Random Variables

• In many situations when an experiment is performed the interest is in some numerical function of the outcome rather than the actual outcome itself.

• If S is the sample space of an experiment then a map $X : S \to R$ is a called a random variable.

• **Additional Requirement:** For any interval $I \subset R$, $X^{-1}(I)$ is an event in S.

Distribution functions

• The distribution function $F : R \to [0, 1]$ of a random variable X is defined as $F(t) = P(X \leq t)$.

Properties of F

- F is non-decreasing, i.e. $t_1 \leq t_2$ implies $F(t_1) \leq F(t_2)$
- $\lim_{t\to\infty} F(t) = 1$
- $\lim_{t\to-\infty} F(t) = 0$
- F is continuous to the right.

Connection between probability and distribution

Question: What is the connection between $P(X \in (a,b])$ and $F(a), F(b)$? How about $P(X = a)$ and $F(a)$? ...

Event concerning X	Probability of the event in terms of F
$X \leq a$	$F(a)$
$X > a$	$1 - F(a)$
$X \geq a$	$1 - F(a-)$
$X = a$	$F(a) - F(a-)$
$a < X \leq b$	$F(b) - F(a)$
$a < X < b$	$F(b-) - F(a)$
$a \leq X \leq b$	$F(b) - F(a-)$
$a \leq X < b$	$F(b-) - F(a-)$

Random selection of points in intervals

• Fix $a < b$ and α, β such that $a \leq \alpha < \beta \leq b$. The probability that a point is randomly selected in the interval (α, β) is

$$\frac{\beta - \alpha}{b - a}$$

• Let C be a fixed point in the interval (a, b). If X is a point randomly selected in the interval (a, b) then the probability that X is selected to be exactly C is

$$P(X = C) = 0$$