**STA 257 – Fall 2002**

**Practice Problems 5**

Recommended preparation for quiz to be held in tutorial on Wednesday, October 23

**Note:** There will be **no** quiz in tutorial on Wednesday, October 16 (Happy Thanksgiving!) and Wednesday, October 30. There **will** be a quiz on Wednesday, October 23. A reminder that the term test is Monday, October 28 from 3:10–5:00 p.m. The test will not be held in the lecture room. Students with surnames beginning with A–Le will write the test in CG 150. Students with surnames beginning with Li–Z will write the test in CG 250.
Sections from Schaeffer covered October 7 through October 16: 2.6, 5.1, 5.2, 5.3

**Questions from the textbook (Schaeffer):**

1. From Section 2.6: 2.39, 2.45(c)
2. From Section 5.3: 5.7, 5.8 [Ans: (b) not independent, (c) 2/3], 5.9, 5.13
3. From Supplementary Exercises: 5.51, 5.53, 5.61, 5.67

**Additional questions:**

4. Questions 10, 11, 12 from Practice Problems 4.
5. A fair coin is tossed four times; $X$ is the number of heads that come up on the first three tosses, and $Y$ is the number of heads that come up on tosses 2, 3, and 4.
   (a) What is the distribution of $X$? of $Y$?
   (b) Make a table of the joint probability function of $X$ and $Y$. (It has 16 entries.)
   (c) Find the marginal probability functions and check that they agree with your answer to (a).
   (d) Are $X$ and $Y$ independent?
   (e) Find the conditional probability that $X = x$, given that $Y = 2$, for $x = 0, 1, 2, 3$.

   [Ans: (a) Each has the binomial distribution with $n = 3$ and $p = 1/2$. (d) No (e) $0, 1/3, 1/2, 1/6$]
6. The joint probability function of $X$ and $Y$ is $p(x, y) = (\lambda/2)^y e^{-\lambda}/x!(y-x)!$ when $x$ and $y$ are integers and $0 \leq x \leq y$.
   (a) Find the marginal probability function of $X$.
   (b) Find the marginal probability function of $Y$. [*Hint*: Multiply and divide $p(x, y)$ by $y!$, producing a binomial coefficient, and then sum from $x = 0$ to $y$.]
   (c) Are $X$ and $Y$ independent?

   [Ans: (a) $X$ is Poisson with parameter $\lambda/2$. (b) $Y$ is Poisson with parameter $\lambda$. (c) No]

7. Let $X$ and $Y$ have the joint density

$$f(x,y) = cx^2y \text{ for } 0 < x < 2 \text{ and } 0 < y < 2-x \text{ and } 0 \text{ otherwise}$$

where $c$ is the appropriate constant.

(a) Draw a picture of the set of possible values of the pair $(X,Y)$.

(b) Show that $c$ must equal $15/8$ if $f(x,y)$ is to integrate to 1 over the set of possible values.

(c) Are $X$ and $Y$ are independent?

(d) Find a density for $X$ by integrating $f(x,y)$. (For any $x$ between 0 and 2, you must integrate over all values of $y$ from 0 to $2-x$ because those are the $y$'s for which $f(x,y)$ is positive.)

(e) For a given possible value of $Y$, what is the set of $x$ for which the joint density is positive?

(f) Find a density for $Y$ by integrating $f(x,y)$ over the values of $x$ found in (e).

[Ans: (a) The triangle with vertices at $(0,0)$, $(0,2)$, and $(2,0)$ (c) No (d) $\frac{15}{16}x^2(2-x)^2$ for $0 < x < 2$ (e) the interval $(0, 2-y)$]

8. Let $X$ and $Y$ be independent random variables with the same distribution; that distribution is not assumed to be continuous or discrete. Show that $P(X < Y) \leq \frac{1}{2} \leq P(X \leq Y)$. [*Hint*: Consider the three quantities $P(X < Y)$, $P(X = Y)$, and $P(X > Y)$.]

9. Let $X$ and $Y$ have the joint density $f(x,y) = \lambda^3 x e^{-\lambda(x+y)}$ for $x > 0$ and $y > 0$.

(a) Find the marginal densities and show that $X$ and $Y$ are independent.

(b) For given positive numbers $a$ and $b$ find $P(X \leq a \text{ and } Y \leq b)$.

(c) Find $P(X \leq a)$.

(d) Find the probability that $X + Y \leq a$ for a given positive number $a$.

(e) Find a density for $Z = X + Y$.

[Ans: (b) $(1-(1+\lambda a)e^{-\lambda a})(1-e^{-\lambda b})$ (c) $1-(1+\lambda a)e^{-\lambda a}$ (d) $1-\left(1+\lambda a+\frac{\lambda^2 a^2}{2}\right)e^{-\lambda a}$ (e) $\frac{\lambda^2 z^2}{2}e^{-\lambda z}$ $(z > 0)$ ]

Relevant sections in Schaeffer for lectures during the week of October 21: 5.4, 5.7, then starting chapter 6