PRACTICE PROBLEMS 4 – Solutions to Additional Questions

#5 $P(Y > a+b \mid Y > a)$

$$= \frac{P(Y > a+b)}{P(Y > a)}$$

$$= \int_{a+b}^{\infty} \frac{1}{\theta} e^{-x/\theta}\, dx \Big/ \int_{a}^{\infty} \frac{1}{\theta} e^{-x/\theta}\, dx$$

$$= e^{-(a+b)/\theta} \Big/ e^{-a/\theta}$$

$$= e^{-b/\theta}$$

$$= P(Y > b)$$

#6 $EZ = E\left(\frac{X-\mu}{\sigma}\right)$

$$= \frac{1}{\sigma}\left[EX - \mu\right]$$

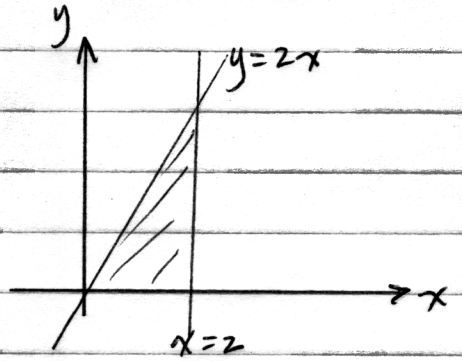
$$= 0$$

$VZ = V\left(\frac{X-\mu}{\sigma}\right)$

$$= \frac{1}{\sigma^2} VX$$

$$= 1$$

#7



D: $0 \le y \le 2x$
$0 \le x \le 2$

$$\iint_D xy\, dA = \int_0^2 \int_0^{2x} xy\, dy\, dx$$

$$= \int_0^2 x \left(\frac{2x}{2}\right)^2 dx$$

$$= 8$$

#8/ $P(A \cup B) = 1 - P(\overline{A \cup B}) = .8$

$P(B) = P(A \cup B) - P(A) + P(AB) = .5$

#9/ X = number of 5's in 10 rolls of fair die.

$X \sim$ Binomial $(10, \frac{1}{6})$

(a) $P(X \geq 2 \mid X \geq 1) = \frac{P(X \geq 2)}{P(X \geq 1)} = \frac{1 - (\frac{5}{6})^{10} - 10(\frac{5}{6})^{9}(\frac{1}{6})}{1 - (5/6)^{10}}$

$= .6148$

$P(X \geq 2) = 1 - (5/6)^{10} - 10(5/6)^{9}(\frac{1}{6}) = .5155$

(b) $P(X = 0 \mid X \leq 2) = \frac{P(X=0)}{P(X \leq 2)} = \frac{(5/6)^{10}}{(5/6)^{10} + 10(5/6)^{9}(\frac{1}{6}) + \binom{10}{2}(5/6)^{8}(\frac{1}{6})^{2}}$

$= .2083$

$Y \sim$ Binomial $(2, \frac{1}{6})$

$P(Y=0) = (5/6)^2 = .6944$

etc.

#10, 11, 12 Will appear on Practice Problems 5