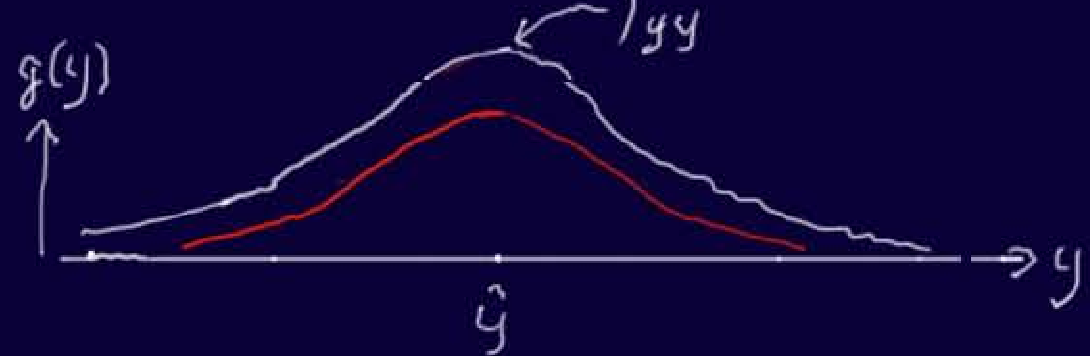


$$g(\hat{\theta};\theta)\,d\hat{\theta} = \frac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} \cdot j_{\hat{\theta}\hat{\theta}}^{1/2} \, d\hat{\theta}$$

Laplace at $\hat{\theta}$ with $\theta = \hat{\theta}$

3) Likelihood to Densities

a) Laplace: Integrate $g(y)$



$\log g(y)$ is $O(n)$ ....

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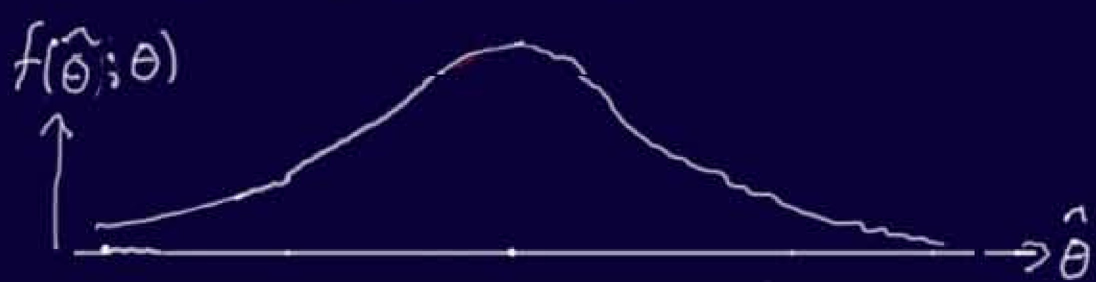


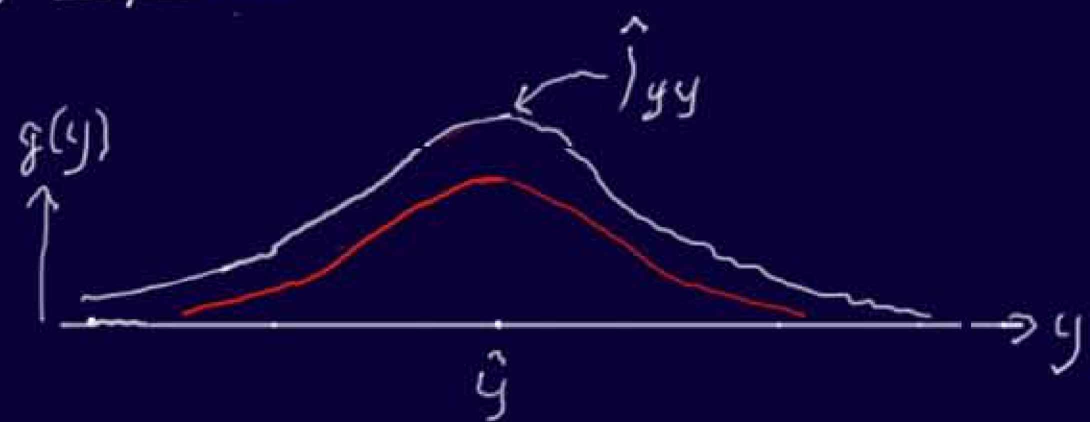
$$g(\hat{\theta};\theta)\,d\hat{\theta} = \frac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} \cdot j_{\theta\theta}^{1/2}\, d\hat{\theta}$$

