Continuous Random Variables

• A random variable $X : S \to R$ is a **continuous random variable** if it can take any value in an interval.

• It does not make sense to ask about the probability of $X = a$; however, one can ask what is the probability of X being in a given interval (a, b).

• The randomness of a continuous random variable X is completely characterized by its **density function** $f : R \to [0, \infty)$.

Density function

The **density function** $f : R \to [0, \infty)$ satisfies the following two properties

1. $f(y) \geq 0$ for all $y \in R$
2. $\int_R f(y)dy = 1$

- If f is the density function of X then

$$P(X \in (a,b)) = \int_a^b f(y)dy = P(X \in (a,b])$$

- For any integrable function $g : R \to R$,

$$E[g(X] = \int_R g(y)f(y)dy$$

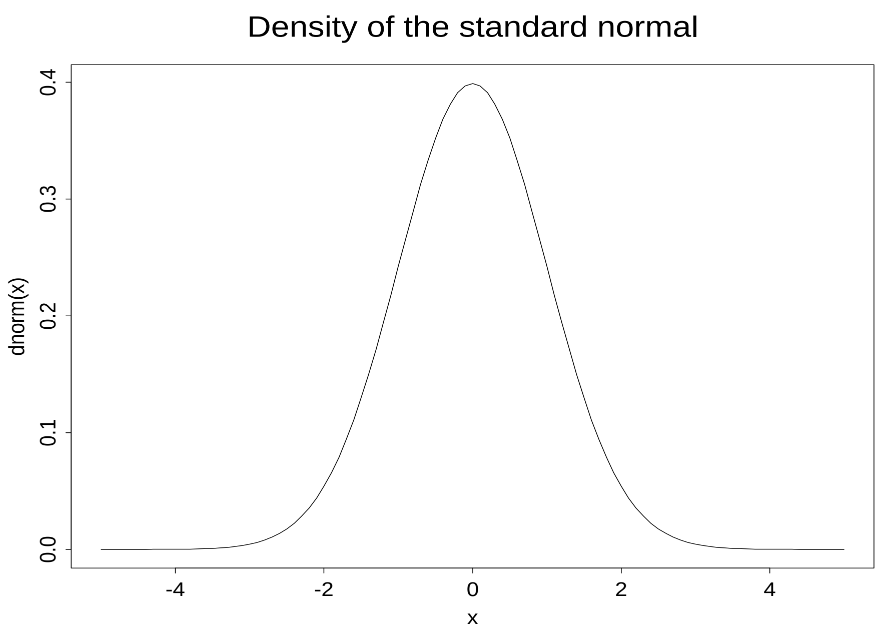
One can then obtain expressions for the $E[X]$ and $Var[X]$.

The standard normal curve

- The standard normal density is

$$f(x) = \frac{1}{\sqrt{2\pi}} exp(-x^2/2),$$

for all $x \in R$.



- If X has a standard normal distribution then we denote $X \sim N(0,1)$.

- If $X \sim N(0,1)$ then $E[X] = 0$, $Var(X) = 1$

- Probabilities like $P(X < a)$ or $P(a \leq X < b)$ are calculated using the table for the standard normal.

General Normal Distribution

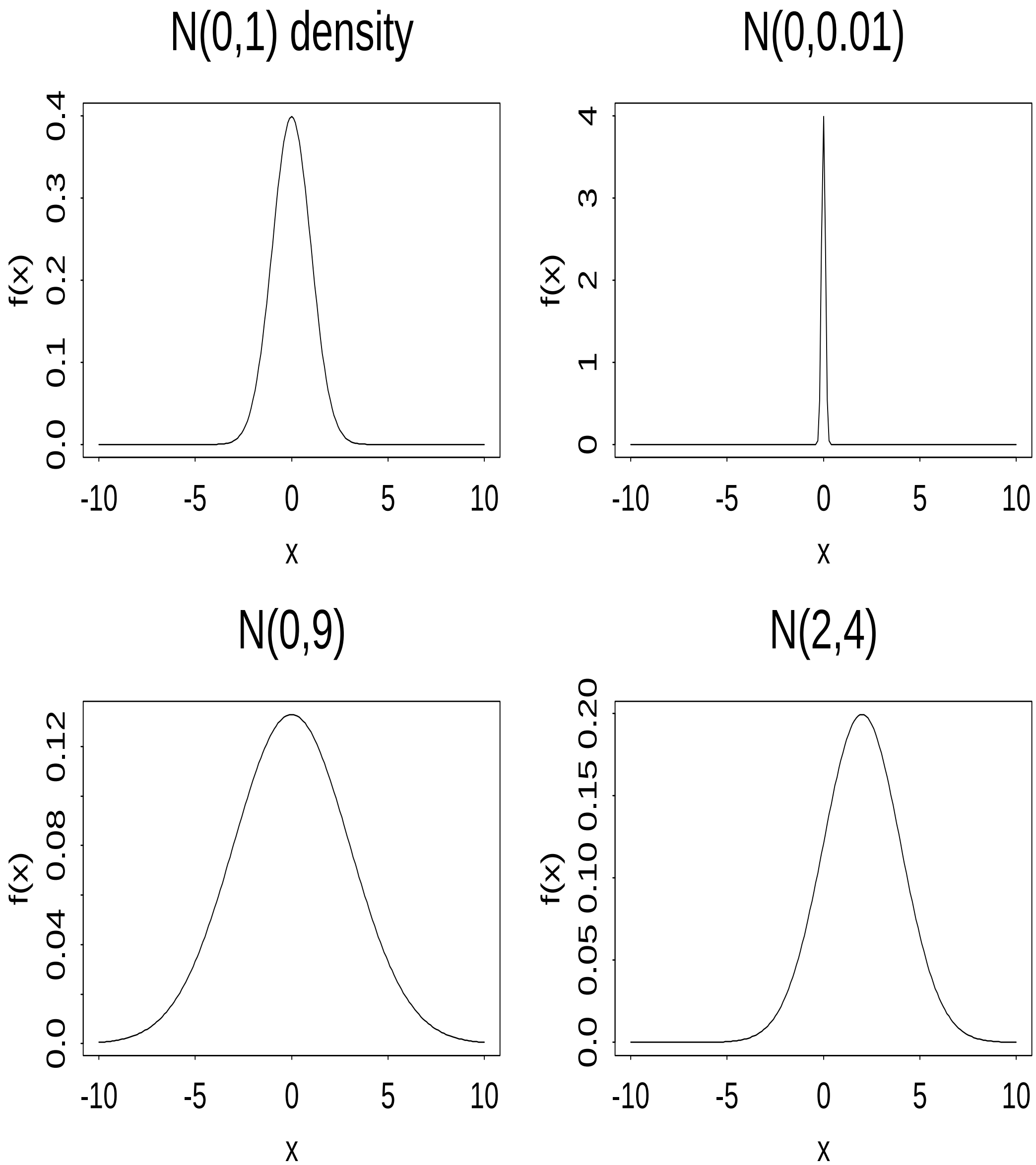
- The general normal distribution $N(\mu, \sigma^2)$ has probability density function

