

## A Description of the PX-HC algorithm

Let  $N = \sum_{c=1}^C N_c$  and write  $\sum_{c=1}^C \sum_{i=1}^{N_c} \sum_{k=1}^{K_{ci}}$  as  $\sum_{c,i,k}$ , the Gibbs sampling algorithm at iteration  $m$  for continuous outcomes:

**Step A:** For  $j = 1, \dots, J$ , draw  $\boldsymbol{\theta}^{(m)}$  in the following steps:

**A1.**

$$\lambda_j^{*(m)} \sim N \left( \boldsymbol{\Omega}_{\lambda_j} \sum_{c,i,k} \left[ y_{cikj}^c - W_{cik}^T \boldsymbol{\beta}_j^{(m-1)} - \xi_j^{(m-1)} b_{cij}^{*(m-1)} \right] \frac{U_{cik}^{*(m-1)}}{\sigma_j^{2(m-1)}}, \boldsymbol{\Omega}_{\lambda_j} \right) \mathbf{1}_{\{\lambda_j^{*(m)} \geq 0\}},$$

$$\text{where } \boldsymbol{\Omega}_{\lambda_j} = \left[ \sum_{c,i,k} \frac{U_{cik}^{*2(m-1)}}{\sigma_j^{2(m-1)}} + 1 \right]^{-1}.$$

**A2.**

$$\boldsymbol{\beta}_j^{(m)} \sim N \left( \boldsymbol{\Omega}_{\boldsymbol{\beta}_j} \sum_{c,i,k} \left[ y_{cikj}^c - \lambda_j^{*(m)} U_{cik}^{*(m-1)} - \xi_j^{(m-1)} b_{cij}^{*(m-1)} \right] \frac{W_{cik}}{\sigma_j^{2(m-1)}}, \boldsymbol{\Omega}_{\boldsymbol{\beta}_j} \right),$$

where  $\boldsymbol{\Omega}_{\boldsymbol{\beta}_j} = \left( \frac{1}{\sigma_j^{2(m-1)}} \sum_{c,i,k} W_{cik} W_{cik}^T + \boldsymbol{\Sigma}_{\boldsymbol{\beta}}^{-1} \right)^{-1}$ , and  $\boldsymbol{\Sigma}_{\boldsymbol{\beta}} = 1000 \mathbf{I}_{p_1}$  is the prior variance for  $\boldsymbol{\beta}_j$ .

**A3.**

$$\xi_j^{(m)} \sim N \left( \boldsymbol{\Omega}_{\xi_j} \sum_{c,i,k} \left[ y_{cikj}^c - W_{cik}^T \boldsymbol{\beta}_j^{(m)} - \lambda_j^{*(m)} U_{cik}^{*(m-1)} \right] \frac{b_{cij}^{*(m-1)}}{\sigma_j^{2(m-1)}}, \boldsymbol{\Omega}_{\xi_j} \right),$$

$$\text{where } \boldsymbol{\Omega}_{\xi_j} = \left( \sum_{c,i} \frac{K_{ci} b_{cij}^{*2(m-1)}}{\sigma_j^{2(m-1)}} + 1 \right)^{-1}$$

**A4.**

$$\psi^{2(m)} \sim IG(A_1^{(m)}, A_2^{(m)}), \text{ where } A_1^{(m)} = \frac{\sum_{c=1}^C \sum_{i=1}^{N_c} K_{ci}}{2} + \frac{1}{2}, \text{ and}$$

$$A_2^{(m)} = \frac{1}{2} \sum_{c,i} \left( A_{ci}^{(m)T} H_{ci}^{-1}(\rho^{(m-1)}) A_{ci}^{(m)} \right) + \frac{1}{2},$$

$$A_{ci}^{(m)} = \mathbf{U}_{ci}^{*(m-1)} - \mu^{*(m-1)} \mathbf{1}_{K_{ci}} - X_{ci} \boldsymbol{\alpha}^{*(m-1)} - \mathbf{1}_{K_{ci}} \boldsymbol{\vartheta}_c^{*(m-1)} - Z_{ci} \otimes \mathbf{1}_{K_{ci}} a_c^{*(m-1)}.$$

**A5.**

$$\boldsymbol{\alpha}^{*(m)} \sim N(\boldsymbol{\mu}_{\boldsymbol{\alpha}}, \boldsymbol{\Omega}_{\boldsymbol{\alpha}}),$$

where

$$\begin{aligned}\mu_\alpha &= \sum_{c,i} X_{ci}^T H_{ci}^{-1}(\rho^{(m-1)}) \left( \mathbf{U}_{ci}^{*(m-1)} - \mu^{*(m-1)} \mathbf{1}_{K_{ci}} - Z_{ci} \otimes \mathbf{1}_{K_{ci}} a_c^{*(m-1)} - g_c^{*(m-1)} \mathbf{1}_{K_{ci}} \right) \\ &\quad \times \Omega_\alpha \frac{1}{\psi^2}, \\ \Omega_\alpha &= \left( \frac{1}{\psi^2} \sum_{c,i} X_{ci}^T H_{ci}^{-1}(\rho^{(m-1)}) X_{ci} + \Sigma_\alpha^{-1} \right)^{-1}.\end{aligned}$$

