Review - Bayes Formula

- Say $\{B_1, B_2, B_3\}$ is a partition of the sample space S and $P(B_i) > 0$ for all $i \in \{1, 2, 3\}$. Then for any event A with $P(A) > 0$

$$P(B_k|A) = \frac{P(A|B_k)P(B_k)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3)}$$

for any $k \in \{1, 2, 3\}$.

- **Generalization**

Say $\{B_1, B_2, \ldots, B_n\}$ is a partition of the sample space S and $P(B_i) > 0$ for all $i \in \{1, 2, \ldots, n\}$. Then for any event A with $P(A) > 0$

$$P(B_k|A) = \frac{P(A|B_k)P(B_k)}{\sum_{i=1}^{n} P(A|B_i)P(B_i)}$$

for any $k \in \{1, 2, \ldots, n\}$.

Review - Independence

- Two events, E and F are *independent* if

$$P(E \cap F) = P(E)P(F).$$

Properties

1. If E and F are independent and $P(F) > 0$ then $P(E|F) = P(E)$.

2. If E and F are independent then E and F^c are independent.

3. If E and F are mutually exclusive and $P(E)P(F) > 0$ then E and F can not be independent.

4. If E and F are independent, and E and G are independent it is not automatically true that E and $F \cap G$ are independent.

5. In the case of three events, E,F, and G, the equality $P(E \cap F \cap G) = P(E)P(F)P(G)$ does not imply that E,F, and G are independent.