What can we get from likelihood?

a new prior for Bayes ---

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BFF5 Foundations of Data Science

U Michigan

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www.utstat.toronto.edu/dfraser/documents/BFF5.pdf

Statistical inference, from $f(y;\theta), y^0$ w. regularity

① For the full θ, must condition from n to p dimensions

- If free of conditioning, then sufficiency is present.
- For conditioning, only directions V $n \times p$ needed at $(y^0, \hat\theta^0)$
- This requires full quantile f'n $y = y(z, \theta)$ & $V = \frac{dy}{d\theta}\Big|_{y^0 \hat\theta^0}$

& gives $\varphi(\theta) = d/dV \, \ell(\theta; y)|_{y^0}$ can. par. of 3rd order equiv expt'l model

② Then for interest $\psi = \psi(\theta)$ marginalize re nuisance λ

Saddle point re φ: $\frac{e^{k/n}}{(2\pi)^{p/2}} e^{-r^2/2} |\hat{j}_{\varphi\varphi}|^{-1/2} ds$

$*$ SP re ψ: $\frac{e^{k/n}}{(2\pi)^{d/2}} e^{-r_\psi^2/2} |\hat{j}_{\varphi\varphi}|^{-1/2} |j_{(\lambda\lambda)}(\hat\varphi_\psi)|^{+\frac{1}{2}} ds$

$\hat{j}_{\varphi\varphi} = \frac{-\partial^2}{\partial\varphi\partial\varphi'} \cdot \ell^0(\hat\varphi)$

$-r^2/2 = \ell^0(\varphi) - \ell^0(\hat\varphi)$

re tilt $\ell^0(\varphi) + s'\varphi$

$\hat\varphi_\psi$ = const. mle

Automatic marginalizing: No need for explicit ancillary or explicit integration

Laplace integration gives the $|j_{(\lambda\lambda)}(\hat\varphi_\psi)|^{+\frac{1}{2}}$. Above is 3rd acc.

③ If ψ is scalar then dist'n f'n of $*$ is given (3rd order) by $\Phi(r_\psi^*)$, which is p-value f'n also called significance f'n

④ Likelihood f'n for ψ also available (3rd order accurate)

$\ell(\psi) = \phi(r_\psi^*)$ but requires use of a $\varphi(\theta)$ with $\hat{j}^0_{\varphi\varphi} = I$.

⑤ Bayes doesn't work beyond primitive first order $O(1)$ widely:

Not for vector parameters

Not for scalar parameters unless ① linear in ② expt'l models with ③ Jeffreys prior (Welch-Peers prior).

Bayes gives 2nd order accuracy for scalar parameter provided

(1) Canonical parameterization $\varphi(\theta)$ with $\hat{j}^0_{\varphi\varphi} = I$ is used

(2) If $\hat{j}^0_{\varphi\varphi}$ is not I, replace by $T\varphi(\theta)$ where $\hat{j}^0_{\varphi\varphi} = T'T$

(3) The one-dim profile contour $P = \{\hat\varphi = \hat\varphi_\psi\}$ is used

(4) The full Jeffreys is used on one-dim profile P

(5) The prior is $|\hat{j}_{\varphi\varphi}|^{1/2} \left|\frac{d\hat\varphi_\psi}{d\psi}\right| \cos(P, O_\psi)$ where O_ψ is gradient of $\psi(\varphi)$ at $\hat\varphi^0$

Themes

1 What is likelihood ?

2 How to get info from Likelihood

3 Welch-Peers (1963) resolution

$\frac{\text{old}}{\text{new}}$

4 Welch-Peers and the New Bayes prior

5 Example 1 p-values and s-values

6 Example 2 Gamma Model

7 Example 3 Bounded set: want posterior probability

8 Suggestions

References

Comment: Regular to Exponential model

1 What is likelihood?

Observed likelihood: $L^o(\theta) = cf(y^o;\theta)$ "Prob sitting on data"

Likelihood map: $L(\theta;\cdot) = cf(\cdot;\theta)$ Map from $\{y\} \to \{L(\cdot;y)\}$

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Shortest, easyist route to MSS (Get characteristics of map)

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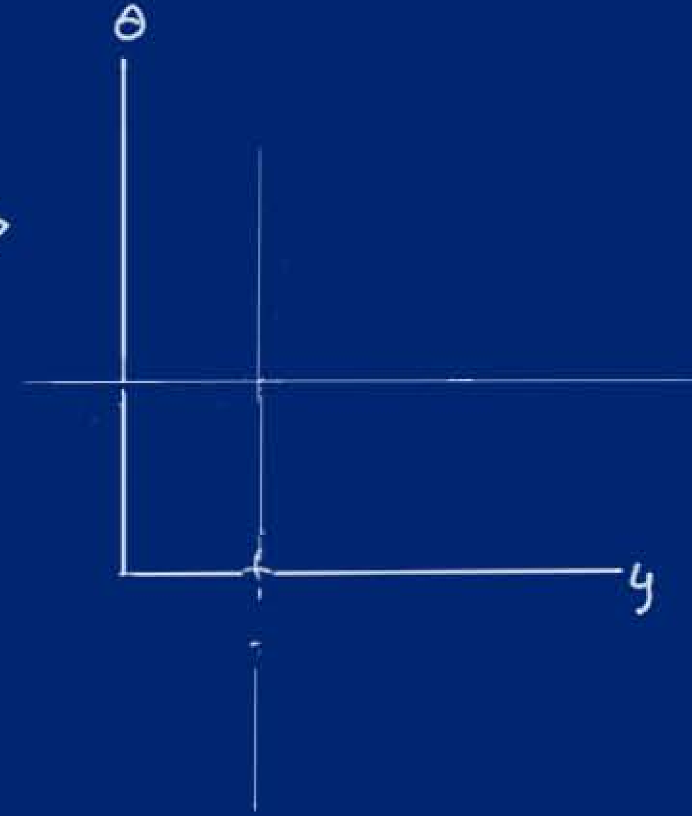
- Shortest, easyist route to MSS (Get characteristics of map)
- Thus gives "Everything about θ"
- But does't separate out the info you need!

2 How to get info from Likelihood

<u>Need</u>: Model $f(y;\theta)$: (pdf)

function on $\{(y,\theta)\}$ $\longrightarrow$

Case: <u>scalar</u>, <u>stoch. inc</u>



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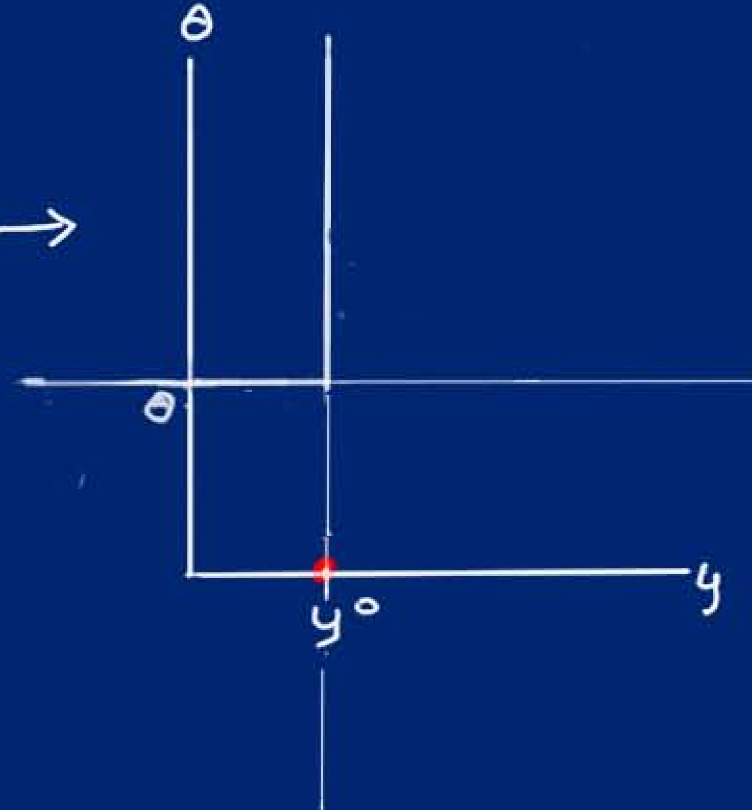
Data y^o



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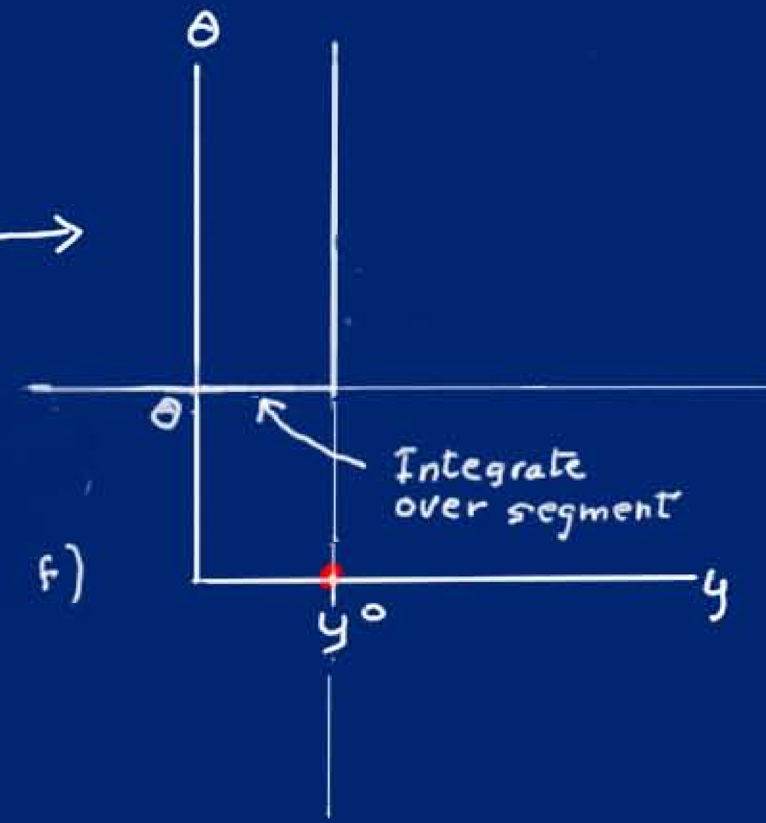

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= where data is (%ile) re θ (current f)

