STA 257 – Fall 2002

Practice Problems 4

Recommended preparation for quiz to be held in tutorial on Wednesday, October 9

Sections from Schaeffer covered in the week of September 30: more on variance from 4.2, 4.6, 2.3 (review), 2.5, 2.6

Questions from the textbook (Schaeffer):

1. From Section 4.4: 4.35
2. From Section 4.6: 4.55
3. From Section 2.6: 2.27, 2.34, 2.39, 2.45
4. From Section 4.1: 4.7(b)(c)
5. From Section 4.4: 4.30

Additional questions:

6. The *standard deviation* of a random variable is the square root of its variance. Suppose $EX = \mu$ and the standard deviation of X is σ. Let $Z = (X - \mu)/\sigma$. Find EZ and VZ. (Subtracting the expected value and dividing by the standard deviation is called *standardizing* the random variable X.) [Ans: $EZ = 0$, $VZ = 1$]

7. Find
$$\int\int_D xy \, dA$$
where D is the region bounded by the x-axis and the lines $x = 2$ and $y - 2x = 0$. [Ans: 8]

8. Given that $P(A) = .4$, $P(AB) = .1$, and $P(\overline{A \cup B}) = .2$, find $P(B)$. [Ans: .5]

9. A fair die is rolled ten times, and the number of 5's is counted.

   (a) Find the conditional probability that there are at least two 5's, given that there is at least one 5. Compare with the unconditional probability that there are at least two 5's.

   (b) Find the condtional probabilities of zero, one, and two 5's, given that there were two or fewer 5's. Compare with the unconditional probabilities of zero, one, and two 5's, if only two rolls are made. (Guess in advance whether they will be the same.)

   [Ans: (a) .6148 (vs. .5155), (b) .2083, .4167, .3750 (vs. .6944, .2778, .0278)]

10. Suppose that the number of children in a randomly chosen family is modeled by the Poisson distribution with $\lambda = 1.8$. (That is, the probability that there are k children is $\lambda^k e^{-\lambda}/k!$.) Suppose that within a family, the genders of the children are determined according to a Bernoulli process with $p = 1/2$. Find the probability that a randomly chosen family has two female children. (*Hint*: Condition on the number of children in the family.) [Ans: $(.9)^2 e^{-.9}/2!$]

11. Let X and Y be independent random variables. Each has possible values 0, 1, and 2, with probabilities .2, .3, and .5, respectively. Find the probabilities of the following events:

   (a) $X = 2$ and $Y = 2$

   (b) $X = Y$

   (c) $XY = 0$

   [Ans: (a) .25, (b) .38, (c) .36]

12. If events A and B are not independent, then either $P(B|A) > P(B)$, in which case we say that A is *favourable for* B, or else $P(B|A) < P(B)$, and we say that A is *unfavourable for* B.

   (a) Show that A is favourable for B if and only if $P(AB) > P(A)P(B)$.

   (b) Show that if A is favourable for B, then B is favourable for A.

   (c) Show that if A is unfavourable for B, then A is favourable for $\overline{B}$.

Relevant sections in Schaeffer for lectures during the week of October 7: 5.1, 5.2, 5.3