[algebra $\hat{\theta} \to \theta$

3) Likelihood to Densities

a) Laplace: Integrate $g(y)$

$\log g(y)$ is $O(n)$ ....



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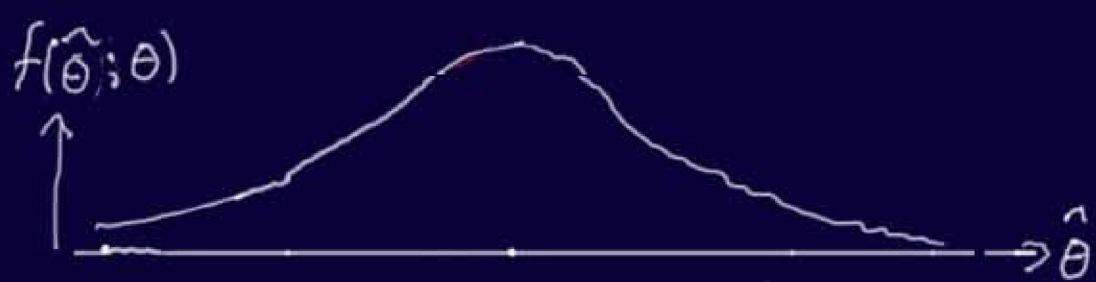
3rd
High Acc.

b) Barndorff-Nielsen p^*



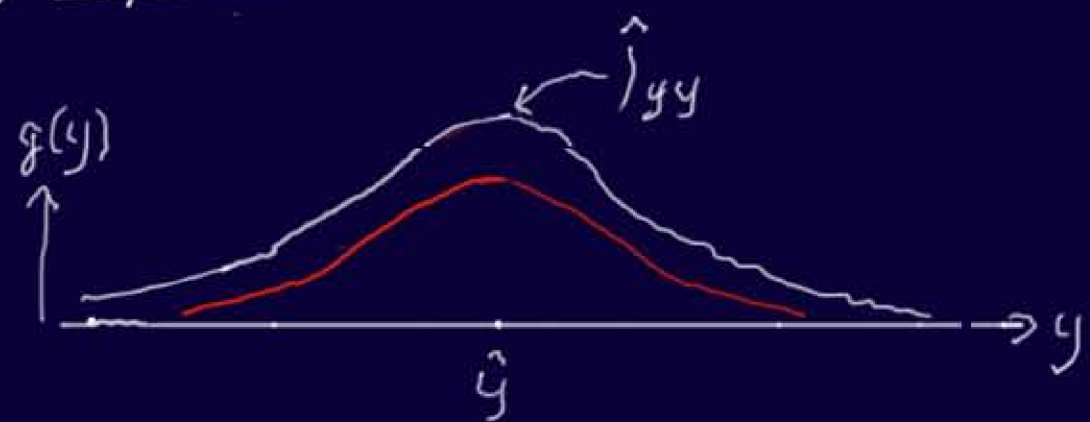
$$g(\hat{\theta};\theta)\,d\hat{\theta} = \frac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} \cdot j_{\theta\theta}^{1/2} \, d\hat{\theta}$$