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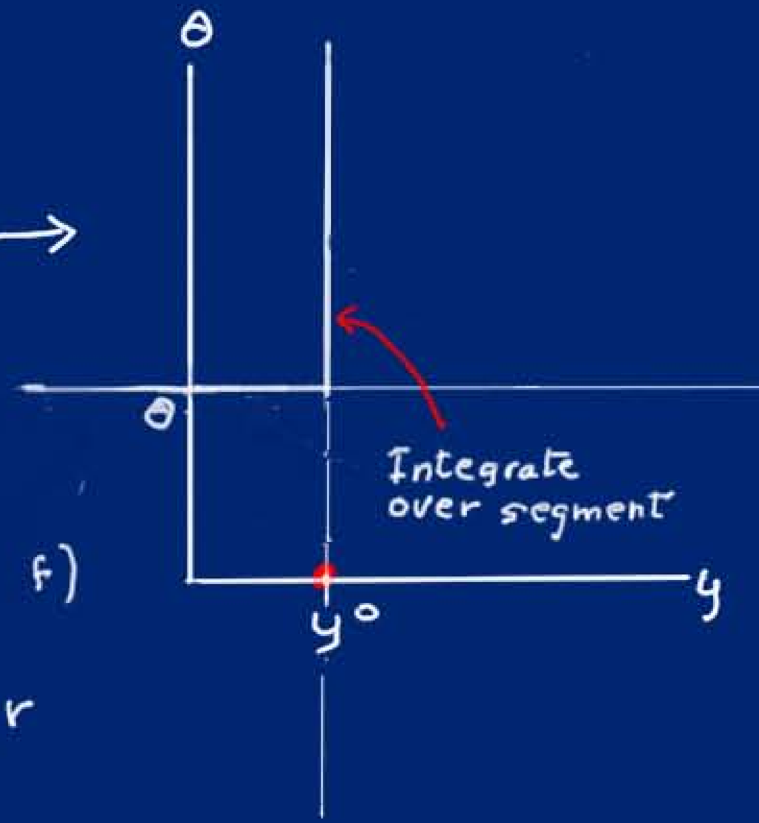
= where θ is re data [6, 7]



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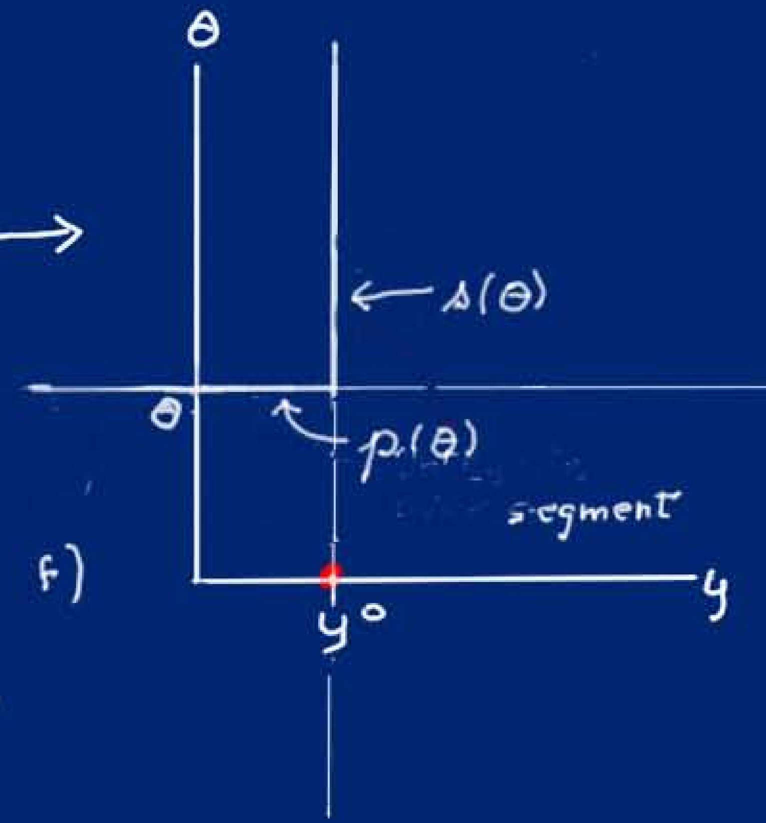

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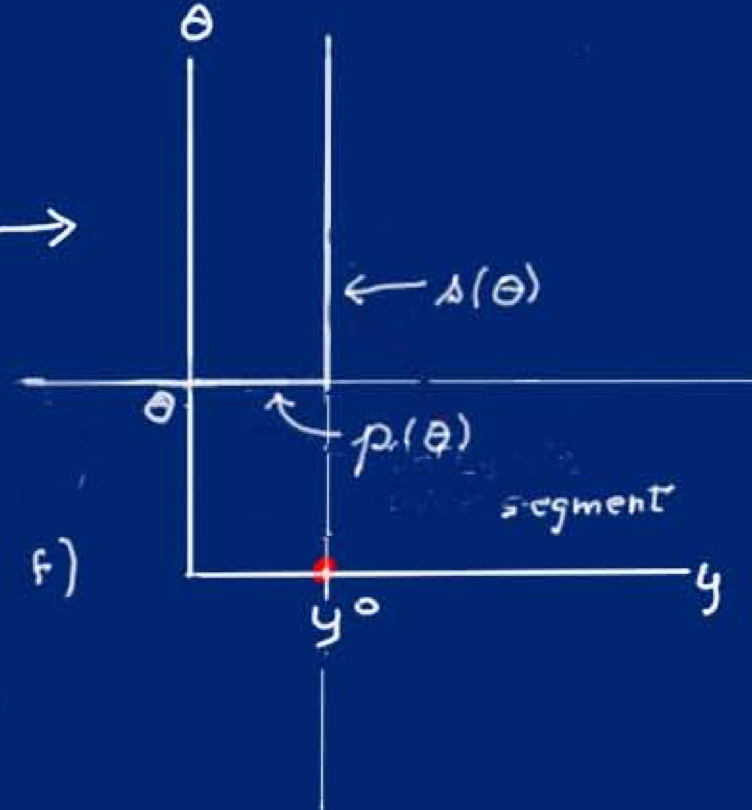

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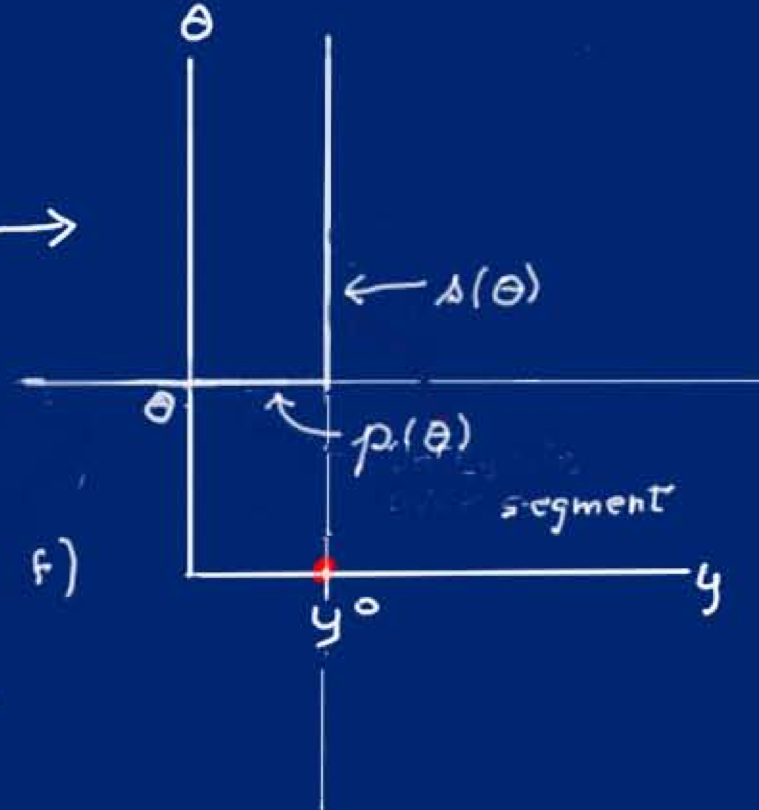

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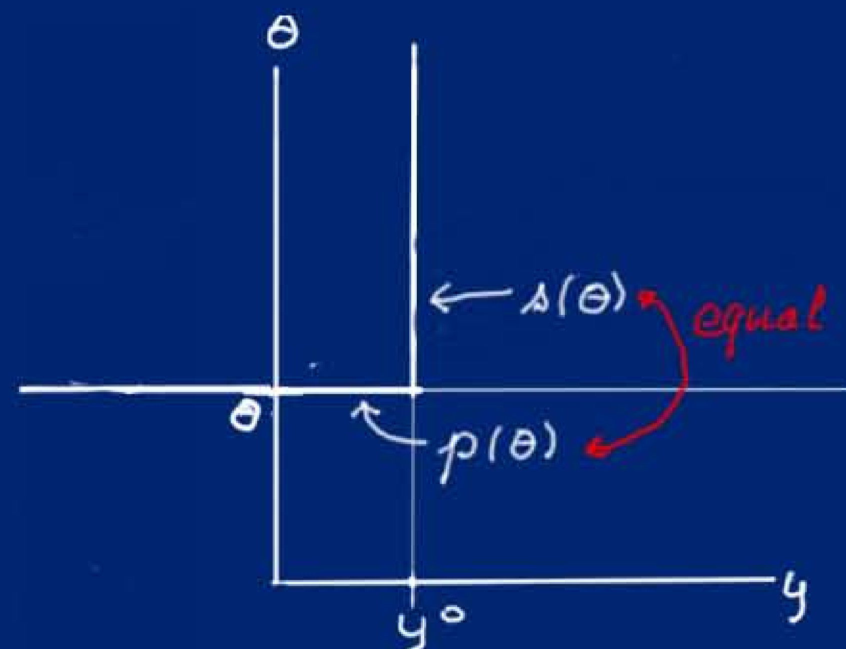

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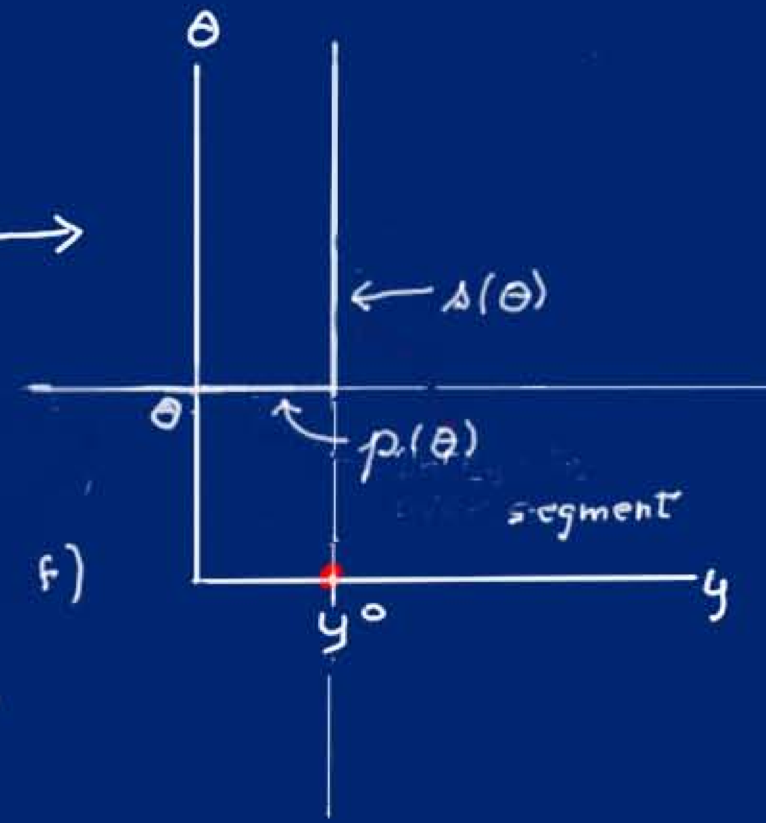

