STA 257 – Fall 2002

Practice Problems 6

Recommended preparation for quiz to be held in tutorial on Wednesday, November 6

Sections from Schaeffer covered October 21 through October 30: 5.4, 5.7, 3.2 (Tchebysheff's (Chebyshev's) Inequality), 7.2

Questions from the textbook (Schaeffer):

1. From Section 5.4: 5.19
2. From Section 5.7: 5.43
3. From Chapter 5 Supplementary Exercises: 5.57
4. From Section 3.2: 3.19
5. From Section 7.3 (although these questions are on material from Section 7.2): 7.1, 7.4

Additional questions:

6. Find an expression for the variance of XY if X and Y are independent. [Ans: $V(XY) = E(X^2) \cdot E(Y^2) - (EX)^2 \cdot (EY)^2$]
7. Verify the following properties of covariance. (Here X, Y, and Z are random variables and a, b, c, and d are constants.)
   (a) $\text{Cov}(aX + b, cY + d) = ac\text{Cov}(X, Y)$
   (b) $\text{Cov}(X + Y, Z) = \text{Cov}(X, Z) + \text{Cov}(Y, Z)$
   (c) $\text{Cov}(Y, X) = \text{Cov}(X, Y)$
8. Let X and Y have the joint distribution shown in the table. Find their covariance and their correlation coefficient.

	$p(x,y)$	y = 1	2	3
	1	.12	.08	.11
x	2	.18	.14	.07
	3	.17	.05	.08

   [Ans: Cov = $-.0821$, $\rho = -.1269$]

9. Let X and Y have the following joint density function:

$$f(x, y) = \frac{6}{5}(x^2 + y) \text{ for } 0 < x < 1 \text{ and } 0 < y < 1 \text{ (and 0 otherwise)}$$

   Find the covariance and correlation coefficient. [Ans: Cov $= -1/100$, $\rho = -.130558$]

10. Show that for any two random variables X and Y, $\text{Cov}(X+Y, X-Y) = VX - VY$.

11. Let Y_1, Y_2, Y_3, ... be independent and identically distributed random variables, with $EY_j = \mu$ and $VY_j = \sigma^2$. Let $Y = Y_1 + Y_2 + Y_3 + \cdots + Y_X$, where X is a positive integer-valued random variable. Show that

$$EY = \mu EX$$

and

$$VY = \mu^2 VX + \sigma^2 EX$$

(provided, of course, that EX and VX exist).

12. Let X have an exponential distribution with mean $1/\lambda$. Given that $X = x$, let Y conditionally hve the normal distribution with expected value μX and variance $\sigma^2 X^2$. Find EY and VY. [Ans: $EY = \mu\lambda$, $VY = (\mu^2 + 2\sigma^2)/\lambda^2$]

13. Suppose A is an event associated with some chance experiment and $P(A) = .35$. Suppose the experiment is repeated 45 times, and on the ith repetition X_i equals 1 if A occurs and 0 if not.

    (a) What is the distribution of $S_{45} = \sum_{i=1}^{45} X_i$?

    (b) What are $E\overline{X}_{45}$ and $V\overline{X}_{45}$?

    (c) What does Chebyshev's inequality say about the probability that $\overline{X}_{45}$ is not between .3 and .4?

    (d) How would your answers change if the experiment was repeated 4500 times instead of 45?

    [Ans: (a) Binomial with $n = 45$ and $p = .35$ (b) .35, .0050506 (c) Less than or equal to 2.02]

Relevant sections in Schaeffer for lectures during the week of November 4: 6.1, 6.2, 6.3, 6.4