Problem - Bus waiting

Buses arrive at a specific stop at 15-minute intervals starting at 5am. If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7:30am, find the probability that he waits:

a) less than 5 minutes

b) more than 10 minutes

for the bus.

Bertrand's Paradox

Consider a "random chord" of a circle. What is the probability that the length of the chord will be greater than the side of the equilateral triangle inscribed in that circle?

It is not clear what a random chord means ....

Solution 1

The size of the chord is determined by the size of D, its distance from the center of the circle.
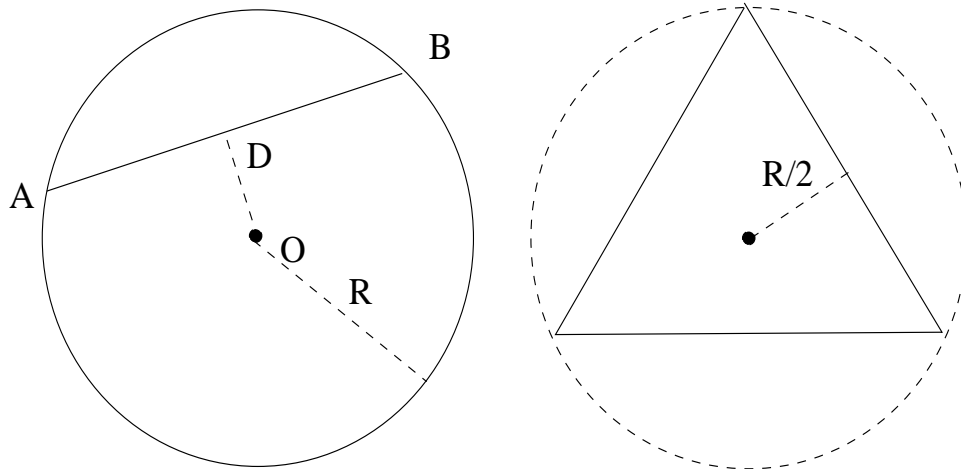

Assume that selecting a random chord is the same as selecting a random distance D uniformly in $[0, R]$ (R is the radius of the circle). Then the length of the chord will be larger than the side of the equilateral triangle if $0 \leq D < R/2$.

Solution 2

Consider an arbitrary point A on the circle. Take all the chords that have one end fixed in A. A random chord in the circle is equivalent to a random chord that passes through A.

Consider the tangent in A; the angle X made by the tangent with a random chord (measured clockwise from the chord) is uniform in $[0, 180]$.
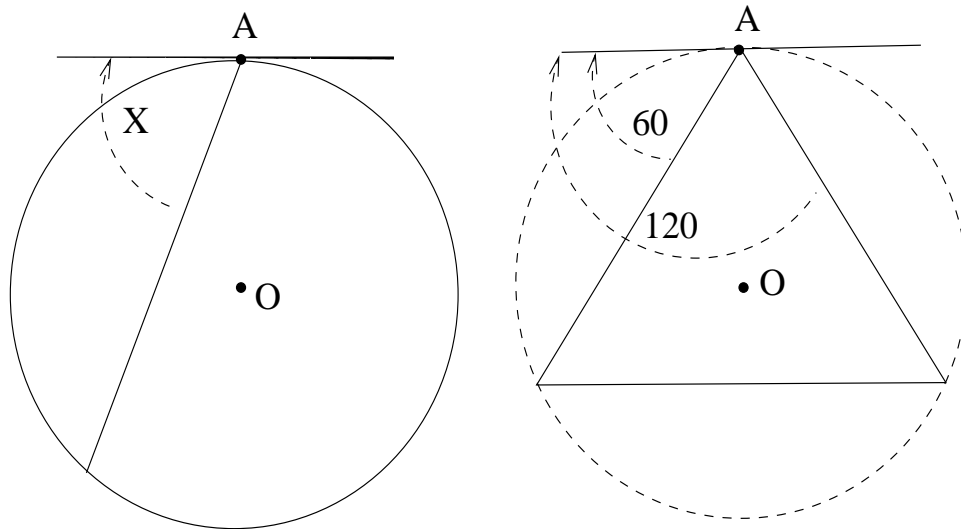
From the figure it results that the random chord will be bigger than the side of the triangle if $60 < X < 120$.