**A6.**

$$\mu^* \sim N(\mu_\mu, \Omega_\mu),$$

where

$$\begin{aligned}\mu_\mu &= \sum_{c,i} \left( \mathbf{U}^{*(m-1)} - X_{ci} a_c^{*(m-1)} - g_c^{*(m-1)} \mathbf{1}_{K_{ci}} - Z_{ci} \otimes \mathbf{1}_{K_{ci}} a_c^{*(m-1)} \right)^T H_{ci}^{-1}(\rho^{(m-1)}) \mathbf{1}_{K_{ci}} \\ &\quad \times \frac{\Omega_\mu}{\psi^2},\end{aligned}$$

$$\text{and } \Omega_\mu = \left[ \frac{1}{\psi^2} \sum_{c,i} \mathbf{1}_{K_{ci}} H_{ci}^{-1} \mathbf{1}_{K_{ci}} + \frac{1}{1000} \right].$$

**A7.** For  $j = 1, \dots, J$ ,

$$\mu_{bj}^{*(m)} \sim N \left( \left( \frac{C}{\sum_{c=1}^C \sum_{i=1}^{N_c} \frac{b_{cij}^{*(m-1)}}{(\tau_j^{*(m-1)})^2}} \right) \left( \frac{N}{(\tau_j^{*(m-1)})^2} + \frac{1}{1000} \right)^{-1}, \left( \frac{N}{(\tau_j^{*(m-1)})^2} + \frac{1}{1000} \right)^{-1} \right).$$

**A8.** For  $j = 1, \dots, J$ ,

$$\tau_j^{*2(m)} \sim IG \left( \frac{N}{2} + \frac{1}{2}, \frac{1}{2} \sum_{c=1}^C \sum_{i=1}^{N_c} (b_{cij}^{*(m-1)} - \mu_{bj}^{*(m)})^2 + \frac{1}{2} \right).$$

**A9.**

$$\sigma_g^{*2} \sim IG \left( \frac{C}{2} + 0.1, \frac{\sum_{c=1}^C (g_c^{*(m-1)})^2}{2} + 0.1 \right).$$

**A10.**

$$\sigma_a^{*2} \sim IG \left( \frac{N}{2} + 0.1, \sum_{c=1}^C \frac{(a_c^{*(m-1)})^T a_c^{*(m-1)}}{2} + 0.1 \right).$$

**A11.** For  $j = 1, \dots, J$ ,

$$\sigma_j^{2(m)} \sim IG \left( \frac{\sum_{c=1}^C \sum_{i=1}^{N_c} K_{ci}}{2} + 0.1, \frac{1}{2} \sum_{c,i,k} \left( y_{cijk}^c - W_{cik}^T \beta_j^{(m)} - \lambda_j^{*(m)} U_{cik}^{*(m-1)} - \xi_j^{(m)} b_{cij}^{*(m-1)} \right)^2 + 0.1 \right)$$

**A12.**

$$p(\rho^{(m)} | \dots) \propto \prod_{c=1}^C \prod_{i=1}^{N_c} |H_{ci}(\rho)|^{-1/2} \exp \left\{ -\frac{1}{2\psi^2} \sum_{c=1}^C \sum_{i=1}^{N_c} A_{ci}^{(m)T} H_{ci}^{-1}(\rho) A_{ci}^{(m)} \right\}.$$

where

$$A_{ci}^{(m)} = \mathbf{U}_{ci}^{(m-1)} - \mu^* - X_{ci}\alpha^{(m)} - g_c^{(m-1)}\mathbf{1}_{K_{ci}} - Z_{ci} \otimes \mathbf{1}_{T_{ci}} a_c^{(m-1)},$$

and  $H_{ci}(\rho)$  is a  $K_{ci} \times K_{ci}$  matrix with  $(r, k)^{th}$  element  $\rho^{|t_{cir} - t_{cik}|}$ . We use parameter transformation to transform  $\rho$  to  $\eta$  via  $\eta = \log(\frac{\rho}{1-\rho})$ . Then the conditional distribution for posterior  $\eta$  is:

$$p(\eta|\dots) \propto \prod_{c=1}^C \prod_{i=1}^{N_c} |H_{ci}(\rho(\eta))|^{-1/2} \exp \left\{ -\frac{1}{2\psi^2} \sum_{c=1}^C \sum_{i=1}^{N_c} A_{ci}^{(m)T} H_{ci}^{-1}(\rho(\eta)) A_{ci}^{(m)} \right\} \frac{\exp(\eta)}{(1 + \exp(\eta))^2},$$

where  $\rho(\eta) = \frac{\exp(\eta)}{1 + \exp(\eta)}$ . Random walk Metropolis-Hastings algorithm is used to sample  $\eta$  with proposal  $N(\eta_{old}, v^2)$ , where  $v^2$  is tuned to have a reasonable acceptance rate.

**Step B** Sample all latent variables:

**B1.** For each  $\{c, i\}$ ,  $\mathbf{U}_{ci}^{(m)} \sim \text{MVN}(\mu_{U_{ci}}, \Omega_{U_{ci}})$  with

$$\begin{aligned} \mu_{U_{ci}} &= \Omega_{U_{ci}} \left[ \sum_{j=1}^J \frac{\lambda_j^{*(m)}}{\sigma_j^{2(m)}} (\mathbf{y}_{cij}^c - \mathbf{W}_{ci} \beta_j^{(m)} - \xi_j^{(m)} b_{cij}^{(m)} \mathbf{1}_{K_{ci}}) \right] \\ &+ \Omega_{U_{ci}} \frac{1}{\psi^2} \left[ \mu^{*(m)} + X_{ci} \alpha^{*(m)} + g_c^{*(m-1)} \mathbf{1}_{K_{ci}} + Z_{ci} \otimes \mathbf{1}_{K_{ci}} a_c^{*(m-1)} \right]^T H_{ci}^{-1}(\rho^{(m)}) \end{aligned}$$

$$\text{and } \Omega_{U_{ci}} = \left( \sum_{j=1}^J \frac{\lambda_j^{*2(m)}}{\sigma_j^{2(m)}} I_{K_{ci}} + \frac{1}{\psi^2} H_{ci}^{-1}(\rho^{(m)}) \right)^{-1}.$$