c) Saddle point Try above for Exponential model

$$e^{y\varphi - \kappa(\varphi)} f(y)\,dy$$

3) Likelihood to Densities

a) Laplace: Integrate $g(y)$



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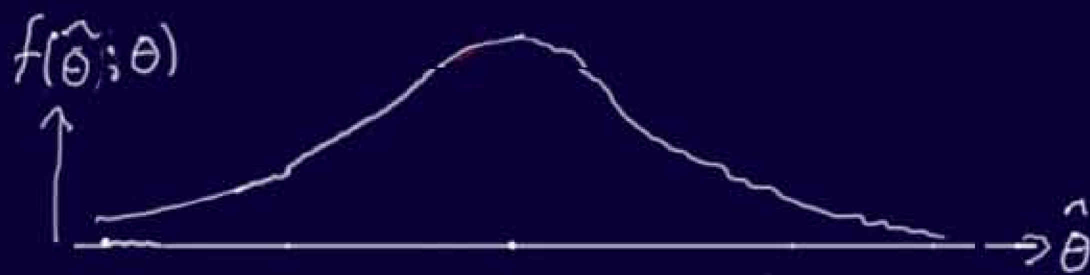
$$\int g(y)\,dy = g(\hat{y}) \frac{(2\pi)^{1/2}}{\hat{j}^{1/2}} \cdot 1 \cdot e^{(3a_4 + 5a_3^2)/24n}$$

3rd High Acc.

b) Barndorff-Nielsen p^*



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$$e^{y\varphi - \kappa(\varphi)} f(y)\,dy = \frac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} j_{\varphi\varphi}^{1/2} \, d\hat{\varphi}$$

SP (from Barndorff-Nielsen)

3) Likelihood to Densities

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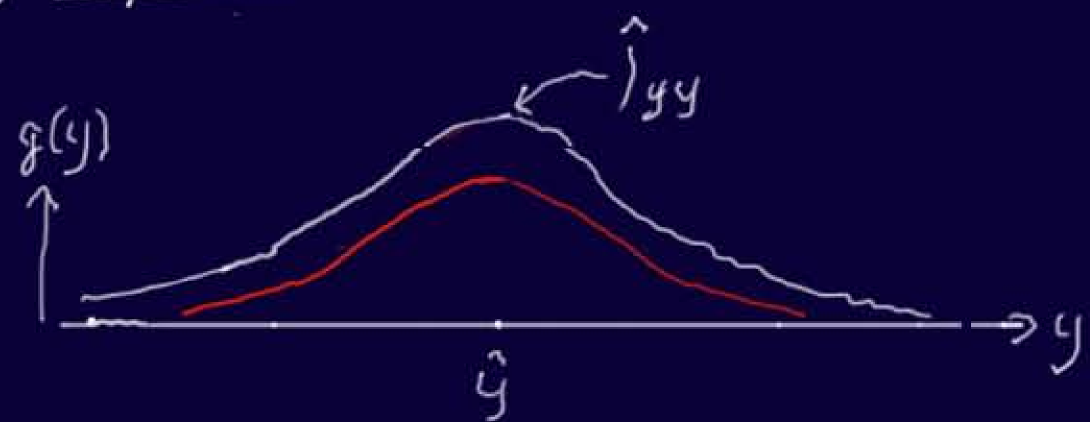

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High Acc.

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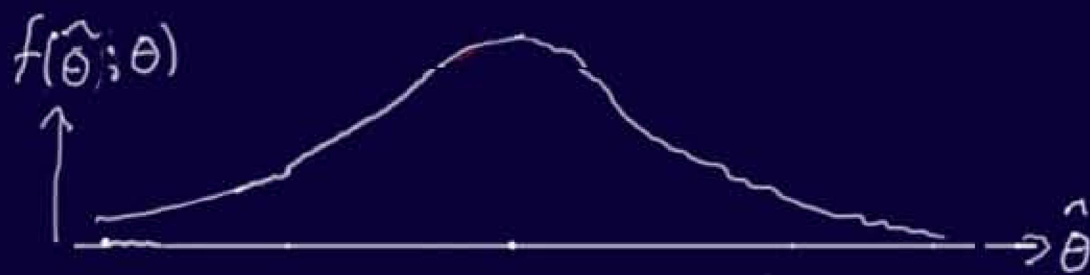