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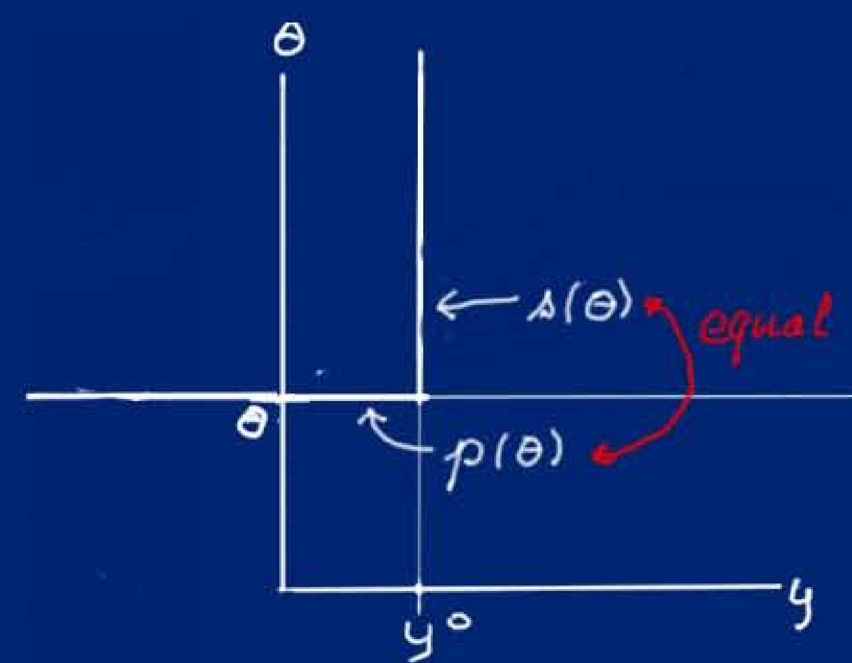

Other models?

Conditional prob. lemma gives nothing!
(except as subjective answer)

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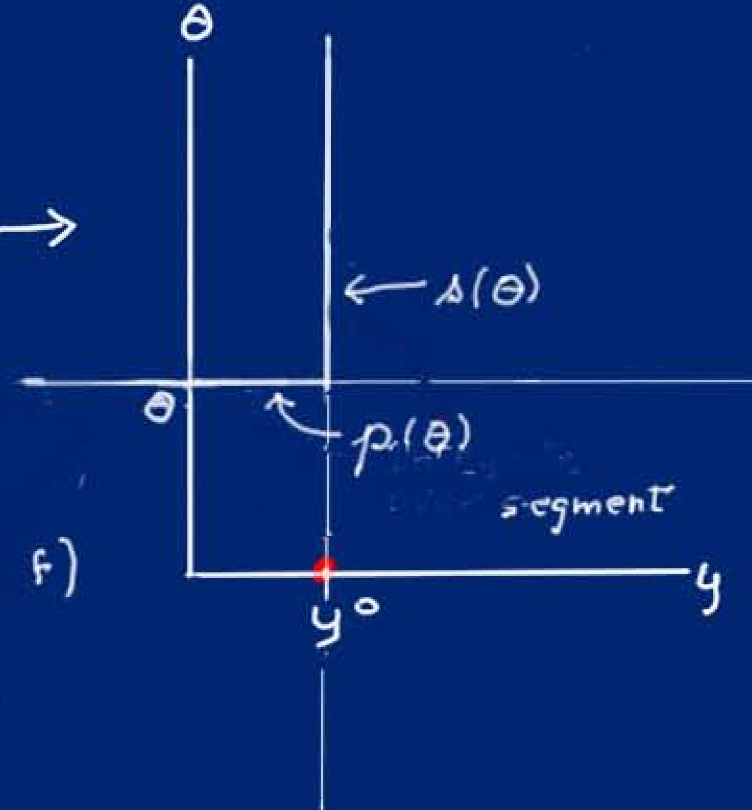

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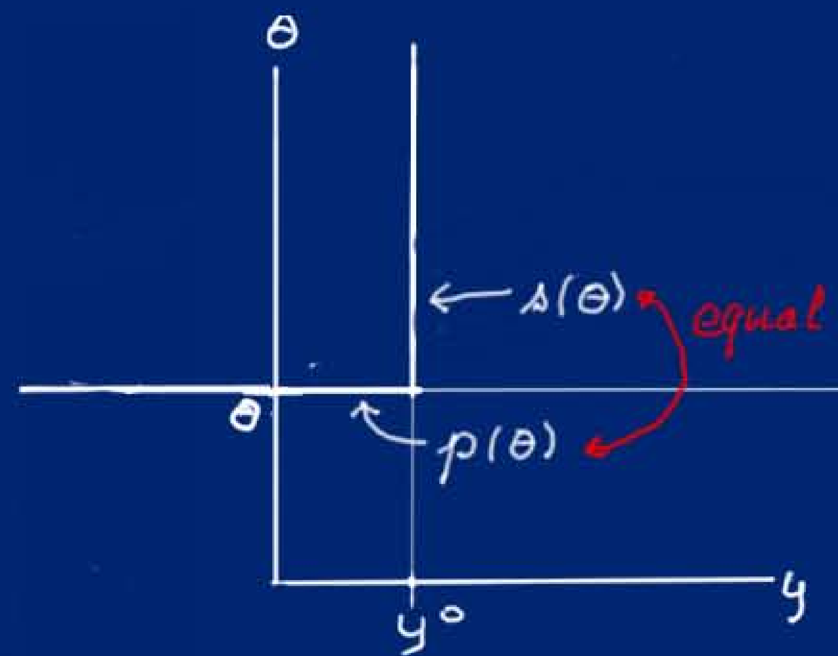

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But "something else" may be going on!   Why we are here

3 Welch-Peers (1963) resolution

Regular model ⇒ Exponential model ⇒ SP approximation ... retain 3rd order!

3 Welch-Peers (1963) resolution

Regular model $\Rightarrow$ Exponential model $\Rightarrow$ SP approximation ... retain 3rd order!

3rd order

Key: Equations $\left\{\begin{array}{l}\text{data generating}\\ \text{structural}\\ \text{quantile}\end{array}\right.$ all the same

Regular $\longrightarrow$ Exponential .... 3rd order accuracy

Summary at end!

3 Welch-Peers (1963) resolution

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Scalar case: SP

$$f(s;\varphi)\,ds \doteq \frac{e^{k/n}}{(2\pi)^{1/2}} \frac{L(\varphi;s)}{L(\hat{\varphi};s)} \left| j_{\varphi\varphi}(\hat{\varphi}) \right|^{-1/2} ds$$

Like. ratio, Neg root info

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$$f(s;\varphi)ds \doteq \frac{e^{k/n}}{(2\pi)^{1/2}} \frac{L(\varphi;s)}{L(\hat\varphi;s)} |j_{\varphi\varphi}(\hat\varphi)|^{-1/2} ds$$ Like. ratio, Neg root info

Change var $t = \int_{s^0}^{s} |j_{\varphi\varphi}(\hat\varphi)|^{-1/2} ds$

Change par

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Change var $\quad t = \int_{s^0}^{s} \left| j_{\varphi\varphi}(\hat\varphi) \right|^{-1/2} ds$

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$$\frac{L}{\hat L} = e^{-(t-\tau)^2/2} + \quad \text{Cross term is } t\tau$$

2nd order

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Change par $\quad \tau = \int_{\varphi^0}^{\varphi} \left|j_{\varphi\varphi}(\varphi)\right|^{1/2} d\varphi$

$g(t;\tau)\,dt \doteq g(t-\tau)$ is location (& Bayes works) $\quad$ 2nd order

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order $O(1)$

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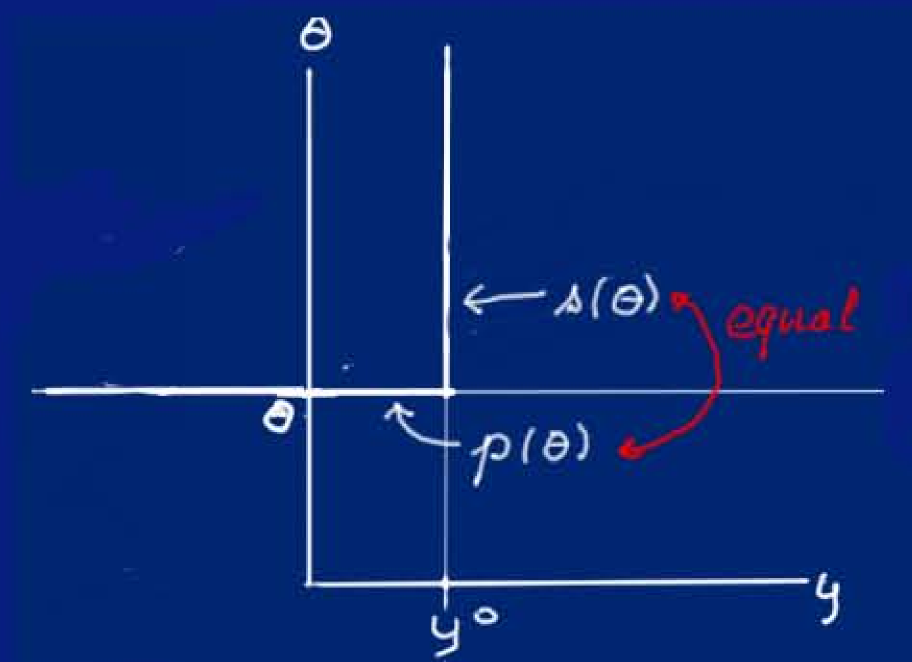

OK for location models : $f(t-\tau)$ 2nd

$p(\theta) \equiv s(\theta)$ ... equal ... reproducibility !

4 Welch-Peers extended $\dim\theta = p$ Scalar Interest (nuisance λ dim $p-1$

Regular model $\Rightarrow$ Exponential model $\Rightarrow$ SP approximation ... retain 3rd order!

[4]

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$$f(s;\varphi)\,ds \doteq \frac{e^{k/n}}{(2\pi)^{p/2}} \frac{L(\varphi;s)}{L(\hat\varphi;s)} \left| j_{\varphi\varphi}(\hat\varphi) \right|^{-1/2} \cdot ds$$

$j_{\varphi\varphi} = p \times p$ matrix SP!

