

What is probability ?

According to the Webster Collegiate Dictionary:

- the chance that a certain event will occur
- a branch of mathematics concerned with the study of probabilities.

This course's answer: It is a (mathematical) tool developed to deal with the randomness around us.

Some problems which can be solved using probability theory

- *Gambling*: The game of craps is played as follows: A player rolls two dice. If the sum of the dice is either a 2, 3 or 12, the player loses; if the sum is either a 7 or an 11 the player wins. If the outcome is anything else, the player continues to roll the dice until he or she rolls either the initial outcome or a 7. If the 7 comes first the player loses. Otherwise he or she wins. What is the probability to win at craps?
- *Party betting*: You are at a party with 50 other people. If you bet that two people at that party have the same birthday are your chances of winning more than 50% or less than 50%?
- *Explain Science*: If $N(t)$ is the number of nuclei contained in a radioactive mass of material at time t , then the *half-life* is defined as the value h such that

$$N(t) = 2^{-t/h}N(0)$$

There appears to be some controversy over whether or not protons decay. Certain theories hypothesize that protons decay with a half-life of about $h = 10^{30}$ years. Using a probabilistic model one can use experimental data to check the aforementioned hypothesis. More precisely if 10^{30} protons have zero decays in 2 years what can one say about the stated hypothesis, is it true or false?

- *Learn Cool Paradoxes:* Suppose that we possess an infinitely large urn and an infinite collection of balls labelled 1, 2, 3, ... and so on. Suppose that at 1 minute to 12pm balls 1 through 10 are placed in the urn and ball 1 is withdrawn (assume withdrawing takes no time). At $\frac{1}{2}$ minute to 12pm balls 11 through 20 are placed in the urn and ball 2 is withdrawn, at $\frac{1}{4}$ minute to 12pm balls 21 through 30 are placed in the urn and ball 3 is withdrawn, and so on. Then one can show that the urn is empty at 12pm.

If we suppose that at 1 minute to 12pm balls 1 through 10 are placed in the urn and ball 10 is withdrawn (assume withdrawing takes no time). At $\frac{1}{2}$ minute to 12pm balls 11 through 20 are placed in the urn and ball 20 is withdrawn, at $\frac{1}{4}$ minute to 12pm balls 21 through 30 are placed in the urn and ball 30 is withdrawn, and so on. Then one can prove that the urn has an infinite number of balls at 12pm.

What happens if the ball extracted is chosen at random from the balls placed?