STA 257 – Fall 2002

Practice Problems 2

Recommended preparation for quiz to be held in tutorial on Wednesday, September 25

Sections from Schaeffer covered in the week of September 16: 3.2, 3.3, 3.4, 3.5, 3.6, 3.7 4.1, 4.2, 4.3, 4.4 (excluding Variance, $E[g(X)]$)

Questions from the textbook (Schaeffer):

1. From Section 3.6: 3.43
2. From Section 3.7: 3.55
3. From Section 4.1: 4.3, 4.7 (a)(d)
4. From Section 4.3: 4.15 (a)(b), 4.19, 4.27
5. From Section 4.4: 4.33, 4.39
6. From Supplementary Exercises: 4.103 (a)-(e)

Additional questions:

7. Describe in some other way the distribution whose distribution function is

$$F(x) = \begin{cases} 0 & \text{if } x < 1, \\ .2 & \text{if } 1 \leq x < 3, \\ .35 & \text{if } 3 \leq x < 4, \\ .6 & \text{if } 4 \leq x < 4.5, \\ 1 & \text{if } x \geq 4.5. \end{cases}$$

   [Ans: Discrete distribution with possible values 1, 3, 4, 4.5 having probabilities .2, .15, .25, .4, respectively]

8. Which of the following are probability density functions? For those that are not, find a number a (if possible) so that $af(x)$ is a density.

   (a) $f(x) = x^4$ for $0 < x < 1$
   (b) $f(x) = \sin x$ for $0 < x < \pi$
   (c) $f(x) = \cos x$ for $0 < x < \pi$
   (d) $f(x) = xe^{-x^2/2}$ for $x > 0$

   [Ans: (a) $a = 5$, (b) $a = 1/2$, (c) Impossible, (d) Yes]

9. We roll a fair die until an odd number comes up; let X be the number of even numbers we get before the first odd number. Find EX. [Ans: 1]

10. Find EX if X has the given density.

   (a) $f(x) = 6x(1 - x)$ for $0 < x < 1$ (and 0 otherwise)
   (b) $f(x) = 3/x^4$ for $x > 1$ (and 0 otherwise)

   [Ans: (a) 1/2, (b) 3/2]

11. The probability function of X is given by $p(k) = (k-1)\left(\frac{1}{2}\right)^k$ for $k = 2, 3, 4, \ldots$ Find EX. [Ans: 4]

12. Suppose the possible values of X are 1, 2, and 1000, with probabilities $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$, respectively.. Find EX. (Notice that EX is not the most probable value of X; it is not even close to any of the possible values of X. All EX has to recommend it is that if we averaged many observations of X, the average would likely be close to EX.) [Ans: 251.25]

13. Let X have the exponential distribution with parameter λ (parametrized as in lecture, i.e. $\lambda = 1/\theta$ in the textbook's parametrization).

    (a) What is the probability that X is greater than EX? (It is not 1/2.)

    (b) Find the number m for which $P(X > m) = \frac{1}{2}$ (this is the *median* of X) and show that regardless of λ, it is less than EX.

    [Ans: (a) $1/e$, (b) $(\ln 2)/\lambda$ which is less than $1/\lambda$]

14. Let X be a nonnegative random variable.

    (a) Suppose the possible values of X are integers. Show that $\sum_{n=0}^{\infty} P(X > n) = EX$.

    (b) Suppose X is continuous. Show that $\int_0^{\infty} P(X > x)dx = EX$. (*Hint*: Write $P(X > x)$ as an integral from x to ∞, then reverse the order of integration.)

Relevant sections in Schaeffer for lectures during the week of September 23: 6.2 (one-dimensional case only), more on expectation and variance from 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 4.2, 4.3, 4.4; and 4.6. If time: more material from Chapter 2.