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For testing $\psi(\theta) = \psi$ there are ancillary contours; integrate over them; via Laplace

$$g(s;\psi)\,ds = \frac{e^{k/n}}{(2\pi)^{1/2}} \frac{L(\hat\varphi_\psi;s)}{L(\hat\varphi;s)} \cdot \left| j_{(\lambda\lambda)}(\hat\varphi_\psi) \right|^{1/2} \cdot \left| j_{\varphi\varphi}(\hat\varphi) \right|^{-1/2} ds$$

On line $\mathcal{L} = \{ \hat{j}_{(\lambda\lambda)} = \hat{j}^{0}_{(\lambda\lambda)} \}$

3rd Unique via continuity

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On line $L=\{\hat j_{(\lambda\lambda)} = \hat j^{0}_{(\lambda\lambda)}\}$

3rd Unique via continuity

split this

p-value $$p(\psi) = \int^{s^0}\frac{e^{k/n}}{(2\pi)^{1/2}}\frac{L(\hat\varphi_\psi;s)}{L(\hat\varphi;s)}\frac{\left|j_{(\lambda\lambda)}(\hat\varphi_\psi)\right|^{1/2}}{\left|j_{(\lambda\lambda)}(\hat\varphi)\right|^{1/2}}\left|j^{\psi\psi}(\hat\varphi)\right|^{1/2}\cdot ds$$ 3rd

1-dim Integration

Dist'n available

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3rd 1-dim Integration

Dist'n available

Do Welch-Peers

Do Welch Peers on ψ

$$s^*(\psi)=\int_{\psi}^{}\frac{e^{k/n}}{(2\pi)^{1/2}}\frac{L(\hat\varphi_\psi;s)}{L(\hat\varphi;s)}\frac{\left|j_{(\lambda\lambda)}(\hat\varphi_\psi)\right|^{1/2}}{\left|j_{(\lambda\lambda)}(\hat\varphi)\right|^{1/2}}\left|j^{(\psi\psi)}(\varphi)\right|^{1/2}\cdot d(\psi)$$

Location to 2nd

Use Welch-Peers 2nd

Requires φ having $\hat j^{\,0}_{\varphi\varphi}=I$

(for computation)

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<u>Regular model</u> $\Rightarrow$ <u>Exponential model</u> $\Rightarrow$ <u>SP</u> approximation ... retain 3rd order! [4]

$$f(s;\varphi)\,ds \doteq \frac{e^{k/n}}{(2\pi)^{p/2}} \frac{L(\varphi;s)}{L(\hat\varphi;s)} \left|j_{\varphi\varphi}(\hat\varphi)\right|^{-1/2} \cdot ds$$

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On line $L = \{\hat j_{(\lambda\lambda)} = \hat j^{0}_{(\lambda\lambda)}\}$
3rd Unique via continuity

p-value $$p(\psi) = \int^{s^0} \frac{e^{k/n}}{(2\pi)^{1/2}} \frac{L(\hat\varphi_\psi;s)}{L(\hat\varphi;s)} \frac{\left|j_{(\lambda\lambda)}(\hat\varphi_\psi)\right|^{1/2}}{\left|j_{(\lambda\lambda)}(\hat\varphi)\right|^{1/2}} \left|j^{(\psi\psi)}(\hat\varphi)\right|^{1/2} \cdot ds$$

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Location to 2nd

Use Welch-Peers <u>2nd</u>
Requires φ having $\hat j^{0}_{\varphi\varphi} = I$
(for computation)

Put Together

$$= \int_\psi c \, \frac{L(\hat\varphi_\psi;s)}{L(\hat\varphi;s)} \left|j_{\varphi\varphi}(\varphi_\psi)\right|^{1/2} d(\psi)$$

Get <u>New Prior</u>
<u>full Jeffrey on profile for ψ</u>

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$$g(s;\psi)\,ds = \frac{e^{k/n}}{(2\pi)^{1/2}} \frac{L(\hat\varphi_\psi;s)}{L(\hat\varphi;s)} \cdot \left|j_{(\lambda\lambda)}(\hat\varphi_\psi)\right|^{1/2} \cdot \underbrace{\left|j_{\varphi\varphi}(\hat\varphi)\right|^{-1/2}}_{\text{split this}} ds$$

On line $L = \{\hat j_{(\lambda\lambda)} = \hat j^{\,0}_{(\lambda\lambda)}\}$
3rd Unique via continuity

p-value $$p(\psi) = \int^{s^0} \frac{e^{k/n}}{(2\pi)^{1/2}} \frac{L(\hat\varphi_\psi;s)}{L(\hat\varphi;s)} \frac{\left|j_{(\lambda\lambda)}(\hat\varphi_\psi)\right|^{1/2}}{\left|j_{(\lambda\lambda)}(\hat\varphi)\right|^{1/2}} \left|j^{(\psi\psi)}(\hat\varphi)\right|^{1/2} \cdot ds$$

3rd 1-dim Integration
Dist'n available

Do Welch-Peers

Do Welch Peers on ψ

$$s^*(\psi) = \int_\psi \frac{e^{k/n}}{(2\pi)^{1/2}} \frac{L(\hat\varphi_\psi;s)}{L(\hat\varphi;s)} \frac{\left|j_{(\lambda\lambda)}(\hat\varphi_\psi)\right|^{1/2}}{\left|j_{(\lambda\lambda)}(\hat\varphi)\right|^{1/2}} \left|j^{(\psi\psi)}(\varphi)\right|^{1/2} \cdot d(\psi)$$

Location to 2nd

Use Welch-Peers 2nd
Requires φ having $\hat j^{\,0}_{\varphi\varphi} = I$
(for computation)

Put Together

$$= \int_\psi c \,\frac{L(\hat\varphi_\psi;s)}{L(\hat\varphi;s)} \left|j_{\varphi\varphi}(\varphi_\psi)\right|^{1/2} d(\psi)$$

Get New Prior
full Jeffrey on profile for ψ

$$= c\int_\psi \frac{L(\hat\varphi_\psi;s)}{L(\hat\varphi;s)} \left|j_{\varphi\varphi}(\varphi_\psi)\right|^{1/2} \frac{d(\psi)}{d\psi} \cdot d\psi$$

Jacobian

4 Welch-Peers extended $\dim\theta = p$ Scalar Interest (nuisance λ dim $p-1$

Regular model ⇒ Exponential model ⇒ SP approximation ... retain 3rd order! [4]

$$f(s;\varphi)ds \doteq \frac{e^{k/n}}{(2\pi)^{p/2}} \frac{L(\varphi;s)}{L(\hat\varphi;s)} |j_{\varphi\varphi}(\hat\varphi)|^{-1/2} \cdot ds$$

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split this

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3rd 1-dim Integration Dist'n available

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(for computation)

Put together $$= \int_\psi c \frac{L(\hat\varphi_\psi;s)}{L(\hat\varphi;s)} |j_{\varphi\varphi}(\varphi_\psi)|^{1/2} d(\psi)$$

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full Jeffrey on profile for ψ

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Jacobian

From iterative calculation of constrained mle $\hat\varphi_\psi$

$$d(\psi) = \left|\frac{d\hat\varphi_\psi}{d\psi}\right| \cos\left\{\frac{d\hat\varphi_\psi}{d\psi}, \frac{d\,\psi(\varphi)}{d\psi}\right\} d\psi$$

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[4]

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Get New Prior full Jeffrey on profile for ψ

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$$= \left\{\frac{d\hat\varphi_\psi}{d\psi} \cdot \frac{d\psi}{d\varphi} \Big/ \left|\frac{d\psi}{d\varphi}\right|\right\} \quad \leftarrow \text{Inner product}$$

gradient of $\psi(\varphi)$

$$\boxed{\begin{array}{l}\text{Use full Jeffreys on profile curve}\\ \text{Get full 2nd order reproducibility}\end{array}}$$

Example 1 p-values and s-values (posterior survival)

Model: Standard Normal location on plane: $\phi(s_1 - \varphi_1, s_2 - \varphi_2)$

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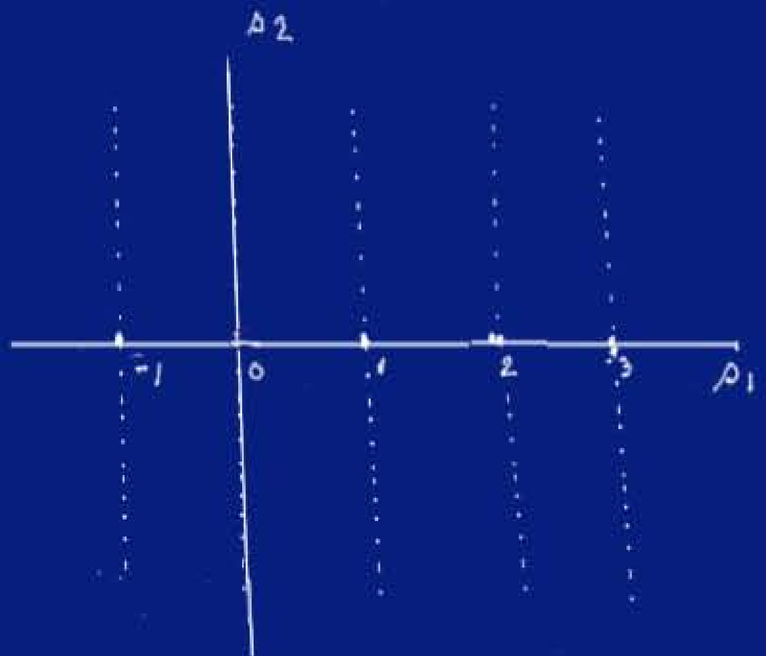
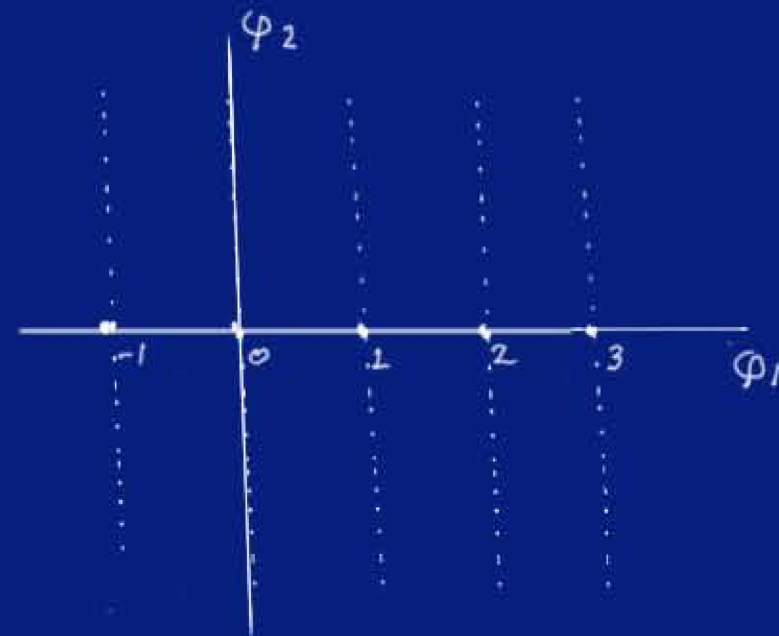
Linear contours ......