**B2.** For each  $\{c, i\}$  draw

$$b_{cij}^{*(m)} \sim N \left( \Omega_{b_{cij}} \left[ \sum_{t=1}^{K_{ci}} \frac{\xi_j^{(m)}}{\sigma_j^{2(m)}} (y_{cikt}^c - W_{cik}^T \beta_j^{(m)} - \lambda_j^{*(m)} U_{cik}^{*(m)}) + \frac{\beta_{0j}^{*(m)}}{(\tau_j^{*(m)})^2} \right], \Omega_{b_{cij}} \right),$$

$$\text{where } \Omega_{b_{cij}} = \left( \frac{K_{ci} \xi_j^{2(m)}}{\sigma_j^{2(m)}} + \frac{1}{(\tau_j^{*(m)})^2} \right)^{-1}.$$

**B3.** For each  $c$ ,  $g_c^* \sim N(\mu_{g_c}, \Omega_{g_c})$  with

$$\mu_{g_c} = \frac{\Omega_{g_c}}{\psi^2} \sum_{i=1}^{N_c} \left( \mathbf{U}_{ci}^{*(m)} - \mu^{*(m)} \mathbf{1}_{K_{ci}} - \mathbf{X}_{ci} \alpha^{*(m)} - \mathbf{Z}_{ci} \otimes \mathbf{1}_{T_{ci}} a_c^{*(m-1)} \right)^T H_{ci}^{-1}(\rho^{(m)}) \mathbf{1}_{T_{ci}},$$

$$\text{and } \Omega_{g_c} = \left( \frac{1}{\psi^2} \sum_{i=1}^{N_c} \mathbf{1}_{T_{ci}} H_{ci}^{-1}(\rho^{(m)}) \mathbf{1}_{T_{ci}} + \frac{1}{\sigma_g^{*2}} \right)^{-1}.$$

**B4.** For  $c = 1, \dots, C$ , For each  $c$ ,  $a_c^{*(m)} \sim MVN(\mu_{a_c}, \Omega_{a_c})$  with

$$\mu_{a_c} = \frac{\Omega_{a_c}}{\psi^2} \sum_{i=1}^{N_c} \left( Z_{ci} \otimes 1_{T_{ci}} \right)^T H_{ci}^{-1}(\rho^{(m)}) \left( \mathbf{U}_{ci}^{*(m)} - \mu^{*(m)} \mathbf{1}_{\mathbf{K}_{ci}} - \mathbf{X}_{ci} \alpha^{*(m)} - \mathbf{g}_c^{*(m)} \mathbf{1}_{\mathbf{T}_{ci}} \right),$$

$$\text{and } \Omega_{a_c} = \left( \frac{1}{\psi^2} \sum_{i=1}^{N_c} (Z_{ci} \otimes 1_{T_{ci}})^T H_{ci}^{-1}(\rho^{(m)}) (Z_{ci} \otimes 1_{T_{ci}}) + \frac{1}{\sigma_a^{*2(m)}} I_{N_c} \right)^{-1}.$$

## B Description of the PX<sup>2</sup>-HC algorithm

The following steps are used to produce the sampling updates:

**Step C** For all parameters that determine the continuous response and latent variable, the Gibbs steps are identical to the ones described in the previous section. Specifically, the conditional distributions used in the Gibbs updates are identical for  $\{(\lambda_j^*, \beta_j, \xi_j, b_{cij}^*, \sigma_j^2) : 1 \leq j \leq J_1\}$ ,  $\psi$ ,  $\alpha^*$ ,  $\mathbf{a}^*$ ,  $g^*$ ,  $\mu^*$ ,  $\boldsymbol{\mu}_b^*$ ,  $\sigma_a^{*2}$ ,  $\sigma_g^{*2}$ ,  $\boldsymbol{\tau}^{*2}$ ,  $\rho$ )

**Step D** For  $j = J_1 + 1, \dots, J$  the following conditional distributions are used to update the chain

**D1.** Draw

$$y_{cijk}^{b*(m)} \sim \begin{cases} TN_+(\mu_{cijk}^*, 1), & \text{if } y_{cijk}^b = 1 \\ TN_-(\mu_{cijk}^*, 1), & \text{if } y_{cijk}^b = 0 \end{cases},$$

where  $TN_+(\mu, \sigma^2)$  and  $TN_-(\mu, \sigma^2)$  are truncated normals with mean  $\mu$  and variance  $\sigma^2$  truncated to  $(0, \infty)$  and  $(-\infty, 0)$ , respectively. Also  $\mu_{cijk}^* = W_{cijk}^T \boldsymbol{\beta}_j^{(m)} + \lambda_j^{*(m)} U_{cijk}^{*(m)} + \xi_j^{(m)} b_{cij}^{*(m)}$ . Put  $\tilde{y}_{cijk}^{b*(m)} = \gamma_j^{(m-1)} y_{cijk}^{b*(m)}$ .