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SP (from Barndorff-Nielsen)

$$= \frac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} j_{\varphi\varphi}^{-1/2} \, dy$$

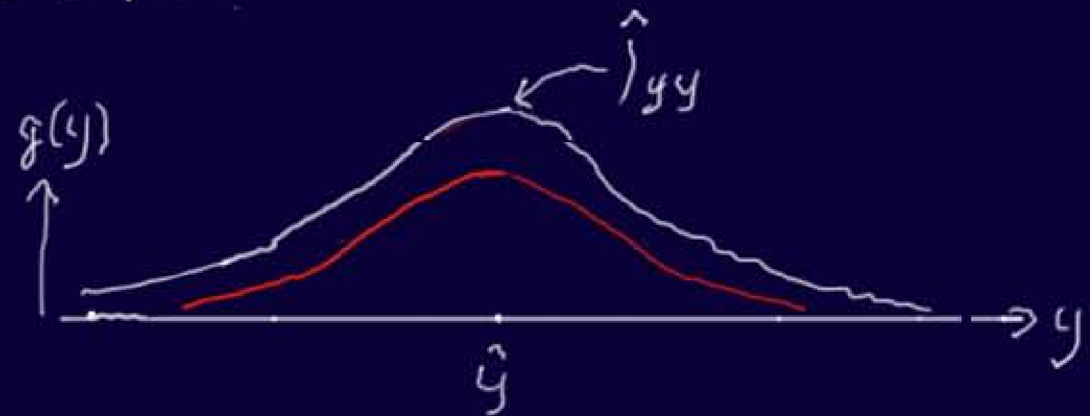
Jacobian $\partial y / \partial \hat{\varphi} = j_{\varphi\varphi}$

3) Likelihood to Densities

a) Laplace: Integrate $g(y)$

$\log g(y)$ is $O(n)$ ....

Fit a Normal $N(\hat{y}, \hat{j}^{-1})$



$$g(y) \approx g(\hat{y}) \frac{(2\pi)^{1/2}}{\hat{j}^{1/2}} \cdot \underline{\frac{\hat{j}^{1/2}}{(2\pi)^{1/2}} e^{-\frac{1}{2}(\cdot\cdot)}} \cdot \underline{e^{a_3 z^3/6n^{3/2} + a_4 z^4/24n}}\, dy$$

cubic, quartic

$$\int g(y)\,dy = g(\hat{y}) \frac{(2\pi)^{1/2}}{\hat{j}^{1/2}} \cdot 1 \cdot e^{(3a_4 + 5a_3^2)/24n}$$

3rd
High Acc.

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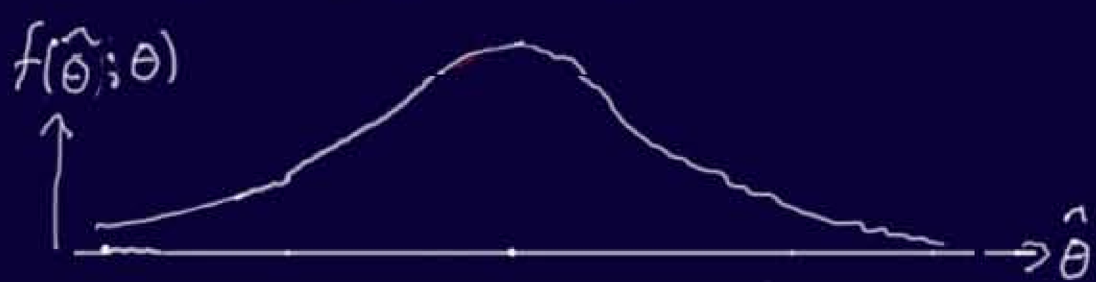

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$$e^{y\varphi - \kappa(\varphi)} f(y)\,dy = \frac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} j_{\varphi\varphi}^{1/2}\, d\hat{\varphi} \qquad \text{SP (from Barndorff-Nielsen)}$$

$$= \frac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} j_{\varphi\varphi}^{-1/2}\, dy \qquad \text{Jacobian } \partial y / \partial\hat{\varphi} = j_{\varphi\varphi}$$

d) All have vector versions
All 3rd order
All highly Accurate

Powerful !

