Review - Conditional Probability

- If A and B are two events such that $P(B) > 0$ then *the conditional probability of* A *given* B is denoted $P(A|B)$ and is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

- For a fixed event B with $P(B) > 0$, if

$$Q(A) = P(A|B),$$

then $Q(\cdot)$ is a probabi lity (it satisfies Axioms I, II, and III). One can use a *sample space reduction* to calculate $Q(A \cap B)$ instead of $P(A|B)$.

Review - Law of Multiplication

- If $P(B) > 0$, $P(A \cap B) = P(A|B)P(B)$
- If $P(B \cap C) > 0$,

$$P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$$

- **Generalization:** If $P(A_1 \cap \ldots \cap A_{n-1}) > 0$

$$P(\cap_{i=1}^{n} A_i) = P(A_n|A_1 \cap \ldots \cap A_{n-1}) \cdot$$

$$\cdot P(A_{n-1}|A_1 \cap \ldots \cap A_{n-2}) \ldots P(A_2|A_1)P(A_1).$$

Review - Partitions

The sets $A_1, A_2, \ldots, A_n$ form a **partition** of the sample space S if

1. The sets $\{A_1, A_2, \ldots, A_n\}$ are mutually disjunctive,

   and

2. $\cup_{i=1}^{n} A_i = S$.

If $A_1, A_2, \ldots, A_n$ is a partition then for any event $B \subset S$

$$B = \cup_{i=1}^{n} (B \cap A_i).$$

Example: $B = (B \cap A) \cup (B \cap A^c)$.

Review - Law of total Probability

- If $P(B) >$ and $P(B^c) >$ then $P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$.

- If $\{B_1, B_2, B_3\}$ is a partition of S and $P(B_i) > 0$ for $i = 1, 2, 3$ then

$$P(A) = P(A|B_1)P(B_1) +$$

$$+ P(A|B_2)P(B_2) + P(A|B_3)P(B_3).$$

- **Generalization** If $\{B_1, B_2, \ldots, B_n\}$ is a partition of S and $P(B_i) > 0$ for $i = 1, 2, \ldots, n$ then

$$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i).$$