**D2.**

$$\gamma_j^{2(m)} \sim IG \left( \frac{\sum_{c,i} K_{ci}}{2} + 0.1, \sum_{c,i,k} \left( \tilde{y}_{cijk}^{b*(m)} - W_{cijk}^T \tilde{\boldsymbol{\beta}}_j^{(m)} - \tilde{\lambda}_j^{*(m)} U_{cijk}^{*(m)} - \tilde{\xi}_j^{(m)} b_{cij}^{*(m)} \right)^2 + 0.1 \right).$$

**D3.**

$$\tilde{\lambda}_j^{*(m)} \sim N \left( \hat{\mu}_{\tilde{\lambda}}, \hat{\Omega}_{\tilde{\lambda}_j} \right),$$

where

$$\hat{\mu}_{\tilde{\lambda}} = \sum_{c,i,k} \left[ \left( \tilde{y}_{cijk}^{b*(m)} - W_{cijk}^T \tilde{\boldsymbol{\beta}}_j^{(m-1)} - \tilde{\xi}_j^{(m-1)} b_{cij}^{*(m-1)} \right) U_{cijk}^{*(m-1)} \right] \left( \sum_{c,i,k} U_{cijk}^{*2(m-1)} + 1 \right)^{-1},$$

$$\text{and } \hat{\Omega}_{\tilde{\lambda}_j} = \left[ \sum_{c,i,k} U_{cijk}^{*2(m-1)} + 1 \right]^{-1} \gamma_j^{2(m)}.$$

**D4.**

$$\tilde{\beta}_j^{*(m)} \sim N\left(\hat{\mu}_{\tilde{\beta}}, \hat{\Omega}_{\tilde{\beta}_j}\right),$$

where

$$\hat{\mu}_{\tilde{\beta}} = \sum_{c,i,k} \left[ \left( \tilde{y}_{cijk}^{b*(m)} - \tilde{\lambda}_j^{*(m)} U_{cik}^{*(m-1)} - \tilde{\xi}_j^{(m-1)} b_{cij}^{*(m-1)} \right) W_{cik} \right] \left( \sum_{c,i,k} W_{cik} W_{cik}^T + \Sigma_{\tilde{\beta}}^{-1} \right)^{-1},$$

$$\text{and } \hat{\Omega}_{\tilde{\beta}_j} = \left[ \sum_{c,i,k} W_{cik} W_{cik}^T + \Sigma_{\tilde{\beta}}^{-1} \right]^{-1} \gamma_j^{2(m)}.$$

**D5.**

$$\tilde{\xi}_j^{(m)} \sim N\left(\left(\sum_{c,i} K_{ci} b_{cij}^{*2(m-1)} + 1\right)^{-1} \sum_{c,i,k} \left[ \tilde{y}_{cijk}^{b*(m)} - W_{cik}^T \tilde{\beta}_j^{(m)} - \tilde{\lambda}_j^{*(m)} U_{cik}^{*(m-1)} \right] b_{cij}^{*(m-1)}, \Omega_{\xi_j}\right),$$

$$\text{where } \Omega_{\xi_j} = \left( \sum_{c,i} K_{ci} b_{cij}^{*2(m-1)} + 1 \right)^{-1} \gamma_j^{2(m)}.$$

**D6.** Set  $\beta_j^{(m)} = \tilde{\beta}_j^{(m)} / \gamma_j^{(m)}$ ,  $\lambda_j^{*(m)} = \tilde{\lambda}_j^{*(m)} / \gamma_j^{(m)}$ , and  $\xi_j^{(m)} = \tilde{\xi}_j^{(m)} / \gamma_j^{(m)}$ .

**D7.** For each  $\{c, i\}$

$$b_{cij}^{*(m)} \sim N\left(\Omega_{b_{cij}} \left[ \sum_{t=1}^{K_{ci}} \xi_j^{(m)} (y_{cikt}^{b*(m)} - W_{cikt}^T \beta_j^{(m)} - \lambda_j^{*(m)} U_{cikt}^{*(m-1)}) + \frac{\beta_{0j}^{*(m)}}{(\tau_j^{*(m)})^2} \right], \Omega_{b_{cij}}^{-1}\right),$$

$$\text{where } \Omega_{b_{cij}} = \left( K_{ci} \xi_j^{2(m)} + \frac{1}{(\tau_j^{*(m)})^2} \right)^{-1}.$$

**D8.** For each  $\{c, i\}$ ,  $U_{ci}^{*(m)} \sim MVN(\mu_{U_{ci}}, \Omega_{U_{ci}})$ , where

$$\begin{aligned} \mu_{U_{ci}} &= \Omega_{U_{ci}} \left[ \sum_{j=1}^{J_1} \frac{\lambda_j^{*(m)}}{\sigma_j^{2(m)}} \left( \mathbf{y}_{cij}^c - \mathbf{W}_{ci} \beta_j^{(m)} - \xi_j^{(m)} b_{cij}^{*(m)} \mathbf{1}_{K_{ci}} \right) \right] \\ &+ \Omega_{U_{ci}} \left[ \sum_{j=J_1}^J \lambda_j^{*(m)} \left( \mathbf{y}_{cij}^{b*} - \mathbf{W}_{ci} \beta_j^{(m)} - \xi_j^{(m)} b_{cij}^{*(m)} \mathbf{1}_{K_{ci}} \right) \right] \\ &+ \Omega_{U_{ci}} \frac{1}{\psi^2} \left[ \mu^{*(m)} + X_{ci} \alpha^{*(m)} + g_c^{*(m-1)} \mathbf{1}_{K_{ci}} + Z_{ci} \otimes \mathbf{1}_{K_{ci}} a_c^{*(m-1)} \right]^T H_{ci}^{-1}(\rho^{(m)}) \end{aligned}$$

$$\text{and } \Omega_{U_{ci}} = \left( \sum_{j=1}^{J_1} \frac{\lambda_j^{*2(m)}}{\sigma_j^{2(m)}} I_{K_{ci}} + \sum_{j=J_1+1}^J \lambda_j^{*2(m)} I_{K_{ci}} + \frac{1}{\psi^2} H_{ci}^{-1}(\rho^{(m)}) \right)^{-1}.$$