4 Density to p-value

$$r^2/2 = \hat{\ell} - \ell$$

Scalar Exp't'l: $e^{y\varphi - \kappa(\varphi)} f(y)\,dy = \dfrac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} \hat{j}_{\varphi\varphi}^{-1/2} \cdot dy$

4 Density to p-value

Scalar Exp't'l: $e^{y\varphi - K(\varphi)} f(y)\, dy = \frac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} \hat{j}_{\varphi\varphi}^{-1/2} \cdot dy$

$r^2/2 = \hat{l} - l$

Differentials:

$r\,dr = (\hat{\varphi} - \varphi)\, dy$

4 Density to p-value

Scalar Exp't'l: $e^{y\varphi - \kappa(\varphi)} f(y)\, dy = \dfrac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} \hat{j}_{\varphi\varphi}^{-1/2} \cdot dy$

$= \dfrac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} \dfrac{r}{q}\, dr$

$r^2/2 = \hat{\ell} - \ell$

Differentials:

$r\,dr = (\hat{\varphi} - \varphi)\, dy$

$q = (\hat{\varphi} - \varphi) \hat{j}_{\varphi\varphi}^{-1/2}$

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$$= \frac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} \frac{r}{q}\, dr$$

$$= \frac{e^{k/n}}{(2\pi)^{1/2}} e^{-\frac{1}{2}\left[r - r^{-1} \ln \frac{r}{q}\right]^2} dr$$

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Integrate: $p(\varphi) = F(y;\varphi) = \Phi\{r - r^{-1} \log \frac{r}{q}\}$

$r^2/2 = \hat{\ell} - \ell$

Differentials:

$r\,dr = (\hat{\varphi} - \varphi)\,dy$

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3rd Acc

Highly Accurate

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$r^2/2 = \hat{\ell} - \ell$

Differentials:

$r\,dr = (\hat{\varphi} - \varphi)\,dy$

$q = (\hat{\varphi} - \varphi)\,\hat{j}_{\varphi\varphi}^{-1/2}$

Integrate: $p(\varphi) = F(y;\varphi) = \Phi\left(r - r^{-1}\log\frac{r}{q}\right)$

$r^* = r - r^{-1}\log\frac{r}{q}$

3rd Acc

Highly Accurate

Normal scaling

4 Density to p-value

Scalar Exp't'l: $e^{y\varphi - \kappa(\varphi)} f(y)\, dy = \dfrac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} \hat{j}_{\varphi\varphi}^{-1/2} \cdot dy$

$$= \frac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} \frac{r}{q}\, dr$$

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$r^2/2 = \hat{l} - l$

Differentials:

$r\,dr = (\hat{\varphi} - \varphi)\, dy$

$q = (\hat{\varphi} - \varphi) \hat{j}_{\varphi\varphi}^{-1/2}$

Integrate: $p(\varphi) = F(y; \varphi) = \Phi\{r - r^{-1} \log \frac{r}{q}\}$

3rd Acc

Highly Accurate

$r^* = r - r^{-1} \log \frac{r}{q}$

Normal scaling

With adjustment $a(y)$ Model $= f(y; \varphi)\, a(\hat{\varphi})\, dy$

4 Density to p-value

$r^2/2 = \hat{\ell} - \ell$

Scalar Exp't'l: $e^{y\varphi - \kappa(\varphi)} f(y)\, dy = \frac{e^{k/n}}{(2\pi)^{1/2}} e^{-r^2/2} \hat{j}_{\varphi\varphi}^{-1/2} \cdot dy$

