STA 257 – Fall 2002

Practice Problems 3

Recommended preparation for quiz to be held in tutorial on Wednesday, October 2

Sections from Schaeffer covered in the week of September 23: 6.2 (one-dimensional case only), more on expectation and variance from 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 4.2, 4.3, 4.4.

Questions from the textbook (Schaeffer):

1. From Section 3.2: 3.11
2. From Section 3.4: 3.37, 3.39
3. From Section 3.7: 3.67
4. From Section 4.2: 4.13(a)
5. From Section 4.3: 4.29
6. From Section 4.4: 4.35
7. From Section 6.2: 6.1

Additional questions:

8. Problems 10, 13, and 14 from Practice Problems 2.
9. Suppose that $EX = \mu$. Show that for any number a, $E(X-a)^2 = E(X-\mu)^2 + (a-\mu)^2$. (*Hint*: Write $(X-a)^2$ as $(X-\mu+\mu-a)^2$.) Conclude that the value of a for which $E(X-a)^2$ is minimized is $a = \mu$.
10. Let the probability mass function of X be $p(k) = kp^2q^{k-1}$ for $k = 1, 2, 3, \ldots$. Find the expected value of $Y = 1/X$. [Ans: p]
11. Two fair dice are rolled and X is the sum of the two numbers that appear. The possible values of X are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, with corresponding probabilities 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, all divided by 36 (why?). Find EX in two different ways:

    (a) Directly from the definition, using the probability function.

    (b) Using the fact that $X = X_1 + X_2$, where X_1 is the score on the first die and X_2 the score on the second. X_1 has possible values 1, 2, 3, 4, 5, 6, with probabilities $\frac{1}{6}$ each, as does X_2, so their expected values are easy to find.

    [Ans: 7]
12. The *standard deviation* of a random variable is the square root of its variance. Suppose $EX = \mu$ and the standard deviation of X is σ. Let $Z = (X-\mu)/\sigma$. Find EZ and VZ. (Subtracting the expected value and dividing by the standard deviation is called *standardizing* the random variable X.) [Ans: $EZ = 0$, $VZ = 1$]

Relevant sections in Schaeffer for lectures during the week of September 30: 4.6, 2.3 (review), 2.5, 2.6