Example - Normal Distribution

The annual rainfall (in cm) in Toronto is normally distributed with $\mu = 140$ and $\sigma = 20$.

What is the probability that starting with this year it will take over 10 years beafore a year occurs with an anual rainfall of 100 cm?

Example - Normal Approximation to Binomial

A factory which produces light bulbs estimates that the probability of a light bulb lighting continuously more than a week is 36%. What is the chance that out of 100 bulbs tested, the number of bulbs still working after a week is stricly larger than 24 and smaller or equal to 42.

Example - Trial of the Pyx (from a paper by Stephen Stigler in the September 1977 issue of *Journal of American Statistical Association*

The procedure described here has been in practice for centuries at the Royal Mint in London. The scheme started around 1200 and was designed to verify the accuracy of the minting process. Such accuracy was important as coins were made of gold andor silver in those times.

The scheme was as follows:

- over a period of time one coin would be taken out of every day of the Mint's production and the coin would be placed in a box called the Pyx (derived from the Greek word for box).

- at irregular times, perhaps as frequently as each year but usually separated by three or four years a trial of the Pyx would be declared and a jury of members of the Worshipful Company of Goldsmiths (an independent tradesman's guild) would assemble.

- at the trial the Pyx would be opened and its contents would be counted, weighted and assayed.

- a given margin of error or *remedy* was allowed for the minting process.

- in its 600 years of history only two masters of the mint have been found in error.

Question: Were people that honest, or was there something wrong about the trial of the Pyx process?

The following is a simplified version of the trial:

- suppose that a margin of error (SD) of 0.1g was allowed per coin. The target weight for a coin would be 10g.

- the trial of the Pyx allowed a cumulative margin of error for the weight of n coins to be $n \times 0.1$.

a) What is the probability that measuring the weight of 100 coins we will be outside the allowed remedy?

Suppose each coin has the weight $X \sim N(10, 0.01)$

Then the sum of independent 100 coins will have a weight $Y \sim N(10 \times 100, 0.01 \times 100)$

What is the probability that the Master of the Mint will fail the trial, that is, what is $1 - P(1000 - 100 \times 0.1 \leq Y \leq 1000 + 100 \times 0.1)$?

b) Suppose a Master of the Mint was dishnonest and the average coin would weigh 9g. Then what is the probability that he will be discovered using 100 coins?

c) What if we correct the allowed remedy to be $\sqrt{n} \times 0.1$?