## C Additional Simulation Plots

### C.1 Model M1

Figure 1: Comparison of Gelman-Rubin diagnostic plots for two loading factors,  $\lambda_1$  and  $\lambda_3$  for models **M1** and **M2**. The solid black line shows the evolution of  $R^2$  for SG, while the dashed red line shows the evolution PX-HC (for **M1**) and  $PX^2$ -HC (for **M2**)

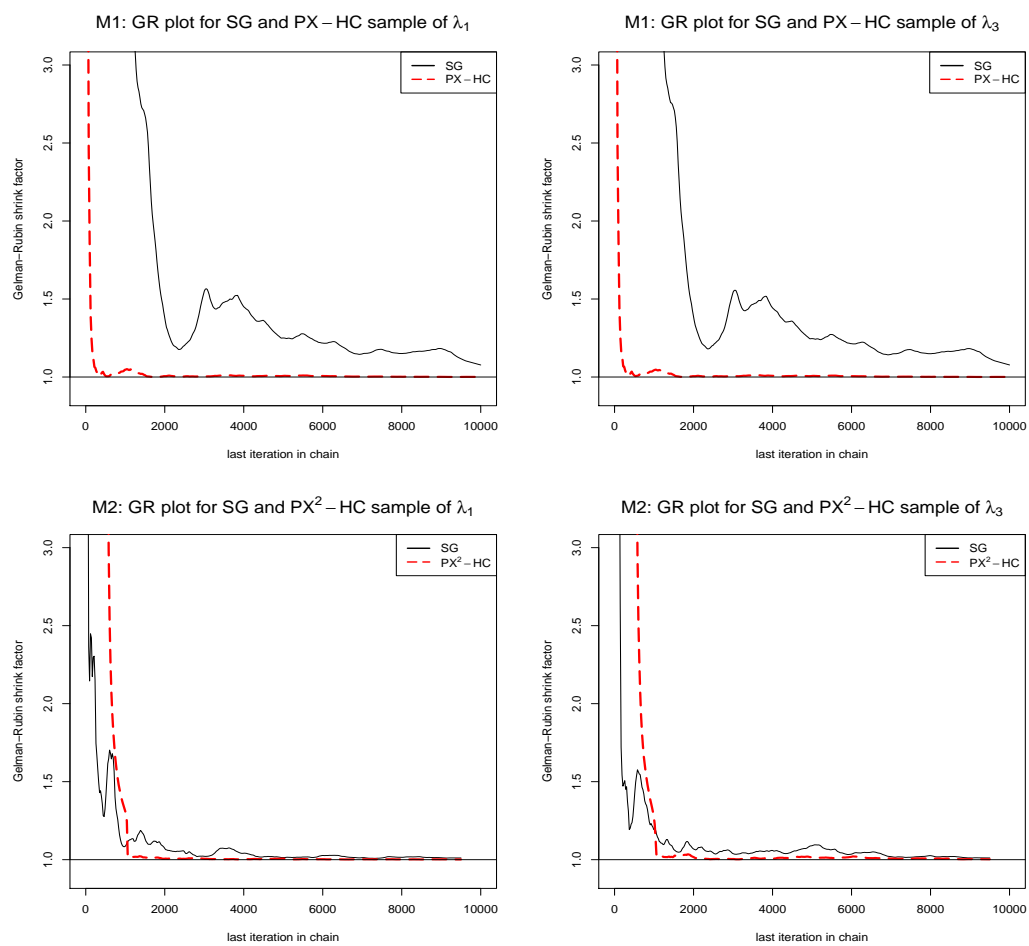


Figure 2: Comparison of trace plots for simulations under model **M1** using *SG* and *PX-HC* scheme. The blue line marks the true value of the parameter, and the red line represents the posterior mean. Left side from top to bottom: trace plots for  $\alpha_1$ ,  $\lambda_1$ , and  $\sigma_a^2$  using standard Gibbs. Right side from top to bottom: trace plots for  $\alpha_1$ ,  $\lambda_1$ , and  $\sigma_a^2$  using *PX-HC*.