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Differentials:

$r\, dr = (\hat{\varphi} - \varphi)\, dy$

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3rd Acc

Highly Accurate

Integrate: $p(\varphi) = F(y; \varphi) = \Phi\{r - r^{-1} \log \frac{r}{q}\}$

$r^* = r - r^{-1} \log \frac{r}{q}$

Normal scaling

With adjustment $a(y)$ Model $= f(y; \varphi)\, a(\hat{\varphi})\, dy$

Use $r^* = r - r^{-1} \log \frac{r}{Q}$ $Q = \frac{a(\varphi)}{a(\hat{\varphi})} q$

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4 Density to p-value

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$$= \frac{e^{k/n}}{(2\pi)^{1/2}} e^{-\frac{1}{2}\left[r - r^{-1} \ln \frac{r}{q}\right]^2} dr$$

Differentials:

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With adjustment $a(y)$ Model $= f(y; \varphi)\, a(\hat{\varphi})\, dy$

Use $r^* = r - r^{-1} \log \frac{r}{Q}$

$Q = \dfrac{a(\varphi)}{a(\hat{\varphi})} q$

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$p(\varphi) = \Phi(r^*)$

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3rd Acc
Highly Accurate

$r^* = r - r^{-1} \log \frac{r}{q}$

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With adjustment $a(y)$ Model $= f(y; \varphi)\, a(\hat{\varphi})\, dy$

Use $r^* = r - r^{-1} \log \frac{r}{Q}$

$Q = \frac{a(\varphi)}{a(\hat{\varphi})} q$

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$p(\varphi) = \Phi(r^*)$

3rd Accuracy
Solves "All" problems!

5 For interest parameter $\psi(\theta)$

a) Model into statistical notation

$$f(s;\varphi)\,ds = \frac{e^{k/n}}{(2\pi)^{p/2}} e^{-\frac{r^2}{2}} |\hat{j}_{\varphi\varphi}|^{-\frac{1}{2}} ds$$

5 For interest parameter $\psi(\theta)$

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$$f(s;\varphi)\,ds = \frac{e^{k/n}}{(2\pi)^{p/2}} e^{-\frac{r^2}{2}} |\hat{j}_{\varphi\varphi}|^{-\frac{1}{2}} ds$$

b) Interest $\psi(\varphi) = \psi_0$, dim d   Examine on $L^{\circ} = \{ s : \hat{\lambda}_{\psi_0} = \hat{\lambda}^{\circ}_{\psi_0} \}$

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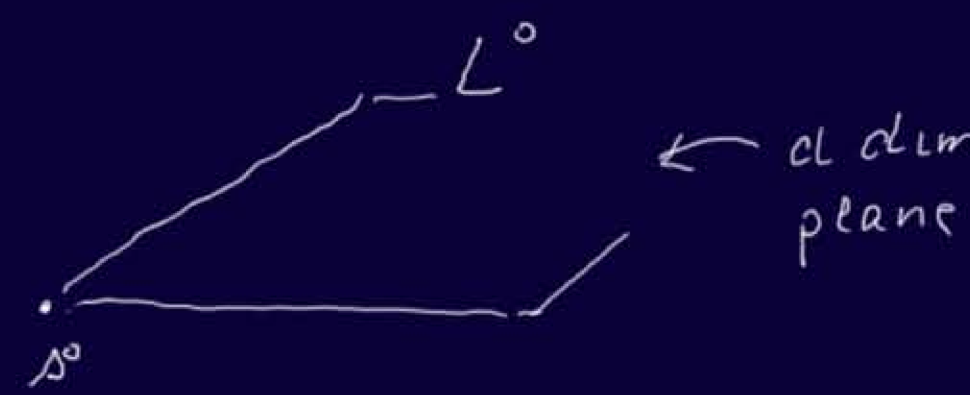

