

PRACTICE PROBLEMS 3 - Solutions to Additional Questions

$$\begin{aligned}\#9/ E(X-a)^2 &= E(X-\mu + \mu - a)^2 = E(X-\mu)^2 + 2E(X-\mu)E(\mu-a) + E(\mu-a)^2 \\ &= E(X-\mu)^2 + 2 \underbrace{E(X-\mu)}_0 (\mu-a) + (\mu-a)^2\end{aligned}$$

Since both terms that are left are ≥ 0 , this expression is minimized when $a = \mu$

$$\#10/ E\left(\frac{1}{X}\right) = \sum_{k=1}^{\infty} \frac{k}{k} p^2 q^{k-1} = \frac{p^2}{1-q} = p$$

$$\#11/(a) EX = 2\left(\frac{1}{36}\right) + 3\left(\frac{2}{36}\right) + 4\left(\frac{3}{36}\right) + \dots + 11\left(\frac{2}{36}\right) + 12\left(\frac{1}{36}\right) = 7$$

$$(b) E(X_1 + X_2) = EX_1 + EX_2 = \frac{1}{6}(1+2+3+4+5+6) + \frac{1}{6}(1+2+3+4+5+6) = 7$$

$$\#12/ EZ = E\left(\frac{X-\mu}{6}\right) = \frac{1}{6}(EX - \mu) = 0$$

$$VZ = V\left(\frac{X-\mu}{6}\right) = \frac{1}{6^2} VX = 1$$