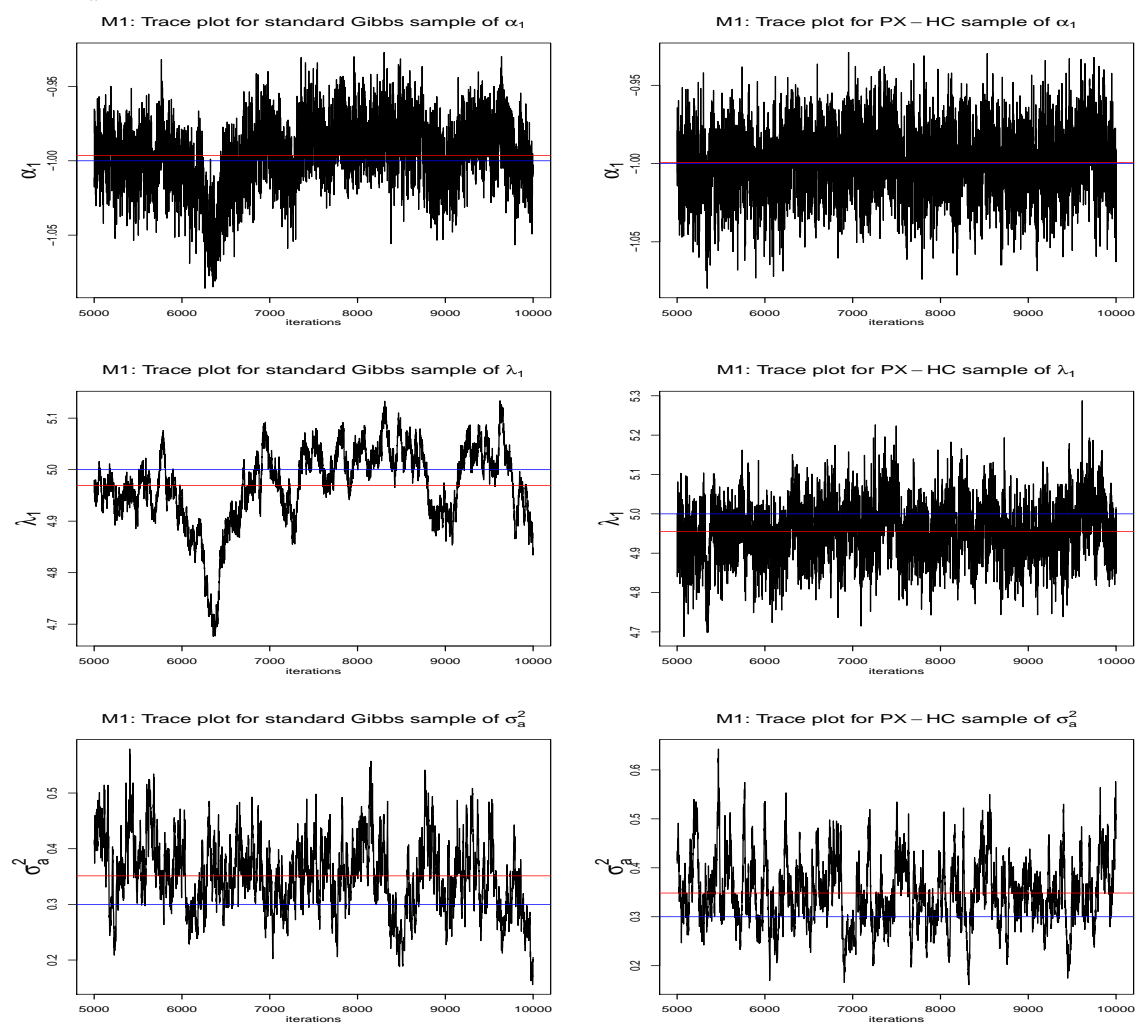


Figure 3: Comparison of ACF plots for the three loading factors  $\lambda_j$ ,  $j = 1, \dots, 3$  for model **M1**. Red line shows the average ACF curve for SG computed from 100 replicated curves which are shown in purple. The blue line shows the average ACF curve for PX-HC computed from 100 replicated curves which are shown in green.