$$f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} exp(-(x-\mu)^2/2\sigma^2),$$

for all $x \in R$.

If $X \sim N(\mu, \sigma^2)$ then $E[X] = \mu$ $Var(X) = \sigma^2$.

The square root of the variance is the *standard deviation* and is denoted **SD**.


N(0,1) density
f(x)
0.0
0.1
0.2
0.3
0.4
-10
-5
0
5
10
x
N(0,0.01)
f(x)
0
1
2
3
4
-10
-5
0
5
10
x
N(0,9)
f(x)
0.0
0.04
0.08
0.12
-10
-5
0
5
10
x
N(2,4)
f(x)
0.0
0.05
0.10
0.15
0.20
-10
-5
0
5
10
x

$\boxed{\text{General Normal Distribution}}$

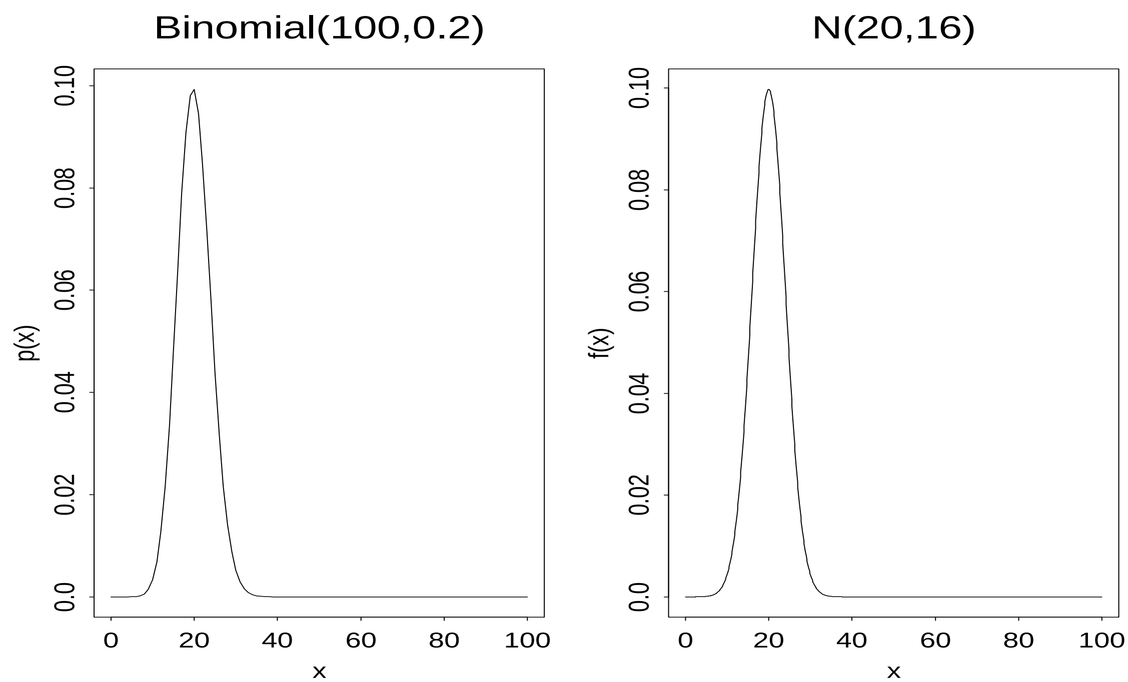
- If $X \sim N(\mu, \sigma^2)$ then $E[X] = \mu$ and $Var(X) = \sigma^2$.

- If $X \sim N(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$ then $Z \sim N(0, 1)$

- Probabilities like $P(X < a)$ or $P(a \leq X < b)$ are calculated using the relationship between X and $Z = \frac{X-\mu}{\sigma}$ as well as the table for the standard normal.

Normal approximation to the Binomial

Let $X \sim Bin(n, p)$. **Rule of thumb:** If $np \geq 5$ and $n(1-p) \geq 5$ approximate X with $Y \sim N(np, np(1-p))$.



The continuity correction

However, a **continuity correction** is necessary. A continuity correction is an adjustment that we make by adding or subtracting 1/2 to a discrete value when we use a continuous distribution to approximate the discrete one.

To apply a continuity correction to the discrete values **included** in an interval, we subtract 1/2 from the smallest value included in an interval, add 1/2 to the largest value included in the interval and then proceed.

Illustration