PRACTICE PROBLEMS 6 — Solutions to Additional Questions

7.4 from Schaeffer It doesn't apply because the moments are infinite.

#6/ Use the result that $E(XY) = (EX)(EY)$ if X, Y are independent and show that $E[f(X)g(Y)] = E(f(X))E(g(Y))$ if X, Y are independent.

#7/ (a) $\mathrm{Cov}(aX+b, cY+d) = E[(aX+b-(aEX+b))(cY+d-(cEY+d))]$

$$= E[a(X-EX)\cdot b(Y-EY)]$$
$$= ab\,E[(X-EX)(Y-EY)]$$

(b) $\mathrm{Cov}(X+Y, Z) = E[(X+Y-EX-EY)(Z-EZ)]$

$$= E[(X-EX)(Z-EZ) + (Y-EY)(Z-EZ)]$$
$$= E[(X-EX)(Z-EZ)] + E[(Y-EY)(Z-EZ)]$$

(c) $\mathrm{Cov}(Y, X) = E[(Y-EY)(X-EX)] = E[(X-EX)(Y-EY)]$

#8/ $EXY = 1(1)(.12) + 1(2)(.08) + \dots + 3(3)(.08) = 3.48$

$EX = 1(.12 + .08 + .11) + 2(.18 + .14 + .07) + 3(.17 + .05 + .08) = 1.99$

$EY = \dots = 1.79$

$\mathrm{Cov}(X,Y) = 3.48 - (1.99)(1.79)$

$EX^2 = 1^2(.12 + .08 + .11) + 2^2(.18 + .14 + .07) + 3^2(.17 + .05 + .08) = 4.57$

$VX = .6099$

$EY^2 = 3.89$, $VY = .6859$

$$\rho = \frac{-.0821}{\sqrt{.6099}\sqrt{.6859}} = -.1269$$

#9/ $EXY = \int_0^1 \int_0^1 xy \, \frac{6}{5}(x^2+y) \, dx \, dy = \dots = \frac{7}{20}$

$f_X(x) = \int_0^1 \frac{6}{5}(x^2+y) \, dy = \dots = \frac{6}{5}(x^2 + \frac{1}{2})$ for $0<x<1$ (0 elsewhe

$EX = \int_0^1 x f_X(x) \, dx = \dots = \frac{3}{5}$

$f_Y(y) = \int_0^1 \frac{6}{5}(x^2+y) \, dx = \dots = \frac{6}{5}(\frac{1}{3}+y)$ for $0<y<1$ (0 elsewhere)

$EY = \int_0^1 y f_Y(y) \, dy = \dots = \frac{3}{5}$

$Cov(X,Y) = \frac{7}{20} - \frac{3}{5}\left(\frac{3}{5}\right) = \frac{-1}{100}$

$EX^2 = \int_0^1 x^2 f_X(x) \, dx$, $EY^2 = \int_0^1 y^2 f_Y(y) \, dy$

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{EX^2-(EX)^2}\sqrt{EY^2-(EY)^2}}$$

#10/ $Cov(X+Y, X-Y) = E\left[(X+Y-EX-EY)(X-Y-EX+EY)\right]$

$= E\left[(X-EX)^2 - (X-EX)(Y-EY) + (Y-EY)(X-EX) - (Y-EY)^2\right]$

$= E\left[(X-EX)^2 - (Y-EY)^2\right] = E\left[(X-EX)^2\right] - E\left[(Y-EY)^2\right]$

#11/ $E(Y|X=x) = E(Y_1 + \dots + Y_x) = xE(Y_i) = x\mu$

$EY = E[E(Y|X)] = E(X\mu) = \mu EX$

$V(Y|X=x) = x\sigma^2$ since Y_i's independent

$VY = V(E(Y|X)) + E(V(Y|X))$

$= V(\mu X) + E(\sigma^2 X)$

$= \mu^2 VX + \sigma^2 EX$

#12/ Moved to Practice Problems 7

#13/ (b) $E(\bar{X}_{45}) = \frac{1}{45} E(S_{45}) = \frac{1}{45}(45)(.35)$

$V(\bar{X}_{45}) = \frac{1}{(45)^2} V(S_{45}) = \frac{1}{(45)^2}(45)(.35)(.65)$

(c) $P(|\bar{X}_{45} - .35| \geq .05) \leq \frac{.0050506}{(.05)^2} = 2.02$

Not very useful!

(d) Then Chebyshev's inequality says the probability that $\bar{X}_{45}$ is not between .3 and .4 is $\leq .0202$