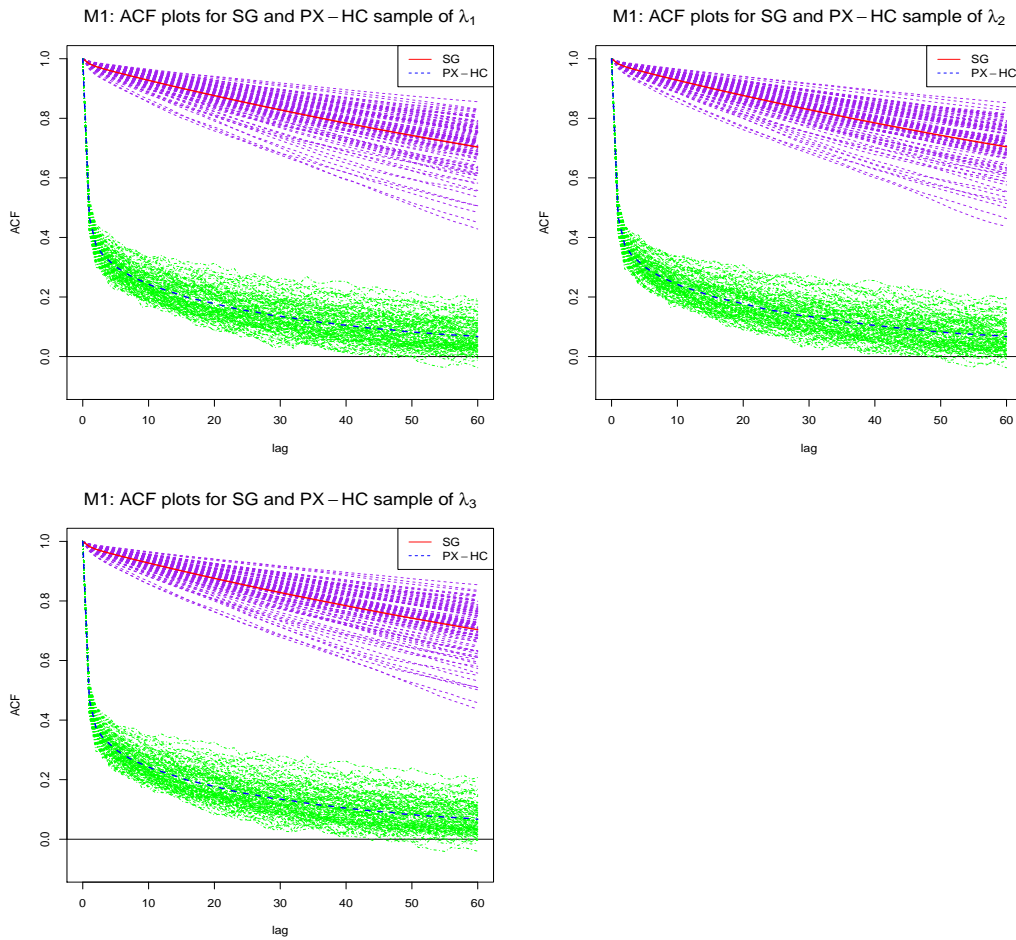
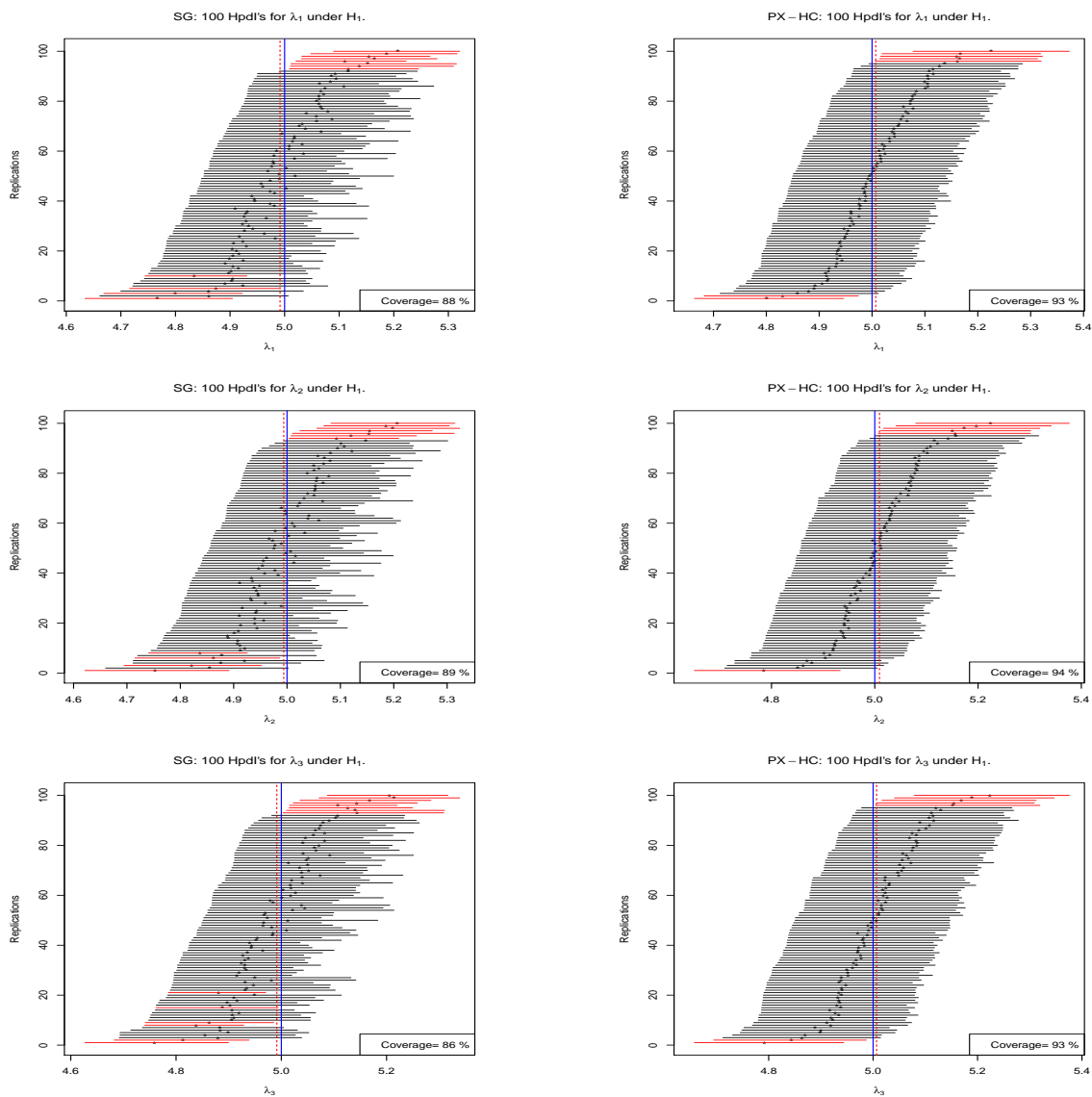




Figure 4: Comparison of highest posterior density interval (HpdI) plots for the three loading factors  $\lambda_j$ ,  $j = 1, \dots, 3$  for model **M1**. The replication number is the order of the lower bound of HpdI's. Left side: HpdI plots for  $\lambda$ s using SG. Right side: HpdI plots for  $\lambda$ s using PX-HC. The blue solid vertical line is the true value, which is 0, of  $\alpha_2$ . The red dashed vertical line is the mean estimation.



**C.2 Model M2**

**D Additional results for the real data example.**

Figure 5: Comparison of highest posterior density interval (HpdI) plots for the three loading factors  $\lambda_j$ ,  $j = 1, \dots, 4$  for model **M2**. The replication number is the order of the lower bound of HpdI's. Left side: HpdI plots for  $\lambda$ s using SG. Right side: HpdI plots for  $\lambda$ s using PX-HC. The blue solid vertical line is the true value, which is 0, of  $\alpha_2$ . The red dashed vertical line is the mean estimation.

