STA 257 – Fall 2002

Practice Problems 7

Recommended preparation for quiz to be held in tutorial on Wednesday, November 13

Sections from Schaeffer covered the week of November 4: 6.1, 6.2, 6.3, 4.5 ($N(\mu, \sigma^2)$ distribution), convolution

Questions from the textbook (Schaeffer):

1. From Section 6.2: 6.1, 6.3, 6.5, 6.7
2. From Section 6.4 (exercises covering material from 6.3): 6.9
3. From Section 4.6: 4.65

Additional questions:

4. Let X have an exponential distribution with mean $1/\lambda$. Given that $X = x$, let Y conditionally have the normal distribution with expected value μX and variance $\sigma^2 X^2$. Find EY and VY. [Ans: $EY = \mu\lambda$, $VY = (\mu^2 + 2\sigma^2)/\lambda^2$]

5. The (standard) lognormal distribution is the distribution of a random variable Y for which $X = \ln Y$ has a standard normal distribution. Find a density for Y. [Ans: $\frac{1}{y\sqrt{2\pi}} e^{-(\ln y)^2/2}$, $y > 0$]

6. Let the cumulative distribution function of X be

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^2/4 & \text{if } 0 \leq x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$$

   Let $Y = X^2/4$. Find the distribution of Y. [Ans: Uniform on $(0, 1)$]

7. Suppose that $f(y)$ is a density, and that we would like to generate observations of a random variable Y having this density using a computer. The computer gives us observations of a random variable X having the uniform distribution on $(0, 1)$.

   (a) Show that if $F(y)$ is the distribution function corresponding to the density $f(y)$, and $y = h(x)$ is the inverse of the function $x = F(y)$, then the random variable $Y = h(X)$ has the desired density.

   (b) Let the desired density be $f(y) = 2/y^3$ for $y > 1$ (and 0 otherwise). Find the transformation $Y = h(X)$ that produces a Y with this density.

   [Ans: (b) $Y = 1/\sqrt{1 - X}$]

8. Let the joint density of X and Y be

$$f(x, y) = \begin{cases} 6e^{-3x-2y} & \text{if } x > 0 \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

   Find a density for $Z = X + Y$ by each of the following approaches:

   (a) Using the cumulative distribution function approach.

(b) Noting that X and Y are independent random variables, finding the marginal densities of X and Y and using the convolution theorem.

[Ans: $6e^{-2z} - 6e^{-3z}$ for $z > 0$]

9. If $X \sim N(15, 30)$ and $Y \sim N(-12, 15)$ find the following:

   (a) The distribution of $X + Y$.

   (b) The distribution of $X - 2Y$.

   (c) The probability that $2X + 3Y \leq 5$.

   (d) The probability that $X > Y$.

   [Ans: (a) $N(3, 45)$ (b) $N(39, 90)$ (c) .7549 (d) Essentially 1]

10. Let $X_1, X_2, \ldots, X_{30}$ be a sample from the $N(20, 25)$ distribution. Find the following:

    (a) The probability that $\overline{X}_{30}$ is between 19 and 21.

    (b) The probability that $S_{30} = X_1 + X_2 + \ldots + X_{30}$ is greater than 650.

    [Ans: (a) .7286 (b) .0336]

11. Find the number a so that the probability is 95% that a normally distributed random variable is within a standard deviations of its expected value. [Ans: 1.96]

Material to be covered during the week of November 11 and relevant sections in Schaeffer: Gamma distribution (4.5) and Chi-square distribution, Multivariate change of variables theorem (not in textbook), other probability distributions which are useful in statistics