On S space, contours $O(n^{-1})$



Canonical variable



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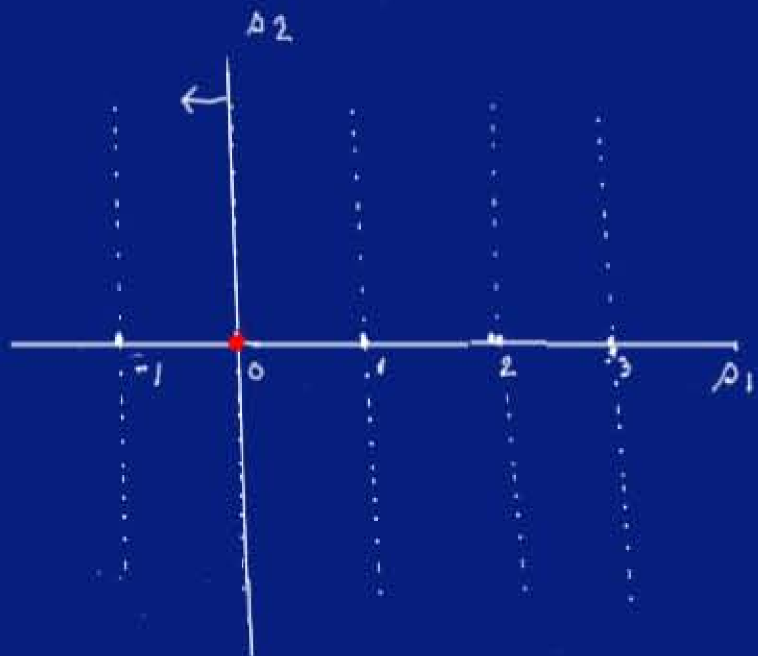

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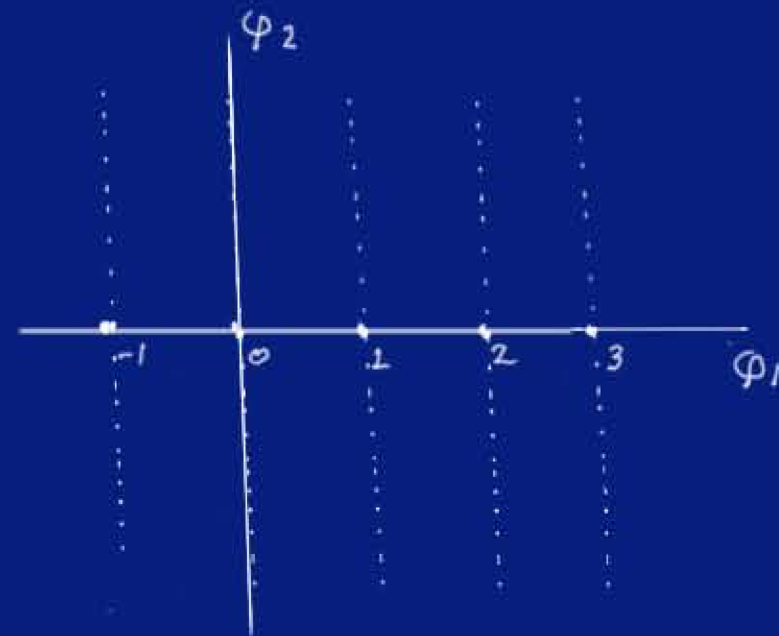

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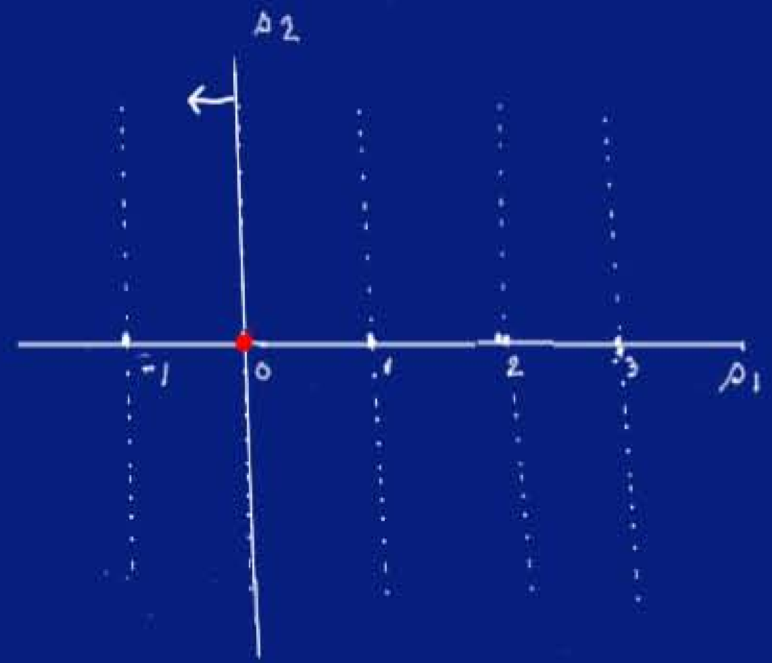

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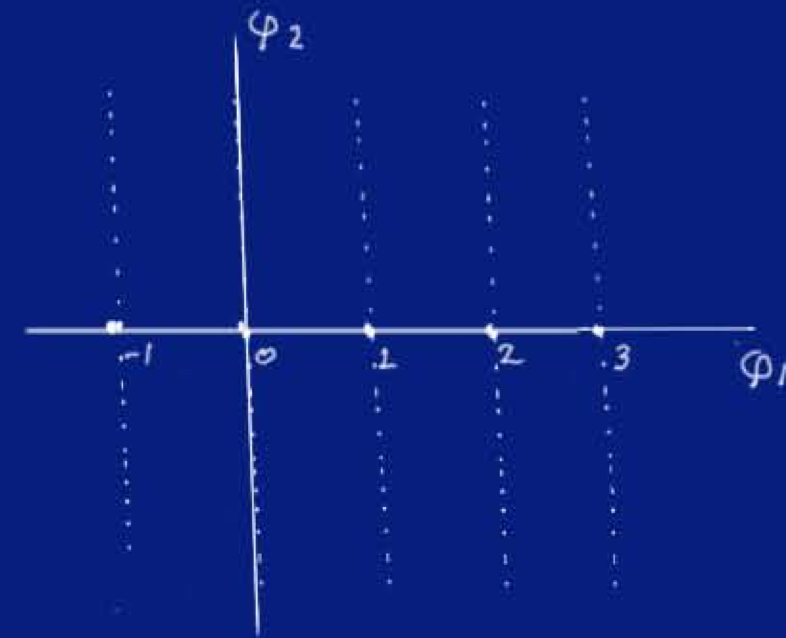

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	ψ	-1	0	1	2
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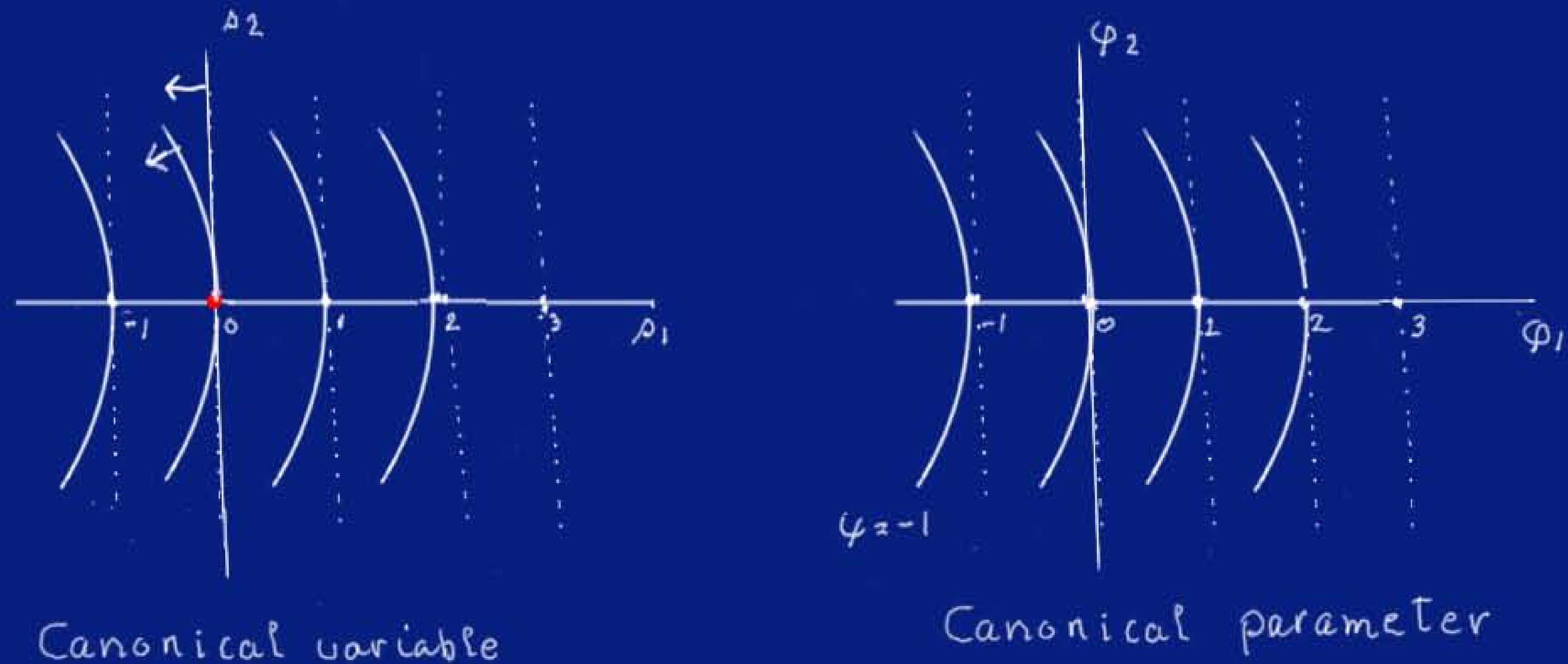

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On S space, contours $O(n^{-1})$

Curved contours: ——

Curved: $\psi=\varphi_1+\gamma\varphi_2^2/2$

Try $\gamma=.3$

$\psi=\varphi_1+.15\varphi_2^2$

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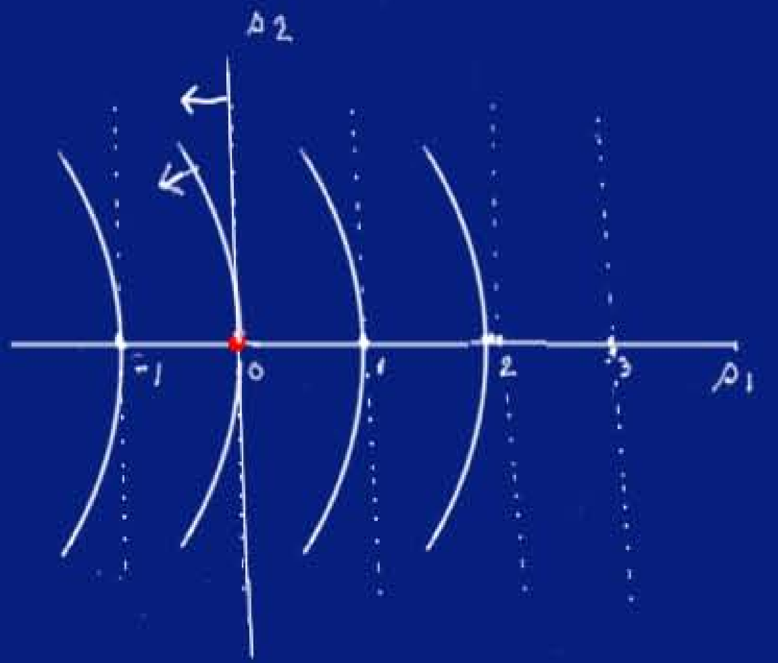