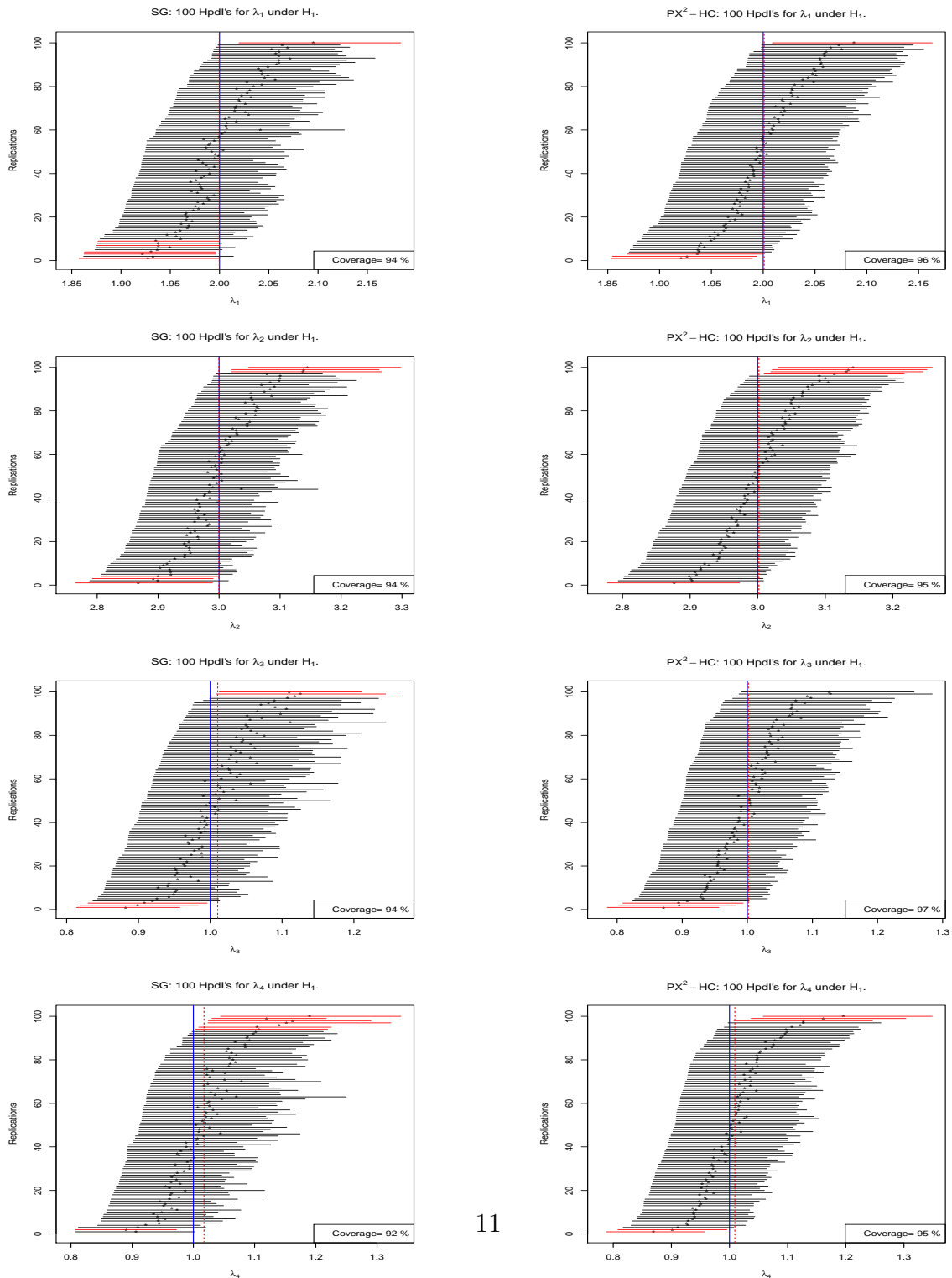


Table 1: The fitting results for the genetic study of type 1 diabetes (T1D) complications dataset using the model proposed by Roy and Lin (2000) Application results. SNP rs7842868 was previously identified to be associated with diastolic blood pressure (DBP) and SNP rs1358030 was previously identified to be associated with HbA1c. Phenotypes of interest are DBP and systolic blood pressure (SBP), two continuous outcomes, and hyperglycemia (HPG, defined as HbA1c greater or equal to 8), a binary outcome. All phenotypes are thought to be related to type 1 diabetes complication severity. The coefficient  $\lambda$ s assess the association between the phenotypes and the latent T1D complication status, and  $\alpha$ s evaluate the association between the latent variable and the genetic marker and the other covariates of interest. See Section 5 for more details.

<b>Analysis of SNP rs7842868</b>				
	Parameter	Estimate	95% HpDI	$\widehat{\log BF}$
SBP	$\lambda_1$	6.621	(6.153, 7.077)	114.85
DBP	$\lambda_2$	3.842	(3.566, 4.110)	112.98
HPG	$\lambda_3$	0.011	$(2.189 \times 10^{-7}, 2.975 \times 10^{-2})$	-1.05
rs7842868	$\alpha_1$	-0.269	(-0.372, -0.164)	10.06
sex	$\alpha_2$	-0.721	(-0.866, -0.584)	62.27
cohort	$\alpha_3$	0.443	( 0.299, 0.585)	20.15
treatment	$\alpha_4$	0.128	(-0.004, 0.263)	0.366
<b>Analysis of SNP rs1358030</b>				
	Parameter	Estimate	95% HpDI	$\widehat{\log BF}$
SBP	$\lambda_1$	6.868	(6.439, 7.302)	128.3
DBP	$\lambda_2$	3.706	(3.491, 3.933)	120.2
HPG	$\lambda_3$	0.010	$(2.566 \times 10^{-7}, 2.740 \times 10^{-7})$	-1.034
rs1358030	$\alpha_1$	- 0.039	(-0.049, 0.122)	-1.104
sex	$\alpha_2$	-0.758	(-0.880, -0.623)	64.86
cohort	$\alpha_3$	0.393	(0.258, 0.532)	18.17
treatment	$\alpha_4$	0.088	(-0.041, 0.220)	-0.18