PRACTICE PROBLEMS 2 - Solutions to Additional Questions

#8/ (a) $f(x) \geq 0$, $\int_0^1 x^4\,dx = \frac{1}{5}$ so $5f(x)$ is a density function.

(b) $f(x) \geq 0$, $\int_0^\pi \sin x\,dx = 2$ so $\frac{1}{2}f(x)$ is a density function

(c) $f(x) < 0$ for $x \in (\frac{\pi}{2}, \pi)$ so $f(x)$ is not a density function and there is no a

(d) $f(x) \geq 0$, $\int_0^\infty x e^{-x^2/2}\,dx = 1$ so $f(x)$ is a density function

#9/ Y = number of rolls until 1st odd number comes up.
Y has a Geometric distribution with parameter $\frac{1}{2}$ so $EY = 2$
$X = Y - 1$ so $EX = 1$

#10/ (a) $EX = \int_0^1 x 6x(1-x)\,dx = \frac{1}{2}$ (b) $EX = \int_1^\infty x \cdot \frac{3}{x^4}\,dx = \frac{3}{2}$

#11/ $EX = \sum_{k=2}^{\infty} k(k-1)\left(\frac{1}{2}\right)^k = \left(\frac{1}{2}\right)^2 \sum_{k=2}^{\infty} k(k-1)\left(\frac{1}{2}\right)^{k-2} = \left(\frac{1}{2}\right)^2 \frac{d^2}{dx^2}\left(\frac{1}{1-x}\right)\Big|_{x=\frac{1}{2}} = 4$

#12/ $EX = 1\left(\frac{1}{4}\right) + 2\left(\frac{1}{2}\right) + 1000\left(\frac{1}{4}\right) = 251.25$

#13/ (a) $EX = \frac{1}{\lambda}$, $P(X > EX) = \int_{1/\lambda}^\infty \lambda e^{-\lambda x}\,dx = \frac{1}{e}$

(b) $P(X > m) = \int_m^\infty \lambda e^{-\lambda x}\,dx = e^{-\lambda m} = \frac{1}{2}$ for $m = \frac{\ln 2}{\lambda} < \frac{1}{\lambda} = EX$

#14/ (a) $\sum_{n=0}^{\infty} P(X > n) = P(X>0) + P(X>1) + P(X>2) + P(X>3) + \cdots$

$= \left(P(X=1) + P(X=2) + P(X=3) + \cdots\right) + \left(P(X=2) + P(X=3) + P(X=4) + \cdots\right)$
$+ \left(P(X=3) + P(X=4) + \cdots\right) + \cdots$
$= 0 \cdot P(X=0) + 1 \cdot P(X=1) + 2 \cdot P(X=2) + \cdots = EX$

(b) done in lecture.