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Integrate out nuisance effect

Get marginal on L^0:

B-N

$$h(s;\psi_0) = \frac{e^{k/n}}{(2\pi)^{\frac{p-d}{2}}}\, e^{\tilde{\ell} - \hat{\ell}}\, |\hat{j}_{\varphi\varphi}|^{-\frac{1}{2}}\, |\tilde{j}_{(\lambda\lambda)}|^{1/2}$$

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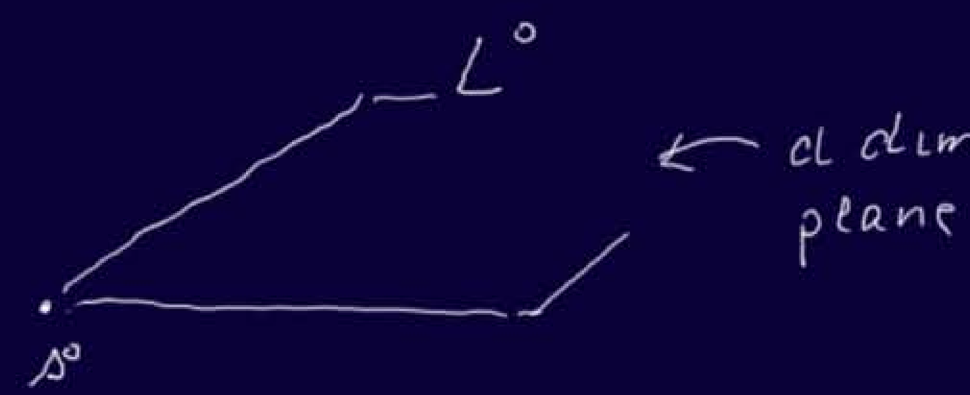

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B-N

scaled re φ

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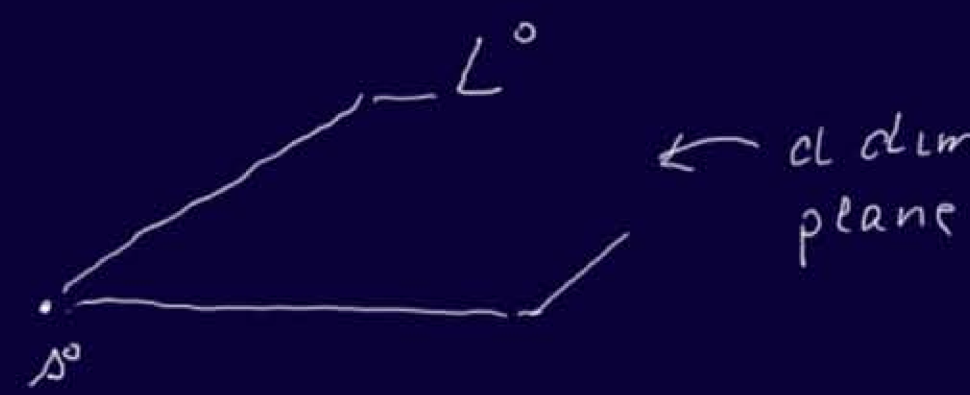

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Integrate out nuisance effect

Get marginal on L°:

B-N

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scaled re φ

Past, present, ...
2014 Ed: X Lin ...
p 237+

5 For interest parameter $\psi(\theta)$

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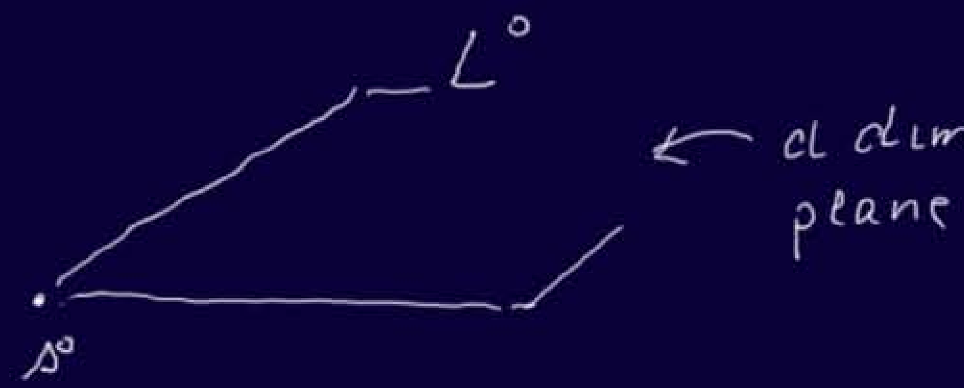

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From above into r^* $p(\psi) = \Phi(r^*)$ JASA 2014 p302+

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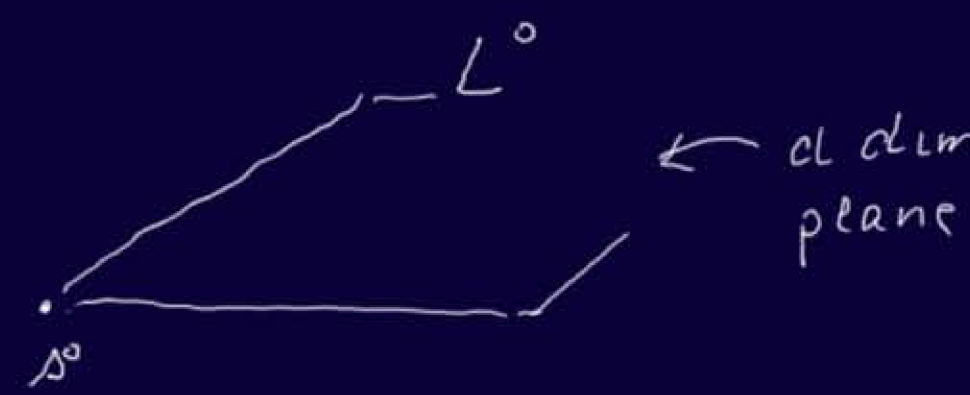

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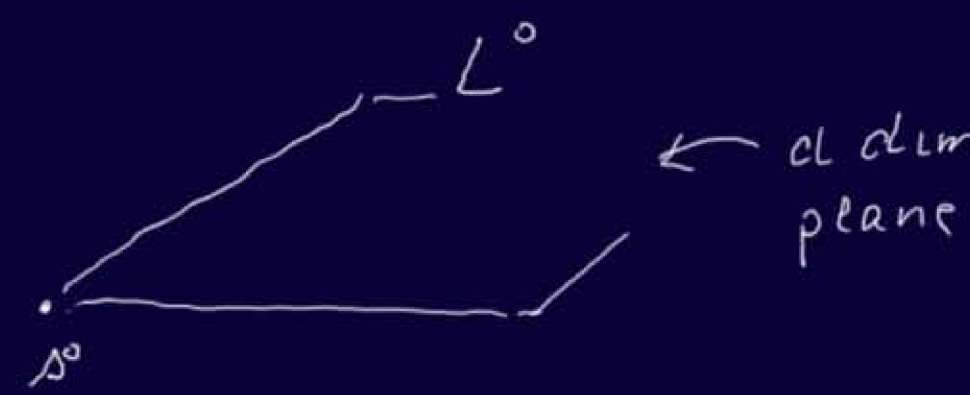

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Directional p-value: "From expectation towards data"

Biometrika 2016

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- Use Jeffreys strictly on C
- Get 3rd order accuracy   F 2016 Stat Sc.

Likelihood inference $O(n^{-3/2})$

p-values

Likelihood

Likelihood to densities

density to p-values

For interest parameter $\psi(\theta)$

Bootstrap

Bayes

Thank you