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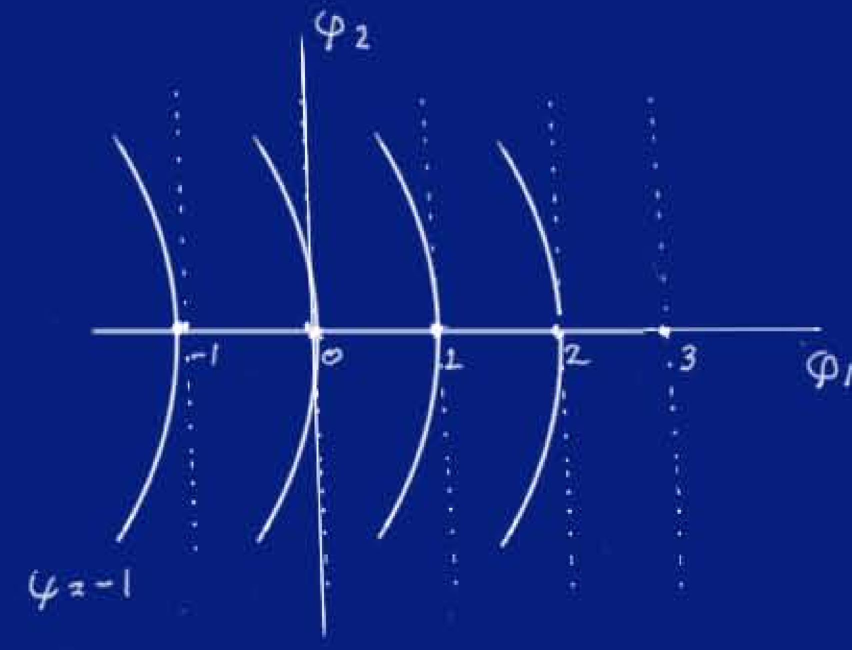

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$s(\psi)$		?		
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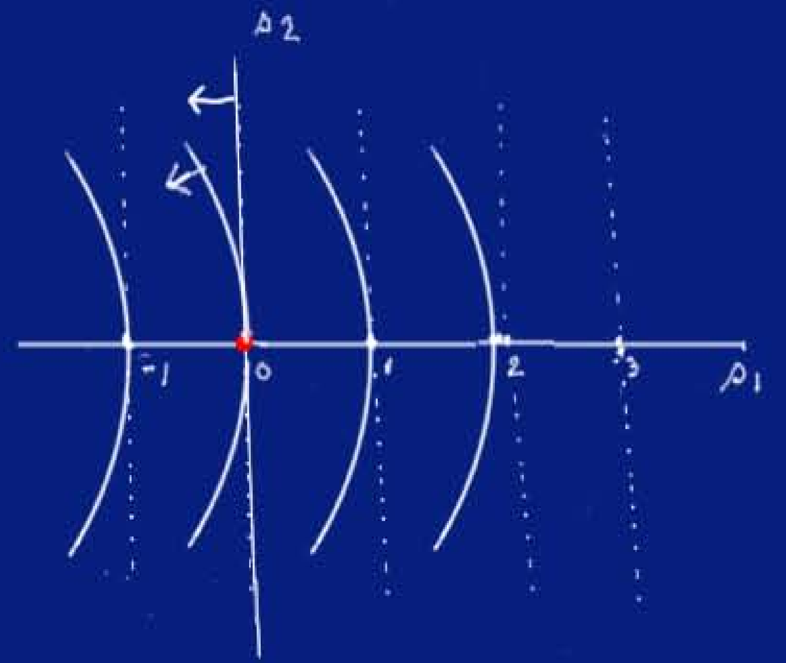

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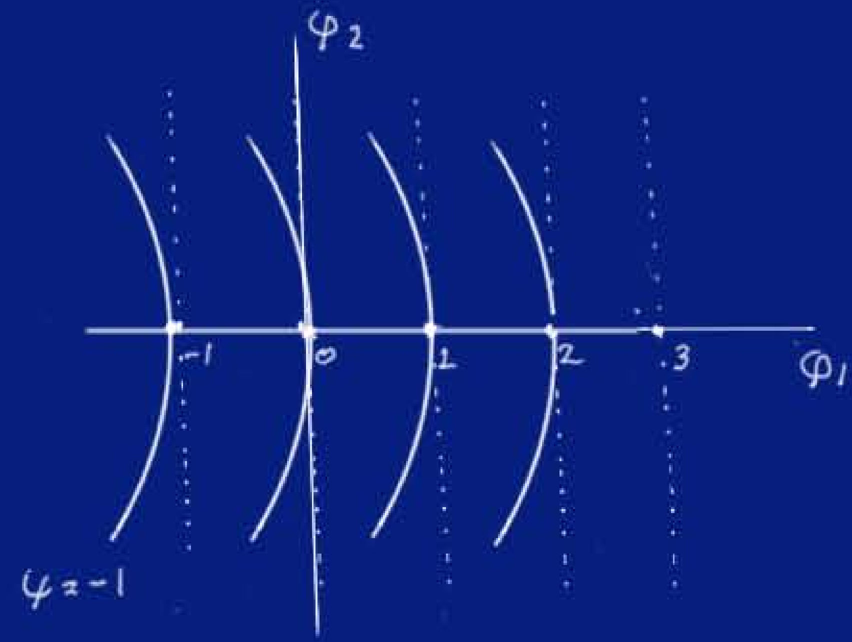

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Curved ψ

ψ	-1	0	1	2	
$p(\psi)$	.80	.44	.13	.02	
$s(\psi)$	.87	.56	.20	.03	← inflated
$s^*(\psi)$	.80	.44	.13	.02	← deflated

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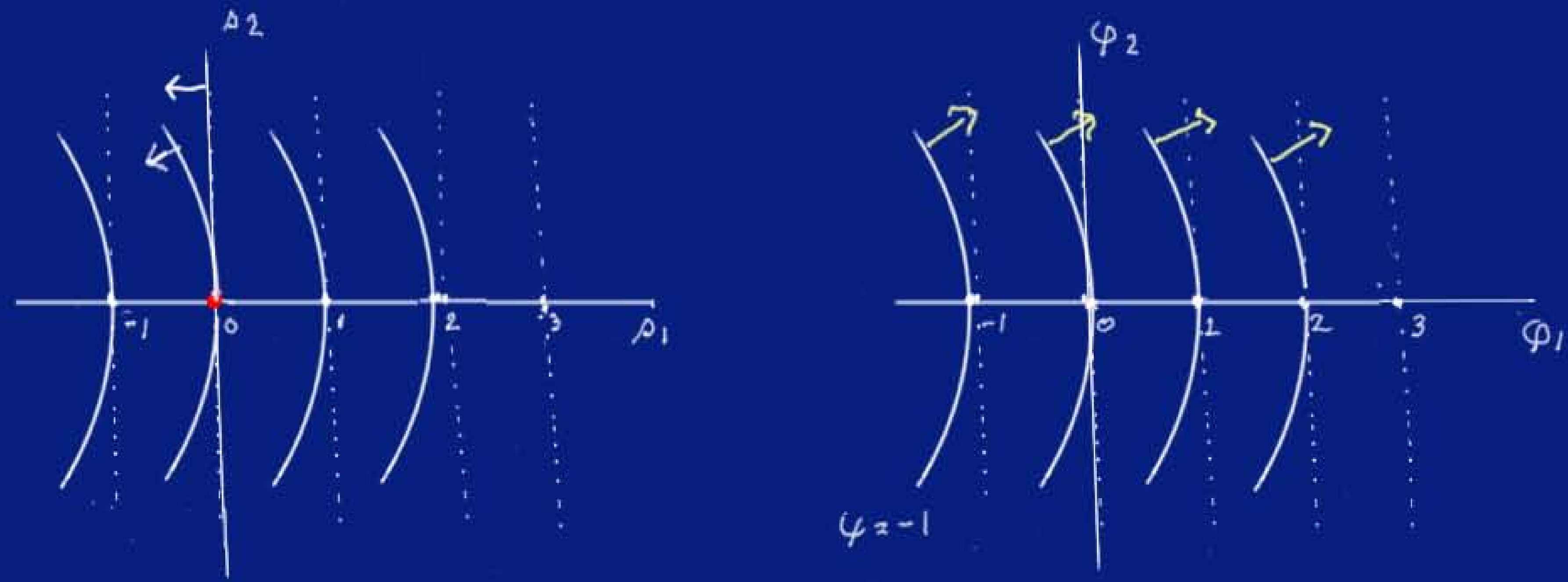

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Bayes survival values are inflated
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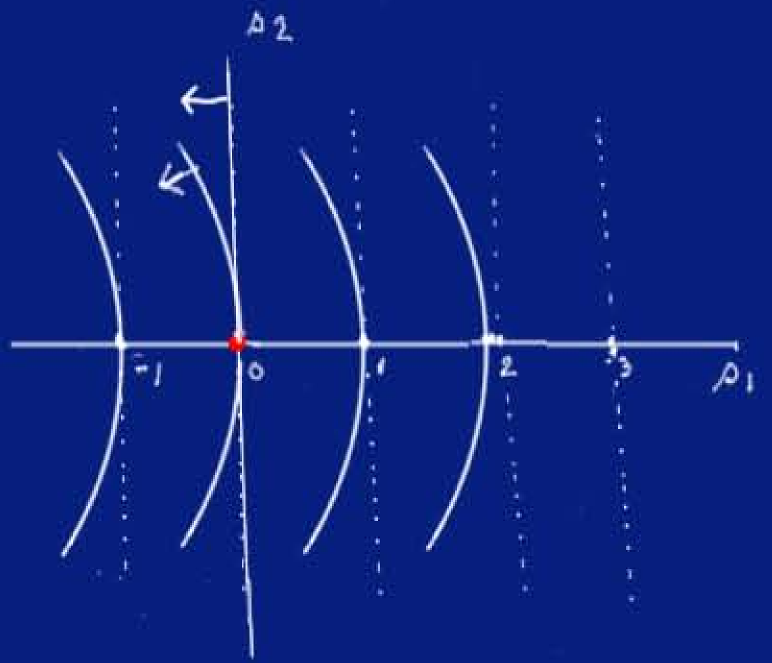

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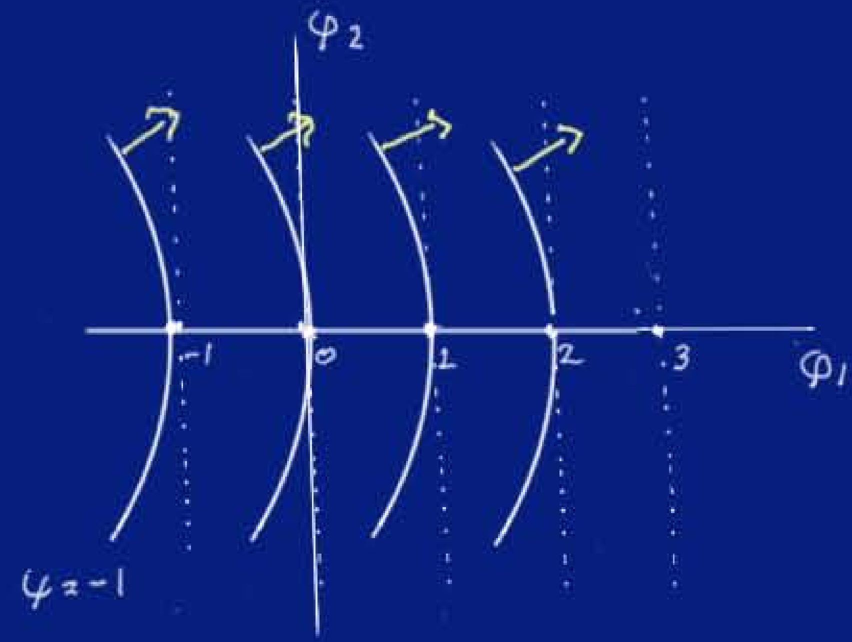

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Bayes survival values are <u>inflated</u>
when parameter contours are <u>cupped left</u>!

But <u>easy</u> correction

$$\boxed{s^*(\psi) = 2s^L(\psi) - s(\psi)}$$

2nd order (round off)

Example 2 Gamma model

$$f(y;\alpha,\beta) = \Gamma^{-1}(\alpha)\, \beta^{\alpha}\, y^{\alpha-1} \exp\{-\beta y\}$$

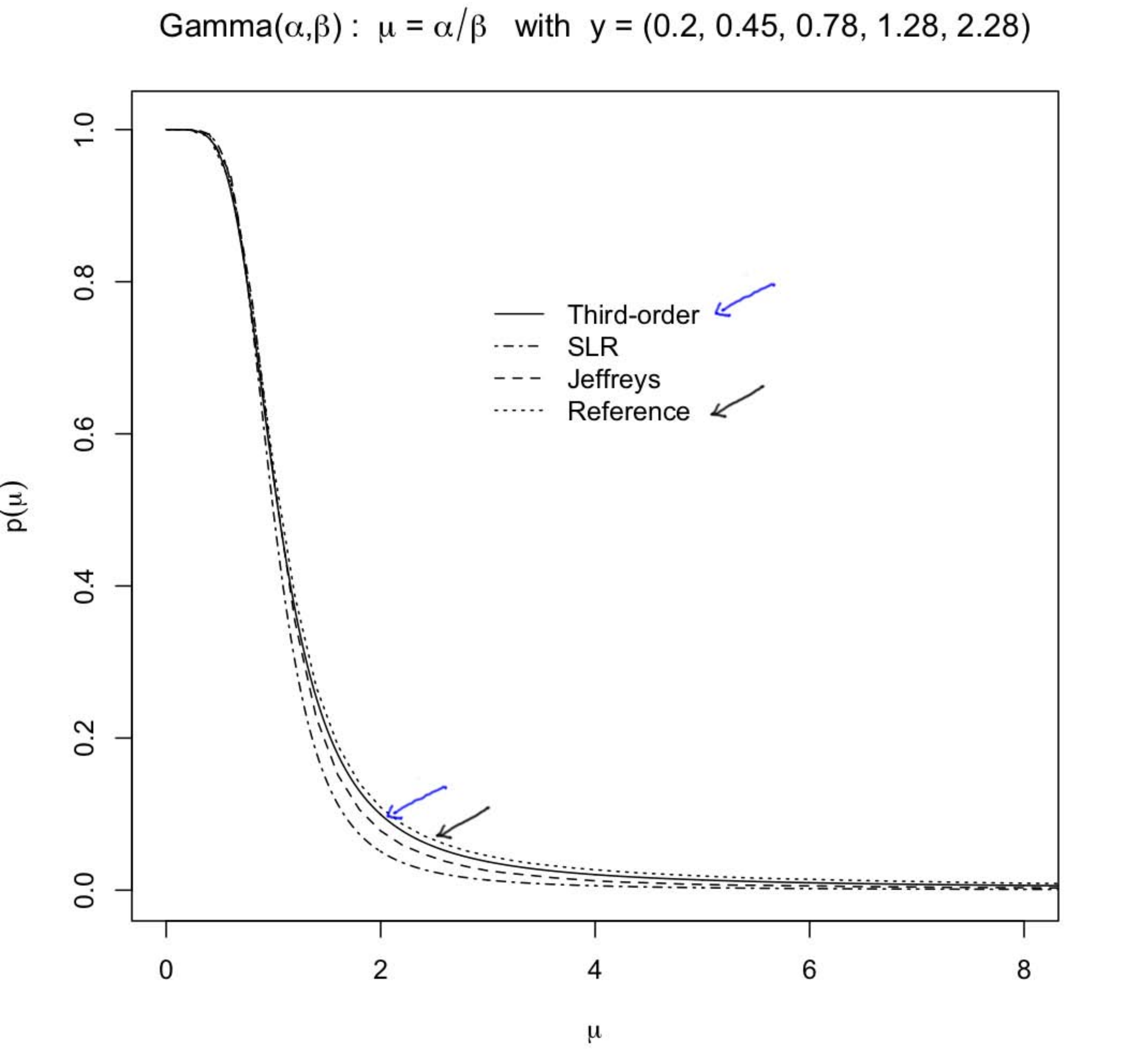
Interest parameters:

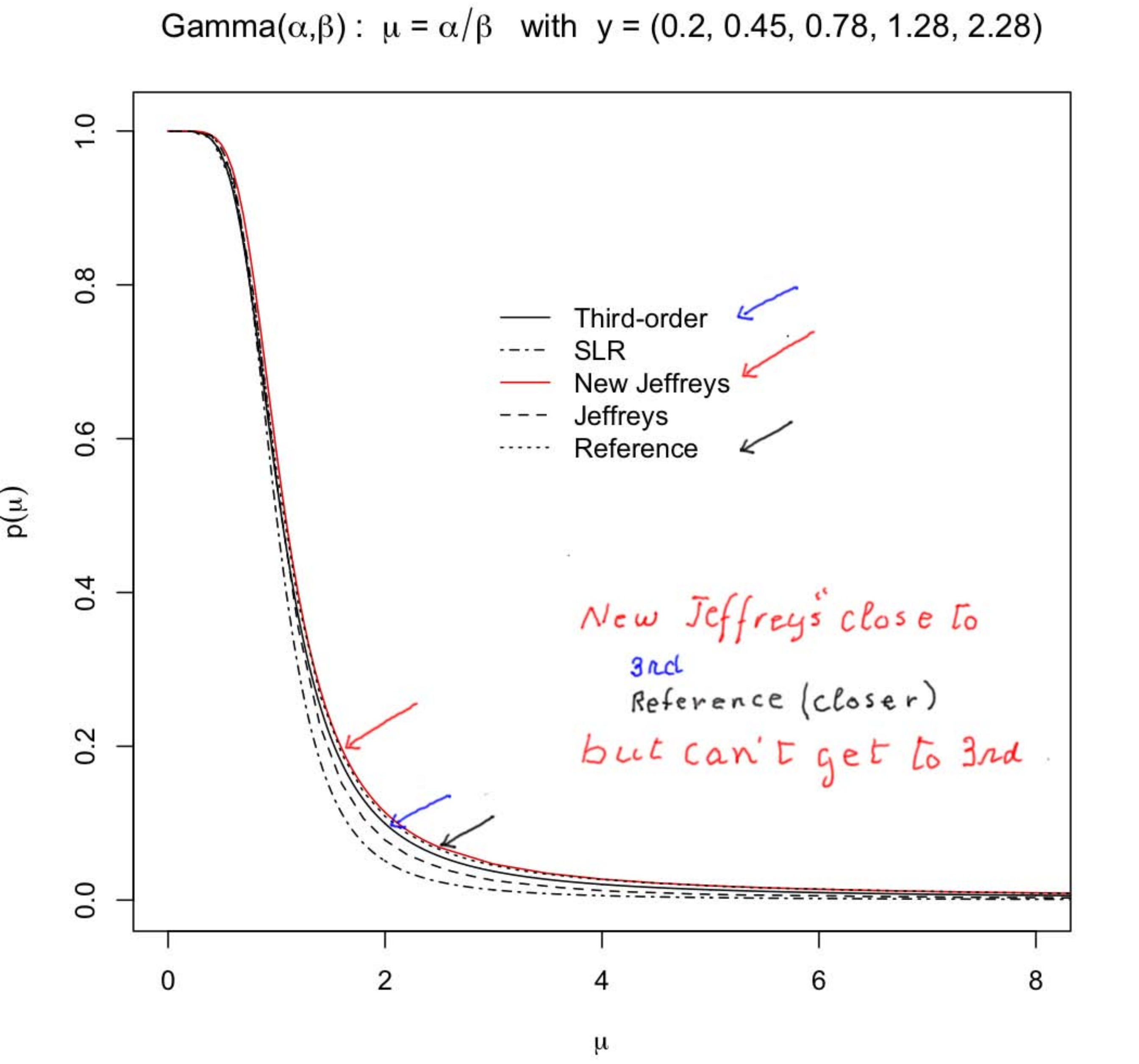
(a) <u>mean</u> $E(y) = \alpha/\beta$

<u>rotating</u> on canonical parameter space

(b) <u>variance</u> $\operatorname{var}(y) = \alpha/\beta^2$

<u>curved</u> on canonical parameter space


Gamma(α,β) : μ = α/β with y = (0.2, 0.45, 0.78, 1.28, 2.28)
Third-order
SLR
Jeffreys
Reference
p(μ)
μ
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8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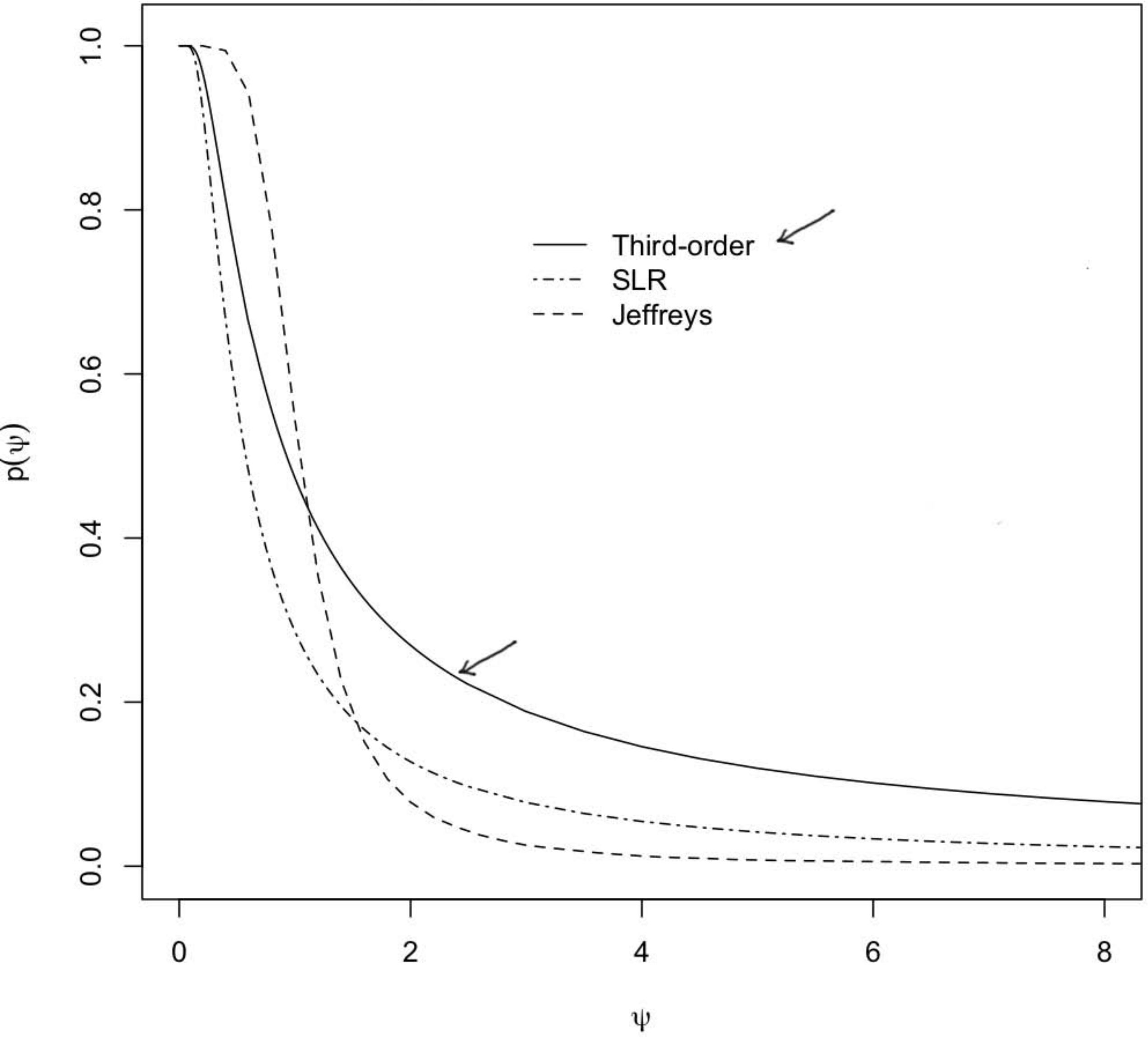
Third-order
SLR
New Jeffreys
Jeffreys
Reference
New Jeffreys" close to
3rd
Reference (closer)
but can't get to 3rd
p(μ)
μ
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8

Gamma(α,β) : $\psi = \alpha/\beta^2$ with y = (0.2, 0.45, 0.78, 1.28, 2.28)

Curved



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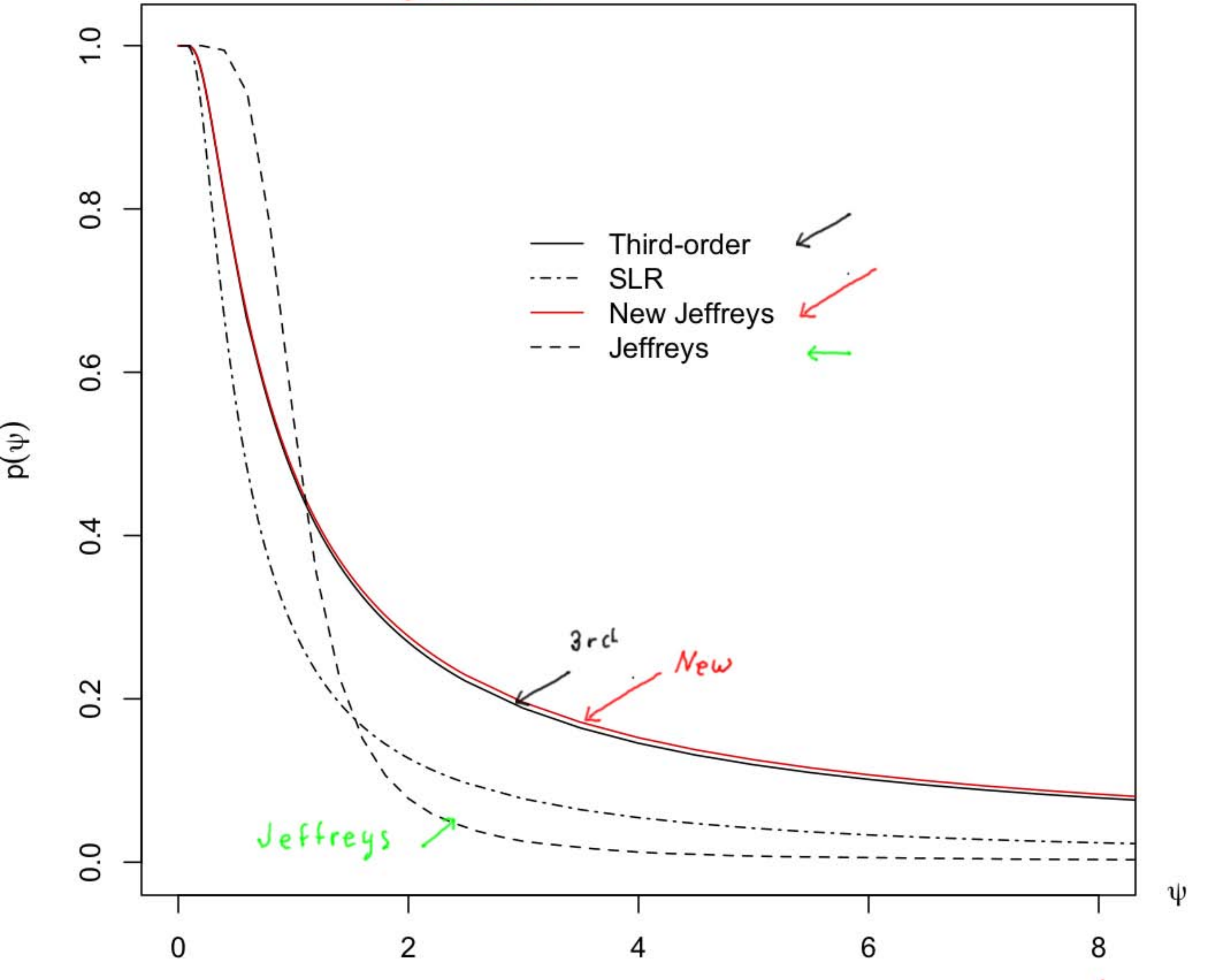

New corrects curvature effects!

BFF schema

1 $f(y;\theta)$ regular

2 $y = y(z;\theta)$ Exact Data generating / Structural / quantile Same

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4 $\varphi(\theta) = \frac{\partial \ell(\theta; y)}{\partial V}$ Canonical parameter of Expontial model 3rd

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on profile (3rd)

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on profile

7 $F(s;\psi) = \int^{s} f(s;\psi)\, ds = \Phi(r^*_\psi) = p$-value fn $= p^0(\psi)$ (3rd)

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2 $y = y(z;\theta)$ Exact Data generating / Structural / quantile Same

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4 $\varphi(\theta) = \frac{\partial \ell(\theta;y)}{\partial V}$ Canonical parameter of Exponential model (3rd)

5 $f(s;\varphi)\,ds = c\,\frac{L^0(\varphi)}{L^0(\hat{\varphi})}\,|j_{\varphi\varphi}(\varphi)|^{-1/2}\,ds$ SP Tilt $\ell^0(\varphi) + s\varphi$ (3rd)

6 $\theta = (\psi, \lambda)$ scalar ψ $f(s;\psi)\,ds = c\,\frac{L^0(\varphi)}{L^0(\hat{\varphi})}\,|j_{\varphi\varphi}(\hat{\varphi})|^{-1/2}\,|J_{(\lambda\lambda)}(\hat{\varphi}_\psi)|^{1/2}\,ds$ on profile (3rd)

7 $F(s;\psi) = \int^s f(s;\psi)\,ds = \Phi(r^*_\psi)$ = p-value fn $= p^0(\psi)$ (3rd)

8 Bayes survivor $= s(\psi) = \int_\psi^\infty L^0(\psi)\,\underbrace{|j_{\varphi\varphi}(\varphi)|^{1/2}\frac{d(\psi)}{d\psi}}_{\text{New prior}}\cdot d\psi$ (2nd)

BFF schema

1 $f(y;\theta)$ regular

2 $y = y(z;\theta)$ <u>Exact</u> Data generating / Structural / quantile <u>Same</u>

3 $V = \frac{dy}{d\theta}\Big|_{y^0 \hat{\theta}^0}$ $n \times p$ Conditioning directions / Tangent to ancillary 2nd

4 $\varphi(\theta) = \frac{\partial \ell(\theta; y)}{\partial V}$ canonical parameter of Exponential model 3rd

5 $f(\underset{\sim}{s};\varphi)\, d\underset{\sim}{s} = c \frac{L^0(\varphi)}{L^0(\hat{\varphi})} \left| j_{\varphi\varphi}(\varphi) \right|^{-1/2} d\underset{\sim}{s}$ SP Tilt $\ell^0(\varphi) + s\varphi$ 3rd

6 $\theta = (\psi, \lambda)$ scalar ψ $f(s;\psi)\, ds = c \frac{L^0(\varphi)}{L^0(\hat{\varphi})} \left| j_{\varphi\varphi}(\hat{\varphi}) \right|^{-1/2} \left| J_{(\lambda\lambda)}(\hat{\varphi}_\psi) \right|^{1/2} ds$ 3rd

on profile

7 $F(s;\psi) = \int^{s} f(s;\psi)\, ds = \Phi(r^*_\psi) = p\text{-value fn} = p^0(\psi)$ 3rd

8 Bayes survivor $= s(\psi) = \int_\psi^\infty L^0(\psi) \underbrace{\left| j_{\varphi\varphi}(\varphi) \right|^{1/2} \frac{d(\psi)}{d\psi}}_{\text{New prior}} \cdot d\psi$ 2nd

9. Bayes can't get to 3rd

Suggestions:

Vector parameter posterior:

Highly unreliabe, avoid! Examples 1, 2a, 2b, 3

Scalar parameter posterior:

Don't integrate a vector case

Use <u>full</u> Jeffreys on scalar <u>profile</u> contour!

$$L(\hat{\varphi}_\psi)\,|j_{\varphi\varphi}(\hat{\varphi}_\psi)|^{1/2}\,d(\psi)$$ Asymptotic

<u>Always</u> report to the client the <u>reliability</u> of any processing

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Thank you